Discussion on event horizon and quantum ergosphere of dynamic rotating black holes in a tunneling framework*

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According to the Parikh–Wilczek tunneling framework, the locations of the local horizons of dynamic rotating black holes can be worked out. The calculations show that the quantum ergosphere of the black hole is identical with the tunneling potential barrier set by particle's tunneling across the relevant horizon. Then, some discussions on the origin of the Hawking radiation will be shown.

Keywords: Hawking radiation, dynamic rotating black holes, tunneling framework, quantum ergosphere

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1. Introduction

Since Hawking discovered black holes' evaporation, the Hawking radiation of different kinds of black holes has been studied.[1−5] In Hawking's paper, the vacuum fluctuation near the horizon is used to describe the Hawking radiation. For containing no less than one horizon in a general black hole, it is necessary for us to ascertain which one is the true origin of the Hawking radiation. Different methods have reached two kinds of conclusions to the problem, both of which are applicable to static black holes, but will be controversial when we study the dynamic ones due to their two horizons' deviating from each other and forming quantum ergosphere.[6]

Given that a dynamic black hole can be described by its stationary correspondence in the tunneling process of particles, Parikh–Wilczek tunneling framework has been applied to research such dynamic spherically symmetric black holes as Vaidya black holes, Vaidya–Bonner black holes.[6−9] The extended application of the tunneling framework displays that the quantum ergosphere is none other than the tunneling potential barrier and the Hawking radiation originates from the vacuum fluctuation near the apparent event horizon.[6] Since the background space time of dynamic rotating black holes is distinct from that of the spherically symmetric ones, their Hawking radiation is worthy of our further studies.

Obtained from the null-hypersurface equation, the local horizons of dynamic rotating black holes will be very complex and their exact locations can hardly be solved using the tunneling framework. Therefore, another scheme is applied in the paper. It is the radial outgoing lightlike geodesic that is adopted to work out the local horizons, which is merely a function of the horizon's contraction (expanding) speed. Then, in terms of the Parikh–Wilczek tunneling framework and General Relativity's correspondence, the contraction (expanding) speed of the event (cosmological) horizon of the nonstationary black holes can be calculated by studying particles' tunneling process in the dragging frame of the stationary space time, which should be verified with York's definition on the horizon.[6,10] According to York's definition, the district should be lightlike, from which photons can hardly flee. In other words, when the Hawking radiation is very trivial, the horizon's contraction speed and its acceleration will, separately, satisfy: $\dot{r}_{\rm H} \approx 0$ and $\ddot{r}_{\rm H} \approx 0$. In this way, we can get the local horizons of dynamic black holes and study the ergosphere and Hawking radiation. The studies on Kerr–Newman, Kerr–Newman–

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de Sitter black holes will be shown below.

2. Kerr–Newman black holes

The line element of a nonstationary Kerr–Newman black hole in the advanced time v is

$$\begin{aligned} \mathrm{d}s^2 &= \left[1-\frac{2M(v)r-Q^2(v)}{\rho^2}\right]\mathrm{d}v^2-2\mathrm{d}v\mathrm{d}r \\ &+\frac{2a[2M(v)r-Q^2(v)]\sin^2\theta}{\rho^2}\mathrm{d}v\mathrm{d}\phi \\ &+2a\sin^2\theta\mathrm{d}r\mathrm{d}\phi-\rho^2\mathrm{d}\theta^2 \\ &-\frac{[(a^2+r^2)^2-a^2\Delta(v)\sin^2\theta]\sin^2\theta}{\rho^2}\mathrm{d}\phi^2, \quad (1) \end{aligned}$$

where a is a constant, $M(v)$ is the mass of the black hole, $Q(v)$ is the charge of the black hole, and

$$\begin{aligned} &\rho^2=r^2+a^2\cos^2\theta, \\ &\Delta(v)=r^2+a^2+Q^2(v)-2M(v)r. \quad (2) \end{aligned}$$

If its null tetrad $Z_{m\nu}=(Z_{1\nu},\,Z_{2\nu},\,Z_{3\nu},\,Z_{4\nu})=(l_\nu,\,n_\nu,\,m_\nu,\,\overline{m}_\nu)$ is constructed as

$$\begin{aligned} l_\nu &= \frac{1}{r^2+a^2}\left\{\frac{\Delta(v)}{2},\,-\rho^2,\,0,\,\frac{-\Delta(v)a\sin^2\theta}{2}\right\}, \\ n_\nu &= \frac{r^2+a^2}{\rho^2}\{1,\,0,\,0,\,-a\sin^2\theta\}, \\ m_\nu &= \{\mathrm{i}a\sin\theta,\,0,\,-\rho^2,\,-\mathrm{i}(r^2+a^2)\sin\theta\}, \\ \bar{m}_\nu &= \frac{1}{2\rho^2}\{-\mathrm{i}a\sin\theta,\,0,\,-\rho^2,\,\mathrm{i}(r^2+a^2)\sin\theta\}, \\ l^\nu &= \frac{1}{r^2+a^2}\left\{r^2+a^2,\,\frac{\Delta(v)}{2},\,0,\,a\right\}, \\ n^\nu &= \frac{r^2+a^2}{\rho^2}\{0,\,-1,\,0,\,0\}, \\ m^\nu &= \left\{\mathrm{i}a\sin\theta,\,0,\,1,\,\frac{\mathrm{i}}{\sin\theta}\right\}, \\ \bar{m}^\nu &= \frac{1}{2\rho^2}\left\{-\mathrm{i}a\sin\theta,\,0,\,1,\,\frac{-\mathrm{i}}{\sin\theta}\right\}, \quad (3) \end{aligned}$$

its radial outgoing lightlike geodesic will read[10]

$$\dot{r}=\frac{\mathrm{d}r}{\mathrm{d}v}=l^\nu\nabla_\nu r=\frac{\Delta(v)}{2(r^2+a^2)}, \quad (4)$$

and the expansion of the null rays Θ will be

$$\begin{aligned} \Theta &= l^\mu_{;\mu}-n^\mu l^\nu l_{\mu;\nu} \\ &= l^\mu_{;\mu}+\frac{r[Q^2(v)-2M(v)r]+M(v)(r^2+a^2)}{(r^2+a^2)^2} \\ &= \frac{r\Delta(v)}{\rho^2(r^2+a^2)}. \quad (5) \end{aligned}$$

Thus, the apparent event horizon R_{AE} is

$$R_{\mathrm{AE}}=M(v)+\sqrt{M^2(v)-a^2-Q^2(v)}, \quad (6)$$

while its local event horizon, obtained from the radial outgoing lightlike geodesic and depending on $\dot{R}_E$, locates at

$$R_{\mathrm{E}}=[M(v)+\sqrt{M^2(v)-a^2y^2-Q^2(v)y}]/y, \quad (7)$$

where $y=1-2\dot{R}_{\mathrm{E}}$ and $\dot{R}_{\mathrm{E}}$ is the contraction speed of the horizon of the dynamic black hole.

In the following, the local event horizon can be obtained from the Parikh–Wilczek tunneling framework and will be verified with York's definition.[6–10]

In the Parikh–Wilczek tunneling framework, the Hawking radiation is regarded as the vacuum oscillation in the inner vicinity of the horizon. In other words, when a couple of virtual particles just generate close to the inner surface of the horizon, the one with the negative energy will drop in and integrate with the black hole for classically prohibited outside, but the other with the positive energy will tunnel across the horizon to infinity and form a real particle, which is none other than the Hawking radiation. While, according to Hawking's initial description, the Hawking radiation happens in the vicinity of the horizon's outer surface. For ignoring the tunneling particles' energy conservation, Hawking's initial theory caused the information loss, which has been solved by the Parikh–Wilczek tunneling framework. According to the framework, when the back reaction of tunneling particles on the geometry of the black hole is considered, the space time will be dynamic and be that of the black hole's dynamic counterpart. In the process, the tunneling potential barrier comes from the particles' self-gravitation or the shrinkage of the black hole. Then, the radiation spectrum will correspond with an underlying unitary theory.[7–35]

What Parikh and Wilczek researched was particles' tunneling process in the spherically symmetric space time, while one can similarly discuss the process in the space time of the rotating black holes. During uncharged massive particles' tunneling across the event horizon of the rotating black hole, if the particles are considered as an ellipsoid shell, the space time will reduce from $r_{\mathrm{E}}(M)$ to $r_{\mathrm{E}}(M-\omega)$ and can be regarded as that of the dynamic black hole, where ω is particles' energy and M is the mass of the space time.[36] Likewise, the tunneling speed is equal to that of the contraction.[6] Thus, the contraction speed and the local event horizon of the dynamic black hole can be worked out. Furthermore, one can examine whether the region defined by the two tunneling points $r_{\mathrm{E}}(M)$

and $r_{\rm E}(M-\omega)$ corresponds to the quantum ergosphere.

Therefore, in order to calculate the dynamic Kerr–Newman black hole's local horizon, we should study the tunneling process in the stationary Kerr–Newman space time. The line element for the space time in the advanced Eddington–Finkelstein coordinates is

$$\begin{aligned} {\rm d}s^2 =& \left[1-\frac{2Mr-Q^2}{\rho^2}\right]{\rm d}v^2-2{\rm d}v{\rm d}r \\ &+\frac{2a[2Mr-Q^2]\sin^2\theta}{\rho^2}{\rm d}v{\rm d}\phi \\ &+2a\sin^2\theta{\rm d}r{\rm d}\phi-\rho^2{\rm d}\theta^2 \\ &-\frac{[(a^2+r^2)^2-a^2\Delta\sin^2\theta]\sin^2\theta}{\rho^2}{\rm d}\phi^2, \quad (8) \end{aligned}$$

where

$$\Delta=r^2+Q^2+a^2-2Mr. \quad (9)$$

Obviously, if $M=M(v)$ and $Q=Q(v)$ in Eq. (8), particles' tunneling process in the space time can demonstrate the evolution of its dynamic counterpart.

It is very convenient for us to choose the line element in the dragging frame. In the frame, the angular velocity satisfies

$$\Omega=\frac{{\rm d}\phi}{{\rm d}v}=-\frac{g_{03}}{g_{33}}=\frac{a(2Mr-Q^2)}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}, \quad (10)$$

and the line element becomes

$$\begin{aligned} {\rm d}s^2 =& \frac{\rho^2(r^2+a^2)}{(r^2+a^2)^2-\Delta a^2\sin^2\theta} \\ &\times\left[\frac{\Delta}{r^2+a^2}{\rm d}v^2-2{\rm d}v{\rm d}r\right]-\rho^2{\rm d}\theta^2 \\ =& \hat{g}_{00}{\rm d}v^2+2\hat{g}_{01}{\rm d}v{\rm d}r+\hat{g}_{22}{\rm d}\theta^2. \quad (11) \end{aligned}$$

Its event horizon, satisfying the null–hypersurface equation:

$$g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu}=0, \quad (12)$$

locates at

$$r_{\rm EH}=M+\sqrt{M^2-a^2-Q^2}. \quad (13)$$

If the charged particles are considered as an ellipsoid shell, the tunneling speed will be[11,12]

$$\dot{r}(M,\,Q,\,r)=-\frac{\hat{g}_{00}}{2\hat{g}_{01}}=\frac{\Delta}{2(r^2+a^2)}. \quad (14)$$

Before the tunneling, the space time will be that of the stationary Kerr–Newman black hole, and the ingoing tunneling point of the tunneling potential will locate at

$$r_{\rm EH}=M+\sqrt{M^2-a^2-Q^2}=R_{\rm E}=R_{\rm AE}. \quad (15)$$

During the tunneling, if particles' back reaction on the black hole geometry is considered, their tunneling speed will be $\dot{r}(M-\omega,\,Q-q,\,r)$, where ω is particles' energy, and q is particles' charge. Thus, the horizon's contraction speed at $r=r_{\rm EH}$ and its acceleration $\ddot{R}_{\rm E}$ will, respectively, read[6,10]

$$\begin{aligned} \dot{R}_{\rm E} =& -\dot{r}(M-\omega,\,Q-q,\,r_{\rm EH}) \\ =& -\frac{(Q-q)^2-Q^2+2\omega r_{\rm EH}}{2(r_{\rm EH}^2+a^2)}, \quad (16) \\ \ddot{R}_{\rm E} =& l^\nu\nabla_\nu\dot{r}|_{r=R_{\rm E},\,\dot{r}=\dot{R}_{\rm E}} \\ =& \frac{(R_{\rm E}-M-2R_{\rm E}\dot{R}_{\rm E})\dot{R}_{\rm E}+Q\dot{Q}-R_{\rm E}\dot{M}}{R_{\rm E}^2+a^2} \\ =& \frac{(R_{\rm E}-M-2R_{\rm E}\dot{R}_{\rm E})\dot{R}_{\rm E}-Qq+R_{\rm E}\omega}{R_{\rm E}^2+a^2}. \quad (17) \end{aligned}$$

When $\omega\to0$ and $q\to0$, Eqs. (16) and (17) will, respectively, satisfy $\dot{R}_{_E}\approx0$ and $\ddot{R}_{\rm E}\approx0$, which accords with York's definition on the horizon.[10] Therefore, substituting Eq. (16) into Eq. (7) will yield the dynamic Kerr–Newman black hole's local event horizon

$$\begin{aligned} R_{\rm E} =& (M+\sqrt{M^2(v)-a^2y^2-Q^2(v)y})/y \\ =& (M+\sqrt{M^2-a^2y^2-Q^2y})/y \\ =& r_{\rm EH}+\frac{(Q^2q/Q-Mr_{\rm EH}\omega/M)}{\sqrt{M^2-a^2-Q^2}} \\ &+O\left(\left(\frac{\omega}{M}\right)^2\right)+O\left(\left(\frac{q}{Q}\right)^2\right). \quad (18) \end{aligned}$$

Because the outgoing tunneling point is

$$\begin{aligned} r_{\rm EHf} =& (M-\omega)+\sqrt{(M-\omega)^2-a^2-(Q-q)^2} \\ =& r_{\rm EH}+\frac{(Q^2q/Q-Mr_{\rm EH}\omega/M)}{\sqrt{M^2-a^2-Q^2}} \\ &+O\left(\left(\frac{\omega}{M}\right)^2\right)+O\left(\left(\frac{q}{Q}\right)^2\right), \quad (19) \end{aligned}$$

we have

$$R_{\rm E}=r_{\rm EHf}. \quad (20)$$

The above calculations show that the apparent horizon and the local horizon can be treated as the ingoing and outgoing tunneling point, respectively. In other words, the quantum ergosphere corresponds to the classically inhibited area, which is also applicable to the massless particle's tunneling.

3. Kerr–Newman–de Sitter black holes

The metric for a dynamic Kerr–Newman–de Sitter black hole in the advanced time v is[37]

$$\begin{aligned}\mathrm{d}s^2 =& \frac{\Delta_1(v)-\Delta_2 a^2\sin^2\theta}{\rho^2\Delta_3^2}\mathrm{d}v^2\\ &-\frac{2(\mathrm{d}v-a\sin^2\theta\mathrm{d}\varphi)\mathrm{d}r}{\Delta_3}-\frac{\rho^2}{\Delta_2}\mathrm{d}\theta^2\\ &+\frac{2a[\Delta_2(r^2+a^2)-\Delta_1(v)]\sin^2\theta\mathrm{d}v\mathrm{d}\varphi}{\rho^2\Delta_3^2}\\ &-\frac{[\Delta_2(r^2+a^2)^2-\Delta_1(v)a^2\sin^2\theta]\sin^2\theta\mathrm{d}\varphi^2}{\rho^2\Delta_3^2},\end{aligned} \tag{21}$$

where a is a constant, $M(v)$ is the mass of the black hole, $Q(v)$ is the charge of the black hole, λ is the cosmological constant, and

$$\begin{aligned}&\Delta_1(v)=\left(1-\frac{1}{3}\lambda r^2\right)(r^2+a^2)-2M(v)r+Q^2(v),\\ &\Delta_2=1+\frac{1}{3}\lambda a^2\cos^2\theta,\quad \Delta_3=1+\frac{1}{3}\lambda a^2,\\ &\rho^2=r^2+a^2\cos^2\theta.\end{aligned} \tag{22}$$

If the null tetrads are

$$\begin{aligned}l_\nu =& \frac{1}{(r^2+a^2)\Delta_3^2}\left\{\frac{\Delta_1(v)}{2},\ -\rho^2\Delta_3,\ 0,\right.\\ &\left.\frac{-\Delta_1(v)a\sin^2\theta}{2}\right\},\\ n_\nu =& \frac{r^2+a^2}{\rho^2}\{1,\ 0,\ 0,\ -a\sin^2\theta\},\\ m_\nu =& \Delta_2\left\{\frac{\mathrm{i}a\sin\theta}{\Delta_3},\ 0,\ -\frac{\rho^2}{\Delta_2},\ -\frac{\mathrm{i}(r^2+a^2)\sin\theta}{\Delta_3}\right\},\\ \bar{m}_\nu =& \frac{1}{2\rho^2}\left\{\frac{-\mathrm{i}a\sin\theta}{\Delta_3},\ 0,\ -\frac{\rho^2}{\Delta_2},\ \frac{\mathrm{i}(r^2+a^2)\sin\theta}{\Delta_3}\right\},\\ l^\nu =& \frac{1}{r^2+a^2}\left\{r^2+a^2,\ \frac{\Delta_1(v)}{2\Delta_3},\ 0,\ a\right\},\\ n^\nu =& \frac{(r^2+a^2)\Delta_3}{\rho^2}\{0,\ -1,\ 0,\ 0\},\\ m^\nu =& \left\{\mathrm{i}a\sin\theta\Delta_3,\ 0,\ \Delta_2,\ \frac{\mathrm{i}\Delta_3}{\sin\theta}\right\},\\ \bar{m}^\nu =& \frac{1}{2\rho^2\Delta_2}\left\{-\mathrm{i}a\sin\theta\Delta_3,\ 0,\ \Delta_2,\ \frac{-\mathrm{i}\Delta_3}{\sin\theta}\right\},\end{aligned} \tag{23}$$

the radial outgoing lightlike geodesic and the expansion of the null rays Θ will read

$$\begin{aligned}\dot{r} =& \frac{\Delta_1(v)}{2\Delta_3(r^2+a^2)},\\ \Theta =& l^\mu_{;\mu}-\frac{\mathrm{d}}{\mathrm{d}r}\frac{\Delta_1(v)}{2\Delta_3(r^2+a^2)}=\frac{r\Delta_1(v)}{\rho^2\Delta_3(r^2+a^2)}.\end{aligned} \tag{24}$$

Therefore, Eq. (24) can yield the local event horizon R_{E}, the local cosmological horizon R_{C}, the apparent event horizon R_{AE}, and the apparent cosmological horizon R_{AC}

$$\begin{aligned}R_{\mathrm{E}} =& \sqrt{Z_1(x,\,M(v),\,Q(v))}\\ &-\sqrt{Z_2(x,\,M(v),\,Q(v))}\\ &+\sqrt{Z_3(x,\,M(v),\,Q(v))},\\ R_{\mathrm{C}} =& \sqrt{Z_1(x',\,M(v),\,Q(v))}\\ &+\sqrt{Z_2(x',\,M(v),\,Q(v))}\\ &-\sqrt{Z_3(x',\,M(v),\,Q(v))},\\ R_{\mathrm{AE}} =& \sqrt{Z_1(1,\,M(v),\,Q(v))}\\ &-\sqrt{Z_2(1,\,M(v),\,Q(v))}\\ &+\sqrt{Z_3(1,\,M(v),\,Q(v))},\\ R_{\mathrm{AC}} =& \sqrt{Z_1(1,\,M(v),\,Q(v))}\\ &+\sqrt{Z_2(1,\,M(v),\,Q(v))}\\ &-\sqrt{Z_3(1,\,M(v),\,Q(v))},\end{aligned} \tag{25}$$

where $x=1-2\Delta_3\dot{R}_{\mathrm{E}}$, $x'=1-2\Delta_3\dot{R}_{C}$, $\dot{R}_{\mathrm{E}}$ ($\dot{R}_{\mathrm{C}}$) is the contraction (expanding) speed, and

$$\begin{aligned}Z_1(x,\,M(v),\,Q(v)) =& \frac{1}{2\lambda}\left[x-\frac{\lambda a^2}{3}+t(x,\,Q(v))\cos\frac{\alpha(x,\,M(v),\,Q(v))}{3}\right],\\ Z_2(x,\,M(v),\,Q(v)) =& \frac{1}{2\lambda}\left[x-\frac{\lambda a^2}{3}-t(x,\,Q(v))\cos\frac{\alpha(x,\,M(v),\,Q(v))+\pi}{3}\right],\\ Z_3(x,\,M(v),\,Q(v)) =& \frac{1}{2\lambda}\left[x-\frac{\lambda a^2}{3}-t(x,\,Q(v))\cos\frac{\alpha(x,\,M(v),\,Q(v))-\pi}{3}\right],\\ \alpha(x,\,M(v),\,Q(v)) =& \arccos\left\{\left[18\lambda M^2(v)-12\lambda\left(x-\frac{\lambda a^2}{3}\right)(Q^2(v)+a^2x)-\left(x-\frac{\lambda a^2}{3}\right)^3\right]t^{-3}(x,\,Q(v))\right\},\\ t^2(x,\,Q(v)) =& \left(x-\frac{\lambda a^2}{3}\right)^2-4\lambda[Q^2(v)+a^2x],\\ \alpha(x,\,M(v),\,Q(v)) \in& \left(\frac{\pi}{2},\quad \pi\right).\end{aligned} \tag{26}$$

Likewise, the tunneling process in the stationary Kerr-Newman-de Sitter space time is applied to proceed with the further calculation and the line element in the dragging frame as follows is preferred

$$\mathrm{d}s^2 = -\frac{\rho^2}{\Delta_2}\mathrm{d}\theta^2 + \frac{\rho^2\Delta_2[\Delta_1\mathrm{d}v^2 - 2\Delta_3(r^2+a^2)\mathrm{d}v\mathrm{d}r]}{[\Delta_2(r^2+a^2)^2 - \Delta_1 a^2\sin^2\theta]\Delta_3^2} = \hat{g}_{00}\mathrm{d}v^2 + 2\hat{g}_{01}\mathrm{d}v\mathrm{d}r + \hat{g}_{22}\mathrm{d}\theta^2, \tag{27}$$

where

$$\Delta_1 = \left(1 - \frac{1}{3}\lambda r^2\right)(r^2+a^2) - 2Mr + Q^2. \tag{28}$$

Based on the null–hypersurface equation, the event (cosmological) horizon r_{EH} (r_{CH}) of the stationary black hole can read

$$\begin{aligned} r_{\mathrm{EH}} &= \sqrt{Z_1} - \sqrt{Z_2} + \sqrt{Z_3}, \\ r_{\mathrm{CH}} &= \sqrt{Z_1} + \sqrt{Z_2} - \sqrt{Z_3}, \end{aligned} \tag{29}$$

where $Z_1 = Z_1(1, M, Q)$, $Z_2 = Z_2(1, M, Q)$, and $Z_3 = Z_3(1, M, Q)$ are the three solutions of the following equation

$$\begin{aligned} &Z^3 + \frac{9(1-\lambda a^2/3)^2 + 12\lambda(a^2+Q^2)}{16\lambda^2}Z \\ &- \frac{3}{2\lambda}\left(1 - \frac{\lambda a^2}{3}\right)Z^2 - \frac{9M^2}{16\lambda^2} = 0. \end{aligned} \tag{30}$$

According to the tunneling framework, if the Hawking radiation comes from the tunneling process of particles with the charge of q and the mass of ω, treated as an ellipsoid shell, the horizon's contraction (expanding) speed will be

$$\begin{aligned} \dot{R}_{\mathrm{E}} &= -\frac{(Q-q)^2 - Q^2 + 2\omega r_{\mathrm{EH}}}{2\Delta_3(r_{\mathrm{EH}}^2 + a^2)}, \\ \dot{R}_{\mathrm{C}} &= -\frac{(Q+q)^2 - Q^2 - 2\omega r_{\mathrm{CH}}}{2\Delta_3(r_{\mathrm{CH}}^2 + a^2)}. \end{aligned} \tag{31}$$

When $\omega \to 0$ and $q \to 0$, further calculations show $\dot{R}_{\mathrm{E}} \approx 0$ and $\ddot{R}_{\mathrm{E}} \approx 0$ ($\dot{R}_{\mathrm{C}} \approx 0$ and $\ddot{R}_{\mathrm{C}} \approx 0$). So the associated R_{E} (R_{C}) can be defined as the local event (cosmological) horizon.

In terms of the following equations, concluded from Eq.(30), and the relation between the roots and coefficients of Eq.(30)

$$\begin{aligned} &\sqrt{Z_1}\sin\frac{\alpha}{3} + \sqrt{Z_2}\sin\frac{\alpha+\pi}{3} - \sqrt{Z_3}\sin\frac{\alpha-\pi}{3} \\ &= \frac{3M}{4r_{\mathrm{EH}}\lambda}\left[\frac{\sin\frac{\alpha}{3}}{\sqrt{Z_1}} + \frac{\sin\frac{\alpha+\pi}{3}}{\sqrt{Z_2}} - \frac{\sin\frac{\alpha-\pi}{3}}{\sqrt{Z_3}}\right], \end{aligned} \tag{32}$$

$$\begin{aligned} &\sqrt{Z_1}\sin\frac{\alpha}{3} - \sqrt{Z_2}\sin\frac{\alpha+\pi}{3} + \sqrt{Z_3}\sin\frac{\alpha-\pi}{3} \\ &= \frac{3M}{4r_{\mathrm{CH}}\lambda}\left[\frac{\sin\frac{\alpha}{3}}{\sqrt{Z_1}} - \frac{\sin\frac{\alpha+\pi}{3}}{\sqrt{Z_2}} + \frac{\sin\frac{\alpha-\pi}{3}}{\sqrt{Z_3}}\right] \\ &= \frac{(1-4\cos^2\frac{\alpha}{3})(\sqrt{Z_2}+\sqrt{Z_3})^2/4}{\sqrt{Z_1}\sin\frac{\alpha}{3} + \sqrt{Z_2}\sin\frac{\alpha+\pi}{3} - \sqrt{Z_3}\sin\frac{\alpha-\pi}{3}} < 0, \end{aligned} \tag{33}$$

the dynamic black hole's local horizons will read

$$\begin{aligned} R_{\mathrm{E}} &= \sqrt{Z_1(x,\,M,\,Q)} - \sqrt{Z_2(x,\,M,\,Q)} \\ &\quad + \sqrt{Z_3(x,\,M,\,Q)} \\ &= r_{\mathrm{EH}} + \frac{4\lambda}{t^2\sin\alpha}\left(Mr_{\mathrm{EH}}\frac{\omega}{M} - Q^2\frac{q}{Q}\right) \\ &\quad \times\left[-\sin\frac{\alpha}{3}\sqrt{Z_1} - \sin\frac{\alpha+\pi}{3}\sqrt{Z_2}\right. \\ &\quad \left. + \sin\frac{\alpha-\pi}{3}\sqrt{Z_3}\right] + O\left(\left(\frac{\omega}{M}\right)^2\right) + O\left(\left(\frac{q}{Q}\right)^2\right) \\ &= r_{\mathrm{EH}} - \frac{4\lambda Q^2}{t^2\sin\alpha}\frac{q}{Q}\left[-\sin\frac{\alpha}{3}\sqrt{Z_1}\right. \\ &\quad \left. - \sin\frac{\alpha+\pi}{3}\sqrt{Z_2} + \sin\frac{\alpha-\pi}{3}\sqrt{Z_3}\right] \\ &\quad + \frac{3M^2}{t^2\sin\alpha}\frac{\omega}{M}\left[-\frac{\sin\frac{\alpha}{3}}{\sqrt{Z_1}} - \frac{\sin\frac{\alpha+\pi}{3}}{\sqrt{Z_2}}\right. \\ &\quad \left. + \frac{\sin\frac{\alpha-\pi}{3}}{\sqrt{Z_3}}\right] + O\left(\left(\frac{\omega}{M}\right)^2\right) + O\left(\left(\frac{q}{Q}\right)^2\right), \end{aligned} \tag{34}$$

$$\begin{aligned} R_{\mathrm{C}} &= \sqrt{Z_1(x',\,M,\,Q)} + \sqrt{Z_2(x',\,M,\,Q)} \\ &\quad - \sqrt{Z_3(x',\,M,\,Q)} \\ &= r_{\mathrm{CH}} + \frac{4\lambda}{t^2\sin\alpha}\left(Mr_{\mathrm{CH}}\frac{\omega}{M} - Q^2\frac{q}{Q}\right) \\ &\quad \times\left[\sin\frac{\alpha}{3}\sqrt{Z_1} - \sin\frac{\alpha+\pi}{3}\sqrt{Z_2}\right. \\ &\quad \left. + \sin\frac{\alpha-\pi}{3}\sqrt{Z_3}\right] + O\left(\left(\frac{\omega}{M}\right)^2\right) + O\left(\left(\frac{q}{Q}\right)^2\right) \\ &= r_{\mathrm{CH}} - \frac{4\lambda Q^2}{t^2\sin\alpha}\frac{q}{Q}\left[\sin\frac{\alpha}{3}\sqrt{Z_1}\right. \\ &\quad \left. - \sin\frac{\alpha+\pi}{3}\sqrt{Z_2} + \sin\frac{\alpha-\pi}{3}\sqrt{Z_3}\right] \\ &\quad + \frac{3M^2}{t^2\sin\alpha}\frac{\omega}{M}\left[\frac{\sin\frac{\alpha}{3}}{\sqrt{Z_1}} - \frac{\sin\frac{\alpha+\pi}{3}}{\sqrt{Z_2}}\right. \\ &\quad \left. + \frac{\sin\frac{\alpha-\pi}{3}}{\sqrt{Z_3}}\right] + O\left(\left(\frac{\omega}{M}\right)^2\right) + O\left(\left(\frac{q}{Q}\right)^2\right), \end{aligned} \tag{35}$$

where $\alpha = \alpha(1, M, Q)$ and $t = t(1, Q)$.

After particles' tunneling across the event (cosmological) horizon, the outgoing tunneling point r_{EHf} (r_{CHf}) locates at

$$\begin{aligned} r_{\mathrm{EHf}} &= \sqrt{Z_1(1,\,M-\omega,\,Q-q)} \\ &\quad - \sqrt{Z_2(1,\,M-\omega,\,Q-q)} \\ &\quad + \sqrt{Z_3(1,\,M-\omega,\,Q-q)}, \end{aligned} \tag{36}$$

$$r_{\rm CHf} = \sqrt{Z_1(1,\, M+\omega,\, Q+q)} + \sqrt{Z_2(1,\, M+\omega,\, Q+q)} - \sqrt{Z_3(1,\, M+\omega,\, Q+q)}, \quad (37)$$

and the Taylor's expansions of the locations indicate that

$$r_{\rm EHf} = R_{\rm E}, \quad r_{\rm CHf} = R_{\rm C}. \quad (38)$$

If $M = M(v)$ and $Q = Q(v)$, the apparent event (cosmological) horizon $R_{\rm AE}$ ($R_{\rm AC}$) will be identical with the ingoing tunneling point $r_{\rm EH}$ ($r_{\rm CH}$). Therefore, the ergospheres of the event (cosmological) horizon coincides with the relevant tunneling potential barriers. The conclusions are also applicable to the tunneling of the massless particles.

4. Conclusions

According to the Parikh–Wilczek tunneling framework, particles' tunneling process in the rotating black holes' space time will be identical with the evolution of its dynamic equivalence. Unlike dynamic spherically symmetric black holes, the horizons are yielded from the radial outgoing lightlike geodesic, while the results should be checked with York's definition on the horizon. By studying the tunneling process in the dragging frame of stationary black holes, horizon's contraction (expanding) speed is calculated, which yields the dynamic black holes' local event (cosmological) horizon. According to the above calculations, it is the quantum ergosphere that is identical with the tunneling potential barrier or the classically inaccessible region set by particles' tunneling.

What about the origin of the dynamic rotating black holes' Hawking radiation? Because the radiation cannot be detected in the nonclassical area or the inside of the horizons, the radiation of a dynamic Kerr–Newman black hole comes from the apparent event horizon, while that of a dynamic Kerr–Newman–de Sitter black hole comes from the apparent event horizon and the local cosmological horizon.

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