An Aharonov–Anandan phase gate in the sub-Hilbert space of a coupling flux qubits system*

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We study two flux qubits with a parameter coupling scenario. Under the rotating wave approximation, we truncate the 4-dimensional Hilbert space of a coupling flux qubits system to a 2-dimensional subspace spanned by two dressed states $|01\rangle$ and $|10\rangle$. In this subspace, we illustrate how to generate an Aharnov–Anandan phase, based on which, we can construct a NOT gate (as effective as a C-NOT gate) in this coupling flux qubits system. Finally, the fidelity of the NOT gate is also calculated in the presence of the simulated classical noise.

Keywords: Aharonov–Anandan phase, sub-Hilbert space, coupling flux qubits

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1. Introduction

The superconducting qubit has attracted much attention over the past ten years, and a variety of different macroscopic quantum phenomena have been observed experimentally. For scalable quantum computation, the coupling between multi qubits is indispensable. For maximal protection from noise, qubits should be biased at their optimal points, where they are not sensitive to certain types of errors.[1] How to couple two qubits biased at their optimal points with different gaps becomes a pivotal problem in experiment. Recently, two strategies for superconducting qubits coupling at their optimal points have been proposed, one is the fixed linear couplings between fixed off-resonant qubits (FLICFORQ) scheme,[2] and the other is the parameter coupling scheme.[3]

In the FLICFORQ scheme, by adding resonant microwaves to each qubit at a power so that the sum of the Rabi frequencies matches the detuning between the two qubits at their optimal points, we can realize the interaction between the two detuned qubits and the resonant microwaves. This scheme also requires that the detuning between the two qubits is small (typically smaller than 500 MHz),[3] which is experimentally feasible for two charge qubits. While for a flux qubit, the energy gap depends exponentially on the critical current of the Josephson junction.[4] Even with nominally identical parameters, two flux qubits will have significantly different gaps.[5] Thus, the FLICFORQ scheme for flux qubits is impracticable, since the detuning between the two flux qubits can easily exceed the Rabi frequencies. While for the parameter coupling scheme, we need only modulate the coupling parameter at the sum or the differential frequency of the two detuned qubits, and theoretically we could realize an arbitrary large coupling between two flux qubits.

It is well known that for universal quantum computation, C-NOT gates can carry out all kinds of tasks.[6] Since in a system with two coupling qubits the $|01\rangle \rightleftharpoons |10\rangle$ or $|00\rangle \rightleftharpoons |11\rangle$ transition is as effective as a C-NOT gate,[7] it is crucial to realize the $|01\rangle \rightleftharpoons |10\rangle$ or $|00\rangle \rightleftharpoons |11\rangle$ transition in practical experiments. Furthermore, as is well known that the geometric phase is independent of the energy level and the operation time, and the quantum gate manipulation based on the geometric phase might be resilient to certain types of errors, since it only depends on the global feature of the path executed on the Bloch

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sphere.[8] Thus, a scheme to realize the $|01\rangle \rightleftharpoons |10\rangle$ transition based on the geometric phase is important. In this paper, we firstly discuss the coupling between two flux qubits. It is shown that the coupling strength in our discussion has been reduced by a factor of $1-\xi$ ($\xi < 1$) in the parameter coupling scheme. Then we propose an experimentally feasible scheme to implement a geometric phase gate manipulation in the sub-Hilbert space spanned by $|01\rangle$ and $|10\rangle$ of the coupling flux qubits system, and finally the fidelity of the geometric phase gate in a sub-Hilbert space subjected to simulated classic noise is also calculated. Our calculation will be constructive to practical experiments of coupling flux qubits systems.

2. Quantum theory in coupling qubits systems

Consider two flux qubits with different gaps Δ_1 and Δ_2 ($\Delta_1 > \Delta_2$) that are biased at their optimal points. The coupling strength $J(\lambda)$ between the two flux qubits can be modulated by adding a microwave to the coupling circuits, as shown in Fig. 1(a). We define the tunable parameter $\lambda(t)$ as $\lambda(t) = \lambda_0 + \mathrm{d}\lambda\cos(\omega t)$, where $\mathrm{d}\lambda\cos(\omega t)$ is the alternating-current (AC) part. The parameter-dependent coupling strength can be denoted as $J(\lambda(t)) = J(\lambda_0) + \mathrm{d}J\cos\omega t$, where $\mathrm{d}J = (\partial J/\partial\lambda)\mathrm{d}\lambda$. We further suppose that the system is in the dispersive coupling regime just as Bertet *et al.* discussed:[3] $J(\lambda_0) = J_0 \ll (\Delta_1 - \Delta_2)/\hbar$. The Hamiltonian of the coupling system reads $H = -\frac{\hbar}{2}\sum_{i=1}^{2}\omega_i\sigma_{zi} + \hbar J(\lambda(t))\sigma_{x1}\sigma_{x2}$, where $\hbar\omega_i = \Delta_i$, and $\sigma_{xi,zi}$ are the Pauli matrices with i denoting the different qubits. Furthermore, we rewrite the interaction Hamiltonian as $H_{\mathrm{I}} = H_{\mathrm{I(dc)}} + H_{\mathrm{I(ac)}} = \hbar J_0\sigma_{x1}\sigma_{x2} + \hbar\mathrm{d}J\cos\omega t\sigma_{x1}\sigma_{x2}$, where subscripts dc and ac denote the DC (direct-current) and the AC parts of the interaction Hamiltonian, respectively. By introducing the Pauli raising and lowering operators, we can rewrite the DC part of the interaction Hamiltonian as $H_{\mathrm{I(dc)}} = \hbar J_0(\sigma_1^+\sigma_2^+ + \sigma_1^-\sigma_2^- + \sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+)$. We adopt the second order perturbation theory to calculate the energy eigenstates and the energy levels. Under the rotating wave approximation (RWA), since the coupling system is in the dispersive coupling regime ($J_0 \ll \omega_1 - \omega_2$), the eigenstates of the coupling system read

$$
\begin{aligned}
|0\widehat{0}\rangle &= |0,\widehat{0}\rangle - \frac{J_0}{\omega_1+\omega_2}|1,\widehat{1}\rangle \approx |0,\widehat{0}\rangle,\\
|0\widehat{1}\rangle &= |0,\widehat{1}\rangle - \frac{J_0}{\omega_1-\omega_2}|1,\widehat{0}\rangle,\\
|1\widehat{0}\rangle &= |1,\widehat{0}\rangle + \frac{J_0}{\omega_1-\omega_2}|0,\widehat{1}\rangle,\\
|1\widehat{1}\rangle &= |1,\widehat{1}\rangle + \frac{J_0}{\omega_1+\omega_2}|0,\widehat{0}\rangle \approx |1,\widehat{1}\rangle,
\end{aligned}
\tag{1}
$$

where $|i,\widehat{j}\rangle$ (i denotes qubit 1, while $\widehat{j}$ denotes qubit 2) denotes the uncoupled state of the system, while $|\widehat{ij}\rangle$ denotes the dressed state of the coupling system. The corresponding eigenvalues of the eigenstates read

$$
\begin{aligned}
E_{|0\widehat{0}\rangle} &\approx E_{|0,\widehat{0}\rangle} = -\hbar\frac{\omega_1+\omega_2}{2},\\
E_{|0\widehat{1}\rangle} &= E_{|0,\widehat{1}\rangle} - \hbar\frac{{J_0}^2}{\omega_1-\omega_2}\\
&= -\hbar\left(\frac{\omega_1-\omega_2}{2} + \frac{{J_0}^2}{\omega_1-\omega_2}\right),\\
E_{|1\widehat{0}\rangle} &= E_{|1,\widehat{0}\rangle} + \hbar\frac{{J_0}^2}{\omega_1-\omega_2}\\
&= \hbar\left(\frac{\omega_1-\omega_2}{2} + \frac{{J_0}^2}{\omega_1-\omega_2}\right),\\
E_{|1\widehat{1}\rangle} &\approx E_{|1,\widehat{1}\rangle} = \hbar\frac{\omega_1+\omega_2}{2},
\end{aligned}
\tag{2}
$$

as illustrated in Fig. 1(b).

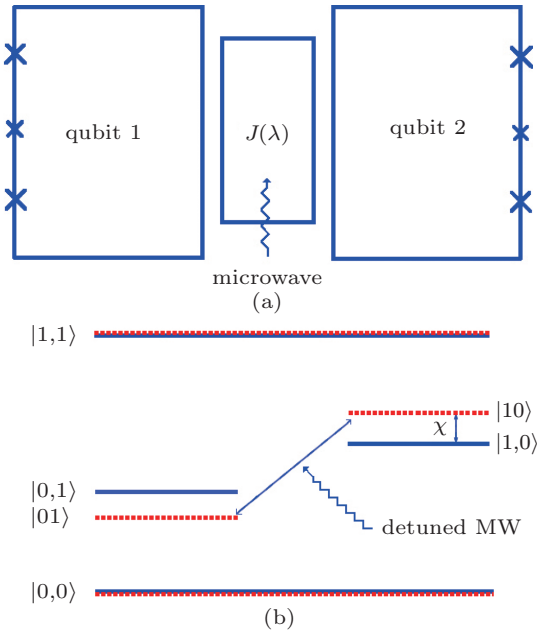


Fig. 1. (colour online) (a) Parameter coupling scheme, the central box represents the coupling circuits. By adding a microwave to the coupler, we can implement a coupling between two flux qubits with different gaps biased at their optimal points. (b) Energy level diagram of a coupling system. Blue lines represent levels of the uncoupled system. Red dashed lines represent levels of the coupled system, which are calculated by using the second order perturbation theory under the rotating wave approximation, where $\chi = J_0^2/(\omega_1 - \omega_2)$.

The AC part of the interaction Hamiltonian can be written as (RWA)

$$\begin{aligned} H_{\mathrm{I(ac)}} &= \hbar \mathrm{d}J\cos\omega t(\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+) \\ &= \hbar \mathrm{d}J\cos\omega t(|0,\widehat{1}\rangle\langle 1,\widehat{0}| + |1,\widehat{0}\rangle\langle 0,\widehat{1}|). \end{aligned} \quad (3)$$

Combining with Eq. (1), we can rewrite the AC part of the interaction Hamiltonian in the dressed basis[3] as

$$\begin{aligned} H_{\mathrm{I(ac)}} &= \frac{\hbar \mathrm{d}J\cos(\omega t)}{1+{J_0}^2/\delta^2}\left[\frac{2J_0}{\delta}(|1\widehat{0}\rangle\langle 1\widehat{0}| - |0\widehat{1}\rangle\langle 0\widehat{1}|)\right. \\ &\quad \left. + \left(1-\frac{{J_0}^2}{\delta^2}\right)(|0\widehat{1}\rangle\langle 1\widehat{0}| + |1\widehat{0}\rangle\langle 0\widehat{1}|)\right] \\ &\quad + \hbar \mathrm{d}J\cos\omega t(|1\widehat{1}\rangle\langle 0\widehat{0}| + |0\widehat{0}\rangle\langle 1\widehat{1}|), \end{aligned} \quad (4)$$

where $\delta = \omega_1 - \omega_2$.

The 4-dimensional Hilbert space of the coupling system can be truncated into two subspaces spanned by two pairs of dressed states $|0\widehat{0}\rangle, |1\widehat{1}\rangle$ and $|0\widehat{1}\rangle, |1\widehat{0}\rangle$ respectively. For simplicity, we denote the four dressed states $|0\widehat{0}\rangle, |0\widehat{1}\rangle, |1\widehat{0}\rangle, |1\widehat{1}\rangle$ as $|d_1\rangle, |d_2\rangle, |d_3\rangle, |d_4\rangle$, respectively, and define the subspace spanned by $|d_1\rangle, |d_4\rangle$ as s_1, while the subspace spanned by $|d_2\rangle, |d_3\rangle$ is s_2. The AC part of the interaction Hamiltonian can also be written as

$$\begin{aligned} H_{\mathrm{I(ac)}} &= \frac{\hbar \mathrm{d}J\cos(\omega t)}{1+{J_0}^2/\delta^2}\left[\frac{2J_0}{\delta}(-\sigma_{s_2}^z) + \left(1-\frac{{J_0}^2}{\delta^2}\right)\sigma_{s_2}^x\right] \\ &\quad + \hbar \mathrm{d}J\cos(\omega t)\sigma_{s_1}^x, \end{aligned} \quad (5)$$

where $\sigma_{s_2}^z = |d_2\rangle\langle d_2| - |d_3\rangle\langle d_3|$, $\sigma_{s_2}^x = |d_2\rangle\langle d_3| + |d_3\rangle\langle d_2|$, $\sigma_{s_1}^x = |d_4\rangle\langle d_1| + |d_1\rangle\langle d_4|$, and the subscript of the Pauli matrix denotes the different subspace. Considering the eigenvalues in Eq. (2), we write the total Hamiltonian of the two subspaces as

$$\begin{aligned} H_{s_1} &= \frac{1}{2}\hbar\Lambda_1\sigma_{s_1}^z + \hbar \mathrm{d}J\cos(\omega t)\sigma_{s_1}^x, \\ H_{s_2} &= \left[\frac{1}{2}\hbar\Lambda_2 - \frac{2\hbar J_0\,\mathrm{d}J}{\delta(1+{J_0}^2/\delta^2)}\cos(\omega t)\right]\sigma_{s_2}^z \\ &\quad + \frac{\hbar[1-({J_0}^2/\delta^2)]\,\mathrm{d}J}{1+({J_0}^2/\delta^2)}\cos(\omega t)\sigma_{s_2}^x, \end{aligned} \quad (6)$$

where $\Lambda_1 = \omega_1 + \omega_2$, and

$$\Lambda_2 = \omega_1 - \omega_2 + \frac{2J_0^2}{\omega_1-\omega_2} = \delta + \frac{2J_0^2}{\delta}.$$

Since $\delta \gg J_0 \gg \mathrm{d}J$, we further have

$$H_{s_2} = \frac{1}{2}\hbar\Lambda_2\sigma_{s_2}^z + \hbar \mathrm{d}J\xi\cos(\omega t)\sigma_{s_2}^x, \quad (7)$$

where

$$\xi = \frac{1-({J_0}^2/\delta^2)}{1+({J_0}^2/\delta^2)}, \quad (\xi < 1).$$

3. Non-adiabatic geometric phase in sub-Hilbert space

For qubit manipulation, one must finish all of the manipulations within the coherence time. Hence fast operations are required. So it would be more feasible to construct a non-adiabatic geometric phase gate rather than an adiabatic geometric phase gate for practical experiments. It is well known that a qubit can be characterized by a unit vector on the Bloch sphere, and the dynamic equation of qubits is governed by the Liouville equation $\mathrm{i}\hbar\partial_t\hat{\rho} = [\widehat{H}, \hat{\rho}]$, where $H = (1/2)\boldsymbol{\sigma}\cdot\boldsymbol{B}$ is the system hamiltonian, and $\widehat{\rho}$ is the density matrix of the qubit state.

Suppose a detuned microwave is added to the coupling circuits as shown in Fig. 1(b). Then the hamiltonian for sub-Hilbert space s_2 in the rotating frame can be written as $H'_{s_2} \approx (\hbar/2)(\Delta\sigma_z + \Omega\cos\varphi\sigma_x + \Omega\sin\varphi\sigma_y)$, with $\Delta = \Lambda_2 - \omega$, and $\Omega = \mathrm{d}J\xi$. We now define a fictitious magnetic field in sub-Hilbert space s_2 as $\boldsymbol{B} = (\hbar\Omega\cos\varphi, \hbar\Omega\sin\varphi, \Delta)$. The $\hbar\Omega$ can be regard as the effective dipolar interaction between the microwave and the coupling qubits system. We must note that the effective interaction strength between the microwave and the two flux qubits coupling system has been reduced by a factor of $1-\xi$ because of the perturbation.

Now we introduce the Bloch sphere corresponding to the projective sub-Hilbert space s_2 (Fig. 2). For an arbitrary pair of orthogonal states $|\phi_1\rangle$, $|\phi_2\rangle$ on this Bloch sphere, $|\phi_1\rangle$, $|\phi_2\rangle$ can be denoted as

$$\begin{aligned} |\phi_1\rangle &= \cos\frac{\vartheta}{2}|d_2\rangle + \sin\frac{\vartheta}{2}|d_3\rangle, \\ |\phi_2\rangle &= -\sin\frac{\vartheta}{2}|d_2\rangle + \cos\frac{\vartheta}{2}|d_3\rangle, \end{aligned} \quad (8)$$

where $|d_2\rangle$, $|d_3\rangle$ can be regard as two eigenstates of Pauli operator σ_z in sub space s_2. It is well known that in a quantum system, if the system state evolves non-adiabatically under the system Hamiltonian, when the state goes back to its initial position, it would accumulate a phase, which is known as the Aharonov–Anandan phase.[9] The phase gate based on the Aharonov–Anandan phase would be a non-adiabatic one, which is defined as the Aharonov–Anandan phase gate. So, if the two orthogonal states each complete a closed loop on the Bloch sphere, they will obtain total phases $\pm\eta$ (including both the dynamic and the geometric phases), respectively. Suppose that the initial state of the coupling system is $|Q(0)\rangle = a|\phi_1\rangle + b|\phi_2\rangle$, after the cyclic evolution, the state would become $|Q(\tau)\rangle = U(\vartheta,\eta)|Q(0)\rangle$, where gate $U(\vartheta,\eta)$ reads[10]

$$U(\vartheta,\eta)=\begin{pmatrix} \mathrm{e}^{\mathrm{i}\eta}\cos^2\frac{\vartheta}{2}+\mathrm{e}^{-\mathrm{i}\eta}\sin^2\frac{\vartheta}{2} & \mathrm{i}\sin\vartheta\sin\eta \\ \mathrm{i}\sin\vartheta\sin\eta & \mathrm{e}^{\mathrm{i}\eta}\sin^2\frac{\vartheta}{2}+\mathrm{e}^{-\mathrm{i}\eta}\cos^2\frac{\vartheta}{2} \end{pmatrix}. \tag{9}$$

Suppose that the orthogonal states are a pair of dark states[11,12] of the system Hamiltonian, after the periodic evolution, they would obtain a pure geometric phase without dynamic parts. For example, if we choose the two orthogonal states as $|OA\rangle$ and $|OC\rangle$

$$\begin{aligned} |OA\rangle &= \frac{\sqrt{2}}{2}(|d_2\rangle+|d_3\rangle), \\ |OC\rangle &= \frac{\sqrt{2}}{2}(-|d_2\rangle+|d_3\rangle), \end{aligned} \tag{10}$$

then we drive state $|OA\rangle$ along the closed loop $\widehat{ABCDA}$, correspondingly the state $|OC\rangle$ would be driven along the closed loop $\widehat{CDABC}$, as shown in Fig. 2. Since these two states are the dark states of

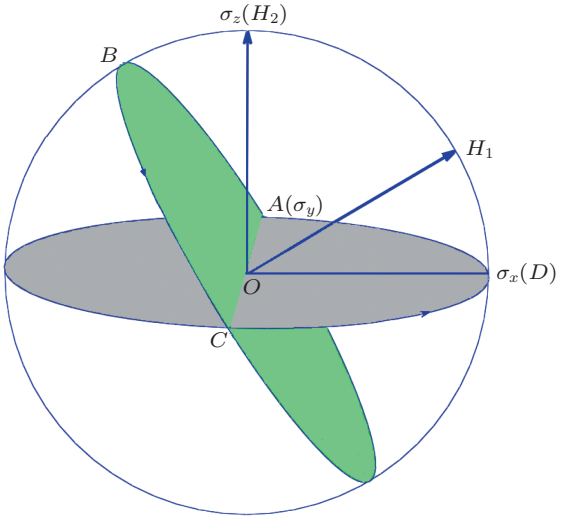


Fig. 2. (colour online) Bloch sphere of projective sub-Hilbert space s_2. After being derived through the closed loop $\widehat{ABCDA}$, state $|OA\rangle$ will accumulate a pure geometric phase without dynamic part.

system Hamiltonian H_1 and H_2, when the two states go back to their initial positions, they would obtain pure geometric phases $\pm\gamma$ respectively. As Fig. 2 shows, since $\vartheta=\pi/2$ for the two dark states $|OA\rangle$ and $|OC\rangle$ of Hamiltonian H_1 and H_2, the U gate now becomes

$$U(\gamma)=\begin{pmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{pmatrix}, \tag{11}$$

where $\gamma=\Omega_{ABCDA}/2$, and Ω_{ABCDA} denotes the solid angle subtended by the closed loop $\widehat{ABCDA}$. Suppose the geometric phase further has $\gamma=\pi/2$, then the U gate would be a NOT gate in sub space s_2, which is as effective as a C-NOT gate, since we can realize the transformation between $|01\rangle$ and $|10\rangle$ in a coupling flux qubit system.

We take the NOT gate in sub space s_2 for example, the pulse sequence in experiment is given as follows.

1) Modulate the amplitude of microwave so that $\mathrm{d}J\xi=\Delta$, then add the microwave to the coupling flux qubits system. Now the Hamiltonian in the rotating frame is $H=\frac{\hbar}{2}(\Delta\sigma_z+\Delta\sigma_x)$ with two eigenvalues $\pm E_1$, as shown in Fig. 2. Wait for a duration of τ_1, $\tau_1=\hbar\pi/|E_1|$, then the path $\widehat{ABC}$ is executed.

2) Switch off the microwave (modulated by pulse ~0.1 ns), correspondingly the Hamiltonian becomes $H_2=\hbar\Delta\sigma_z/2$. Wait for a duration of τ_2, $\tau_2=\hbar\pi/|E_2|$, where E_2 is the eigenvalue of H_2, $E_2=\hbar\Delta$, then the path $\widehat{CDA}$ is executed.

3) After step 2) is executed, we immediately repeat steps 1) and 2).

After the pulse sequence from 1) to 3), the total geometric phase accumulated by state $|OA\rangle$ during the whole process would be $\gamma=\pi/2$, and correspondingly, we will get a NOT gate in sub space s_2. Since in the whole process we only implement a switch off and a switch on of the microwave at proper time, which is actually modulated by the pulse and can virtually be very fast and exact in experiment, our gate manipulation will have a relatively high fidelity.

4. Fidelity

As mentioned above, the Hamiltonian in sub space s_2 is $H'_{s_2}=\hbar/2(\Delta\sigma_z+\Omega\sigma_x)$ (for our scheme, $\varphi=0$), with $\Delta=\Lambda_2-\omega$, and $\Omega=\mathrm{d}J\xi$. Generally, parameter Δ and the amplitude of microwave $\mathrm{d}J$ would have time-dependent fluctuations, while factor ξ is assumed to be a constant without fluctuation. we suppose that the time-dependent parameter fluctuations have $\Delta(t)=\Delta_0+\mathrm{d}\Delta(t)$, $\Omega(t)=\Omega_0+\mathrm{d}\Omega(t)$. We further define $\mathrm{d}H'(t)=\hbar/2[\mathrm{d}\Delta(t)\sigma_z+\mathrm{d}\Omega(t)\sigma_x]$, then the Hamiltonian with time-dependent parameter fluctuation reads $H'(t)=H'_0+\mathrm{d}H'(t)=\hbar/2[\Delta(t)\sigma_z+\Omega(t)\sigma_x]$. The dynamics of a qubit on the Bloch sphere is governed by the Liouville equation, in the interaction representation, it becomes $\partial\rho_{\mathrm{I}}(t)/\partial t=-\mathrm{i}[\mathrm{d}H'_{\mathrm{I}}(t),\rho_{\mathrm{I}}(t)]$, with $\rho_{\mathrm{I}}(t)=$

$e^{iH_0't}\rho(t)\,e^{-iH_0't}$, $dH_I'(t) = e^{iH_0't}dH'(t)\,e^{-iH_0't}$, and $\rho(t)$ is the time-dependent density matrix. The formalistic solution of the Liouville equation in the interaction representation has been worked out by using the Kubo's cumulant expansion method.[13,14] For the classic Gaussian Markov stochastic process, the Liouville equation obtained by using Kubo's cumulant expansion method can be directly written as[15]

$$\partial\rho_I(t)/\partial t = -\int_0^t dt_1\langle[dH_I'(t),[dH_I'(t_1),\rho_I(t)]]\rangle, \quad (12)$$

where a δ-type time correlation function is defined as the white noise

$$\langle d\xi_i(t)d\xi_j(t')\rangle = \Gamma_{\xi_i}\delta_{ij}\delta(t-t') \quad (i=1,2;j=1,2), \quad (13)$$

with $\xi_1 = \Delta$, $\xi_2 = \Omega$, and zero mean $\langle d\xi_i\rangle = 0$. We rewrite $\Gamma_{\xi_i} = \mu_i\xi_i$, where μ_i denotes the coupling factor between the environment and parameter ξ_i. By simple algebraic manipulation, finally Eq. (12) can be simplified as

$$\partial\boldsymbol{M}(t)/\partial t = \Re\cdot\boldsymbol{M}(t). \quad (14)$$

Here we have defined the superoperator $\Re$ of the form

$$\Re = \begin{pmatrix} -\frac{\Gamma_\Delta}{8} & i\Omega & -i\Omega \\ i\frac{\Omega}{2} & -\frac{\Gamma_\Delta}{2}-\frac{\Gamma_\Omega}{4}-i\Delta & \frac{\Gamma_\Omega}{4} \\ -i\frac{\Omega}{2} & \frac{\Gamma_\Omega}{4} & -\frac{\Gamma_\Delta}{2}-\frac{\Gamma_\Omega}{4}+i\Delta \end{pmatrix}, \quad (15)$$

and the time-dependent matrix $\boldsymbol{M}(t)$ of the form

$$\boldsymbol{M}(t) = \begin{pmatrix} \rho_{11}-\rho_{22} \\ \rho_{12} \\ \rho_{21} \end{pmatrix}, \quad (16)$$

where ρ_{ij} is the density matrix element of the state in sub-Hilbert space s_2. By solving Eq. (14), we have $\boldsymbol{M}(t) = \Sigma_{i=1}^3\nu_i\,e^{\varsigma_i t}\boldsymbol{r}_i$, where ς_i and $\boldsymbol{r}_i$ are the eigenvalues and the eigenvectors of superoperator $\Re$, respectively, and coefficient ν_i is determined by the initial state in sub space s_2. By using computer simulation, we can obtain the density matrix of the final state after evolving under the Hamiltonian system. Suppose, initially, that the state in sub space s_2 is $\cos(\theta/2)|d_2\rangle + \sin(\theta/2)|d_3\rangle$, after the evolution (suppose the evolving time is τ), we can obtain $\boldsymbol{M}(\tau)$, and furthermore we can obtain the density matrix ρ of the final state after evolution. The fidelity of the geometric gate reads $F = \mathrm{Tr}[\rho_{\mathrm{id}}\cdot\rho_{\mathrm{no}}]$, where ρ_{id} and ρ_{no} denote the density matrices without and with the parameter fluctuation, respectively.

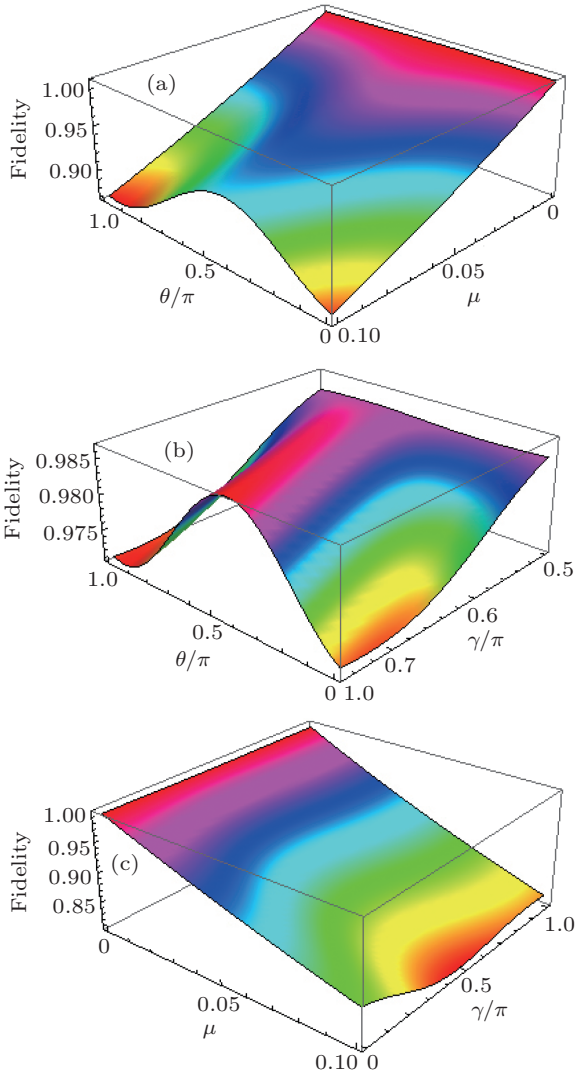


Fig. 3. (colour online) Fidelity of the geometric phase gate in the sub-Hilbert space of a coupling flux qubits system. (a) Fidelity with different coupling factor μ and different θ while $\Omega = \Delta$. (b) Fidelity with different θ and different geometric phase γ while $\mu = 0.02$. (c) Fidelity with different μ and different geometric phase γ while $\theta = \pi/2$.

For simplicity, we casually use $\delta = 1000$ MHz, $dJ = 20$ MHz, $J_0 = 120$ MHz. The fidelity simulation result is shown in Fig. 3. We further suppose $\mu_1 = \mu_2 = \mu$ (actually in experiment, the amplitude of microwave can be controlled precisely, and correspondingly μ_2 might be much smaller than μ_1) for simplicity. Figure 3(a) shows the fidelity of the geometric phase gate in sub space s_2 with different coupling factor μ and different dressed state θ while $\Omega = \Delta$. Figure 3(a) shows that under a certain noise strength, if the initial state is $\frac{\sqrt{2}}{2}(|d_2\rangle + |d_3\rangle)$, the geometric phase gate will have a relative high fidelity. Figure 3(b) shows the fidelity with different initial states and different geometric phase γ while coupling factor $\mu = 0.02$; it illustrates that for a certain geo-

metric phase γ, if the initial state is the equal superposition of $|d_2\rangle$ and $|d_3\rangle$, the geometric phase gate has a relative high fidelity, which actually has the same conclusion as that of Fig. 3(a). Besides, we also calculate the fidelity with different geometric phase γ and different μ while $\theta = \pi/2$, as shown in Fig. 3(c). It is shown that the fidelity would be relative low for a NOT gate. Typically, in sub space s_2 when $\mu = 0.02$, the fidelity for the NOT gate is bigger than 0.98 in our simulation.

5. Conclusion

In summary, we have proposed a feasible experimental scheme to implement geometric phase gates in the sub-Hilbert space of a coupled two flux qubis system. The calculation based on the perturbation theory shows that the effective dipolar interaction between the microwave and the controlling circuits of the coupling flux qubits system has a reduction of $(1-\xi)$. By truncating the 4-dimensional Hilbert space, we discuss the Aharonov–Anandan geometric phase in this sub-Hilbert space and simulate the fidelity of the geometric phase gate. Our simulation shows that under a certain noise strength, if the initial state of the coupling flux qubits system is the equal superposition of $|01\rangle$ and $|10\rangle$, the transition $|01\rangle \rightleftharpoons |10\rangle$ will have a relatively high fidelity, which is actually as effective as a C-NOT gate. Our scheme will be constructive for the gate manipulation in a coupling flux qubits system.

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