Two methods to deal with an inhomogenous Rabi frequency in microwave transition*

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We analyse the influence of an inhomogenous microwave field on the coherence of atom ensembles. Two methods are proposed to suppress the dephasing generated by the inhomogenous Rabi frequency. One of them is realized by using a spin echo, and the other one is based on the identical spin rotation effect. The calculation results show that the contrast of a signal acquired in experiment can be improved by using the two methods. Their advantages and drawbacks are discussed. We hope they can be used to improve the contrast of experimental signals in situations where microwave fields are very inhomogenous. Finally, we discuss the case of a continuous working microwave field and show that the dipole force raised with the inhomogeneity can be eased by spin flip.

Keywords: inhomogenous microwave field, spin echo, identical spin rotation effect

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1. Introduction

Spin manipulation is an important technique in atom optics, which is usually realized by hyperfine transitions between ground states. For most cases, a homogenous microwave field is required. However any electromagnetic field created by current carrying wires can not be homogenous but only close to that in specific places. This limits the contrast of experimental results, because atoms in different places may experience different Rabi frequencies which are proportional to the magnitudes of the electromagnetic fields. And well prepared pure ensembles will become mixed ones after the microwave transitions (Fig. 1(a)). So the inhomogeneity of the Rabi frequency is also a main source of dephasing. To resolve this problem, efforts have been made to design and simulate kinds of cavities and horns to create microwave fields that areas homogenous as possible. In some setups, such as the atom chip atomic clock,[1,2] cavities are replaced by several current carrying wires mounted in a vacuum chamber for simplicity, which results in a low initial contrast.[3] Here we present two proposals to recover the coherence that is affected by the inhomogenous Rabi frequency. The first one is realized by manually reversing the Bloch vectors via spin manipulation, which is similar to the spin echo.[4] The other one is increasing the atomic interaction to suppress the dephasing.

2. The spin echo method

It is well known that one needs a resonant $\pi = \Omega T$ pulse to completely pump atoms from $|0\rangle$ to $|1\rangle$, where Ω is the Rabi frequency. In the Bloch sphere (rotating frame), such a process can be considered as rotating the Bloch vector around axis x (S_x) by π. The pulse can also be taken as a sum of two $\pi/2$ pulses. Due to the inhomogeneity of Ω_x, Bloch vectors of some atoms are rotated more (less) than $\pi/2$ after the first pulse, which means they dephase during the first pulse. This dephasing will become worse after the second pulse. Our first method involves inserting a rotation around the y axis by π between the two pulses. Bloch vectors which are over (less) rotated after the first pulse become less (over) rotated (Fig. 1(b)). So the slow ones take the lead, and the fast ones fall behind. Their deviations from each other will be compensated after the second pulse. In experiments, to switch the axis of rotation between x and y, we can change the phase of the microwave by $\pi/2$.

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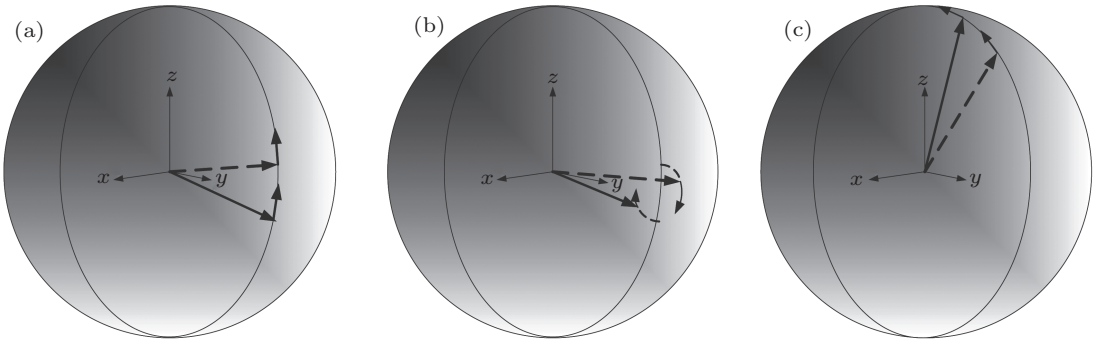


Fig. 1. (a) Two atoms (solid and dashed) initially in state $|z;-\rangle$ start to rotate around the x axis. Due to an inhomogenous microwave field, they start to dephase. The dashed one is faster than the solid one. (b) When they are symmetry about the y axis, a rotation around the y axis is given to reverse them. (c) Then the fast one starts to chase the slow one, and they will rephrase.

In reality, several unfavorable factors limit the performance of this method. The most important one is the inhomogeneity of Ω_y, which influences the rotation speed of S_y. For most cases, $\Omega_y = \Omega_x$, which means Ω_y would also be a very inhomogenous Rabi frequency (otherwise we can use S_y instead of S_x). This makes the inserted rotation also be an imperfect π pulse. So the over and the less rotated Bloch vectors will not be well reversed (no longer in the same meridian). So, after the second $\pi/2$ pulse, their deviations can not be compensated exactly. But the contrast of the whole ensemble can still be improved. For a single atom initially in state $|0\rangle$, after the two $\pi/2$ pulses, the population of state $|0\rangle$ is

$$\rho_{00} = \cos^2(\theta/2), \tag{1}$$

where θ is the angle rotated around the x axis in practice. If the inserted pulse is used, the population will be

$$\rho'_{00} = \cos^2(\theta/2)\cos^2(\phi/2), \tag{2}$$

where ϕ is the angle rotated around the y axis by the inserted pulse in practice. It can be easily seen that ρ'_{00} will be always smaller than ρ_{00} unless ϕ is zero (the inserted pulse is not exerted) or θ is π (the Bloch vectors have already been well rotated). So the contrast, which is defined as

$$C = \frac{\langle\rho_{11}\rangle - \langle\rho_{00}\rangle}{\langle\rho_{11}\rangle + \langle\rho_{00}\rangle} = 1 - 2\langle\rho_{00}\rangle, \tag{3}$$

will be higher if the inserted pulse is used. To calculate it, we make three assumptions for simplicity. The first one is that the possible value of θ is equally distributed between $\pi - \Delta$ and $\pi + \Delta$. The second one is $\theta = \phi$, which means Ω_x and Ω_y are equal with each other for each atom. We also neglect the influence caused by the Doppler shift. For ultracold atoms, the velocities of the atoms are very low, as is the Doppler shift of the microwave. Take ^{87}Rb for example, the temperature of the atom cloud cooled by optical molasses can be several micro Kelvin, and the Doppler shift of the microwave field which couples the two hyperfine ground states is only several Hertz. Usually the duration of microwave transition is no more than a few mini seconds. So the Doppler shift contributes a disturbance of less than 0.01 to θ, which can be neglected in our calculation. Then the contrast of the whole ensemble with or without the inserted pulse is given by

$$C' = \frac{1}{4} - \frac{\sin(2\Delta)}{8\Delta} + \frac{\sin(\Delta)}{\Delta}, \tag{4}$$

or

$$C = \sin(\Delta)/\Delta. \tag{5}$$

It can be seen from Fig. 2 that C' is always bigger than C except when Δ is zero.

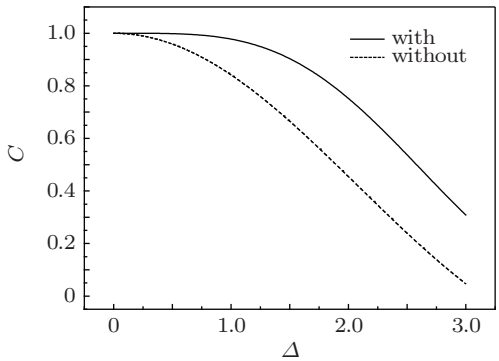


Fig. 2. Contrasts of the whole ensemble with or without the inserted pulse calculated with the assumptions given in the text.

In the discussion above, we have assumed that the Rabi frequencies experienced by the atoms do not

change too much during the whole process. For ultracold atoms and high Rabi frequency, such an assumption is appropriate. However, for the case of a Ramsey process, the Ramsey free evolution time is so long that the locations of atoms may change randomly and the Rabi frequencies that they experience during the two $\pi/2$ pulses maybe different. To calculate the contrast in this situation, we define α and β as the angles rotated around the x axis in practice during the first and the second $\pi/2$ pulses, respectively. Then the final population of state $|0\rangle$ with or without the inserted pulse will be

$$\rho'_{00} = \cos^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\phi}{2}\right) + \sin^2\left(\frac{\alpha-\beta}{2}\right)\sin^2\left(\frac{\phi}{2}\right) \quad (6)$$

or

$$\rho_{00} = \cos^2\left(\frac{\alpha+\beta}{2}\right), \quad (7)$$

respectively. To make ρ'_{00} smaller than ρ_{00}, the inequality

$$\sin^2\left(\frac{\alpha-\beta}{2}\right) < \cos^2\left(\frac{\alpha+\beta}{2}\right) \quad (8)$$

must be satisfied. For atoms trapped in sites of optical[5] or magnetic[6,7] lattices, the microwave fields they experience change little during the whole process, so α and β are almost the same as each other. Then inequality (8) will be satisfied, and the inserted pulse is useful. But, for most cases, the two angles are independent. It can be easily proved that if the distributions of α and β satisfy $f(\pi/2+x) = f(\pi/2-x)$, then

$$\int_{\pi/2-\infty}^{\pi/2+\infty}\int_{\pi/2-\infty}^{\pi/2+\infty}\left[\sin^2\left(\frac{\alpha-\beta}{2}\right) - \cos^2\left(\frac{\alpha+\beta}{2}\right)\right] f(\alpha) f(\beta)\,\mathrm{d}\alpha\mathrm{d}\beta = 0. \quad (9)$$

The equation above shows that if α and β are independent of each other, the inserted pulse will not change the contrast, which is also verified by numerical simulations. To resolve this problem, we can divide the first $\pi/2$ pulse into two $\pi/4$ pulses and insert a rotation of π around axis $(y-z)/2$ (Fig. 3(a)). Like the method mentioned above, the deviation between Bloch vectors created during the first $\pi/4$ pulse is compensated during the second $\pi/4$ pulse. So the dephasing will be suppressed before the Ramsey free evolution. Likewise, the same procedure should be done to the second $\pi/2$ pulse, but the axis of the inserted rotation must be changed to $(y+z)/2$. The simulation (Fig. 3(b)) shows that by using this method we can still improve the contrast of the Ramsey process.

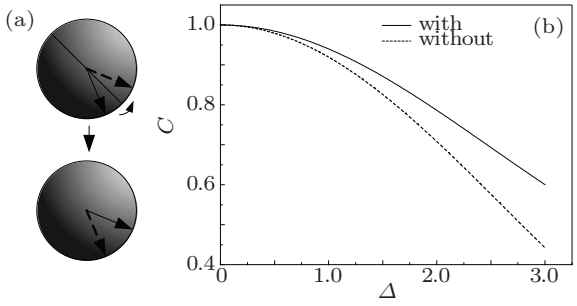


Fig. 3. (a) Rotate Bloch vectors around axis $(y-z)/2$ to reverse them. (b) Simulation results of the contrast with or without the two inserted pulses.

3. The identical spin rotation effect method

The first method involves manually inserting a π pulse between two microwave pulses to reverse the Bloch vectors. There is a mechanism based on the identical spin rotation effect,[8] which can reverse them automatically via strong atomic interaction. When a resonant microwave is exerted, the eigenstates of a single two-level atom are $|x;+\rangle = |\uparrow\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|x;-\rangle = |\downarrow\rangle = (|1\rangle - |0\rangle)/\sqrt{2}$ with eigenvalues being $\pm\hbar\Omega/2$, respectively. If we consider a system composed of two atoms a and b (without interaction between them), and their Rabi frequencies are Ω_a and Ω_b, respectively, the Hamiltonian can be written as (in the rotating frame)

$$H_0 = \hbar\Omega_0 |\uparrow\uparrow\rangle\langle\uparrow\uparrow| - \hbar\Omega_0 |\downarrow\downarrow\rangle\langle\downarrow\downarrow| + \hbar\delta\left(|S\rangle\langle A| + |A\rangle\langle S|\right), \quad (10)$$

where $\Omega_0 = (\Omega_\mathrm{a}+\Omega_\mathrm{b})/2$, $\delta = (\Omega_\mathrm{a}-\Omega_\mathrm{b})/2$, $|S\rangle = (|11\rangle - |00\rangle)/\sqrt{2}$, and $|A\rangle = (|10\rangle - |01\rangle)/\sqrt{2}$. The last term in Eq. (10) shows that there is a Rabi oscillation between $|S\rangle$ and $|A\rangle$, which is considered as the main source of dephasing.[9] For example, if the initial state of the system is

$$|\psi(0)\rangle = |0\rangle_\mathrm{a}\otimes|0\rangle_\mathrm{b} = \frac{1}{2}\left(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle - \sqrt{2}\,|S\rangle\right), \quad (11)$$

the state of the system will evolve as

$$|\psi(t)\rangle = \frac{1}{2}\{\,\mathrm{e}^{\mathrm{i}\Omega_0 t}|\uparrow\uparrow\rangle + \mathrm{e}^{-\mathrm{i}\Omega_0 t}|\downarrow\downarrow\rangle - \sqrt{2}\left[\cos(\delta t)\,|S\rangle + \mathrm{i}\sin(\delta t)\,|A\rangle\right]\}. \quad (12)$$

If $\Omega_0 = 2\delta$ ($\Omega_\mathrm{a} = 3\Omega_\mathrm{b}$), at time $T = \pi/\Omega_0$,

$$|\psi(T)\rangle = \frac{1}{2}\left(-|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle - \sqrt{2}\mathrm{i}\,|A\rangle\right)$$

$$= -\frac{1}{2}\left(|11\rangle + |00\rangle - \mathrm{i}\,|01\rangle + \mathrm{i}\,|10\rangle\right)$$
$$= \frac{|1\rangle - \mathrm{i}\,|0\rangle}{\sqrt{2}} \otimes \frac{|1\rangle + \mathrm{i}\,|0\rangle}{\sqrt{2}}. \quad (13)$$

It can be seen that because of δ the two identical states evolve into two states that are orthogonal to each other. This Rabi oscillation, which transfers $|S\rangle$ to $|A\rangle$, is always resonant as long as the energy expectation values of $|S\rangle$ and $|A\rangle$ are equal to each other. To suppress it, a large detuning is needed (Fig. 4), which means the degeneracy of $|S\rangle$ and $|A\rangle$ must be destroyed.

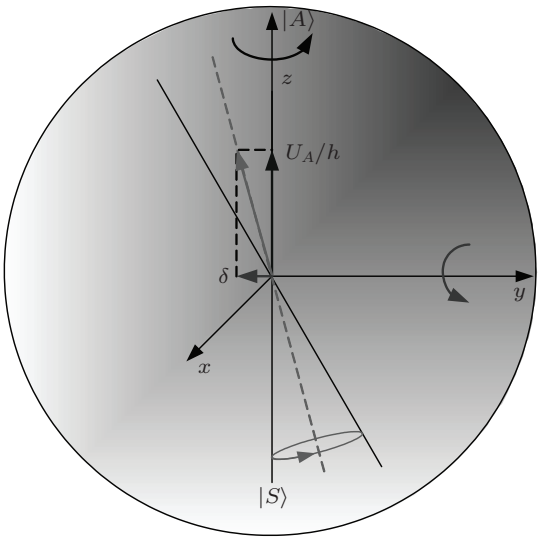


Fig. 4. The inhomogenous of Rabi frequency induces an oscillation between $|S\rangle$ and $|A\rangle$, which is a rotation around axis y at the speed of δ. A detuning is made to tilt the axis of rotation close to axis z and suppress the oscillation.

One of the mechanisms available is using the atomic interaction. When the interaction between atoms a and b is taken into consideration, the collision shift must be added to the Hamiltonian. Here we assume that the atoms are ultracold, so the excitation of $|1\rangle$ from $|0\rangle$ during a collision is energetically impossible. Then the effective Hamiltonian is

$$H = H_0 + U_{\uparrow\uparrow}\,|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + U_{\downarrow\downarrow}\,|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + U_S\,|S\rangle\langle S| + U_A\,|A\rangle\langle A|, \quad (14)$$

where $U_{\uparrow\uparrow}$, $U_{\downarrow\downarrow}$, U_S, and U_A are collision shifts of $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, $|S\rangle$, and $|A\rangle$, respectively. It can be seen that $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, and $|S\rangle$ are symmetrical under the interchange, while $|A\rangle$ is antisymmetrical. For bosons (fermions), the wave functions of $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, and $|S\rangle$ are symmetrical (antisymmetrical) and that of $|A\rangle$ is antisymmetrical (symmetrical). In the limit of low temperature, it is sufficient to consider only the s-wave scattering, then the total cross sections of $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, and $|S\rangle$ are nontrivial (zero) and that of $|A\rangle$ is zero (nontrivial). So $U_{\uparrow\uparrow}$, $U_{\downarrow\downarrow}$, and U_S are nontrivial (zero), but U_A is zero (nontrivial). Anyway, the atomic interaction makes a gap between the energy expectation values of $|S\rangle$ and $|A\rangle$. Take fermions for example, Eq. (12) should be rewritten as

$$|\psi(t)\rangle = \frac{1}{2}\Big\{\mathrm{e}^{\mathrm{i}\Omega_0 t}\,|\uparrow\uparrow\rangle + \mathrm{e}^{-\mathrm{i}\Omega_0 t}\,|\downarrow\downarrow\rangle - \sqrt{2}\left[c_S(t)\,|S\rangle + c_A(t)\,|A\rangle\right]\Big\}, \quad (15)$$

where

$$c_S(t) = \left[\cos\left(\frac{\Omega' t}{2}\right) - \frac{\mathrm{i}U_A}{\hbar\Omega'}\sin\left(\frac{\Omega' t}{2}\right)\right]\mathrm{e}^{\mathrm{i}\frac{U_A t}{2\hbar}}, \quad (16)$$

$$c_A(t) = \mathrm{i}\frac{\delta}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right)\mathrm{e}^{-\mathrm{i}\frac{U_A t}{2\hbar}}, \quad (17)$$

$$\Omega' = \sqrt{\left(\frac{U_A}{\hbar}\right)^2 + \delta^2}. \quad (18)$$

It can be seen that the modulus of $c_A(t)$ is limited by factor δ/Ω'. If we manage to keep $U_A \gg \hbar\delta$, it will be close to zero. So $|A\rangle$ will play a trivial role in Eq. (15), which makes Eq. (15) close to Eq. (12) when δ is zero.

For bosons, we can obtain almost the same result

$$|\psi(t)'\rangle = \frac{1}{2}\Big\{\mathrm{e}^{\mathrm{i}\left(\Omega_0+\frac{U_{\uparrow\uparrow}}{\hbar}\right)t}\,|\uparrow\uparrow\rangle + \mathrm{e}^{-\mathrm{i}\left(\Omega_0-\frac{U_{\downarrow\downarrow}}{\hbar}\right)t}\,|\downarrow\downarrow\rangle - \sqrt{2}\left[c'_S(t)\,|S\rangle + c'_A(t)\,|A\rangle\right]\Big\}, \quad (19)$$

where

$$c'_S(t) = \left[\cos\left(\frac{\Omega' t}{2}\right) + \frac{\mathrm{i}U_S}{\hbar\Omega'}\sin\left(\frac{\Omega' t}{2}\right)\right]\mathrm{e}^{\mathrm{i}\frac{U_S t}{2\hbar}}, \quad (20)$$

$$c'_A(t) = \mathrm{i}\frac{\delta}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right)\mathrm{e}^{-\mathrm{i}\frac{U_S t}{2\hbar}}, \quad (21)$$

$$\Omega' = \sqrt{\left(\frac{U_S}{\hbar}\right)^2 + \delta^2}. \quad (22)$$

For bosons, $U_A = 0$. So the dephasing term $c'_A(t)$ can be suppressed via increasing U_S. However, it can be seen that extra phases of states $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, and $|S\rangle$ appear due to the collision shifts. For most of the cases, $U_{\uparrow\uparrow}$, $U_{\downarrow\downarrow}$, and U_S are different from each other. Take the s-wave scattering into consideration, the collision shift is proportional to the density of atoms. For different atoms, different locations of them result in random evolvement of phases of their internal states, which leads to the decoherence. Such a problem can be eased by using fermionic-like atoms, such as ^{87}Rb. The ground-state hyperfine levels $|F = 1, m_F = -1\rangle$ and $|F = 2, m_F = 1\rangle$ are an ideal choice for $|0\rangle$ and $|1\rangle$ states, respectively. Because $a_{00} : a_{01} : a_{11} = 100.4 : 97.7 : 95.0$, $U_{\uparrow\uparrow}$, $U_{\downarrow\downarrow}$, and U_S are almost the

same as each other. Then the decoherence caused by the collision shift can be neglected. Another feature of $|F=1, m_F=-1\rangle$ and $|F=2, m_F=1\rangle$ is that both of them can be trapped by a magnetic trap and their first order Zeeman shifts are the same, which makes them a good candidate for atomic clocks.

4. Continuously working microwave field

The discussion above mainly deals with microwave pulses. If the microwave is exerted continuously (rotating the Bloch vectors continuously around the x axis), the inhomogeneity of the Rabi frequency may result in another problem. When the transition starts, the single atom initially in $|0\rangle$ evolves as

$$|\phi(t)\rangle = \frac{1}{\sqrt{2}}\left(\mathrm{e}^{\mathrm{i}\Omega t}|\uparrow\rangle + \mathrm{e}^{-\mathrm{i}\Omega t}|\downarrow\rangle\right). \qquad (23)$$

Due to the inhomogeneity of Ω, $|\uparrow\rangle$ and $|\downarrow\rangle$ experience inhomogenous light shifts. So the maximum of Ω will trap the wave function of $|\downarrow\rangle$ and repel that of $|\uparrow\rangle$. Then the two kinds of wave functions start to become separated due to the dipole force of the microwave field. In the end, the Rabi oscillation disappears, and only half of the atoms in state $|\uparrow\rangle$ or $|\downarrow\rangle$ are left. In this process, all the information about the coherence of microwave and atoms is lost. Such a decoherence process can also be slowed down by using the spin echo method. If we insert a microwave pulse to rotate the Bloch vectors around the y axis by π, states $|\uparrow\rangle$ and $|\downarrow\rangle$ will be exchanged. Then the wave functions once repelled (trapped) by the microwave field will be trapped (repelled) in the following rotation of S_x. The wave functions once separated will be coincided again. To avoid the inserted pulse from affecting the rotation of S_x, it should be given when the Bloch vectors have been rotated $(n+1/2)\,\pi$ (n is a positive integer) around the x axis. The Rabi frequency of S_y is also inhomogenous, which will also separate the wave functions during the inserted pulse. So this method can only slow down the separation and not resolve the problem perfectly.

5. Conclusion

In conclusion, the two methods can be used to improve the contrast of an atomic ensemble with the presence of an inhomogenous microwave field. The first method, based on spin echo, can be easily realized, but is still limited by the inhomogeneity. The second one, based on the identical spin rotation effect, can keep the contrast at a high level. However, the condition of high collision shift is hard to fulfil. Finally the reduction of the dipole force of inhomogenous microwave fields is discussed.

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