Influence of pumping laser bandwidth on the quantum fluctuation chracteristic of a non-degenerate optical parametric amplifier

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Usually the quantum fluctuation characteristic of a non-degenerate optical parametric amplifier is analysed under the assumption of monochromatic pumping. However, in experiments, a driving beam with finite bandwidth is used to obtain the non-degenerate signal and idler beam amplifications. On account of that, we derive an analytical solution for the non-degenerate optical parametric amplification system with finite bandwidth laser pumping, and evaluate the associated quantum fluctuation. Finally, the application of the V_1 criterion to bipartite entanglement is discussed.

Keywords: pumping laser bandwidth, squeezing, entanglement

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1. Introduction

The interest in continuous variable entanglement has been aroused by successful experiments on multichannel quantum cryptographic communication,[1] quantum holographic teleportation,[2] quantum dense coding,[3,4] and so on. It is very difficult to maintain the pump light in a high squeezing state for a long time.[5−12] We always hope to obtain high squeezing and entanglement. We have investigated the nonlinear effects, such as the the phase mismatch and the pump depletion, on the quantum fluctuation characteristic of the non-degenerate optical parametric amplification (NOPA) system,[13−19] whereas the single frequency periodic pumping and the multi-frequency periodic pumping were also included to obtain the squeezing and the high entanglement.[16−19] In this paper, we consider another problem. Usually the quantum fluctuation characteristic of an NOPA system is analysed under the assumption of monochromatic pumping; the signal and the idler beams may be treated as frequency degenerate but polarization non-degenerate. However, in NOPA experiments, a driving beam with finite bandwidth $\Delta\omega = 100$ kHz is used.[20] On account of that, we derive an analytical solution for the NOPA system with finite bandwdith laser pumping, and the quantum fluctuation of frequency non-degenerate beams is included. Finally, the application of the V_1 criterion to bipartite entanglement is discussed. In Section 2, we outline the formalism to be used. In Sections 3 and 4, we evaluate squeezing $4\langle(\Delta y_1)^2\rangle$ and quantum fluctuation variances $V_1$and $\tilde{V}_{1\mathrm{A}}$ for bipartite entanglements of the beams coming out of the NOPA system. The physical explication and the discussion are giving in Section 5.

2. Solutions for the NOPA with finite bandwidth pumping

Following the previous paper, we start by solving the semi-classical equations describing the interaction between the pump wave α_0 and two degenerate parametric modes α_1 and α_2[13−19]

$$\begin{aligned}
&\frac{\mathrm{d}\alpha_1}{\mathrm{d}\tau} = -\alpha_1 + \frac{\chi}{\gamma_1}\alpha_0\alpha_1^*,\\
&\frac{\mathrm{d}\alpha_2}{\mathrm{d}\tau} = -\alpha_2 - \frac{\chi}{\gamma_1}\alpha_0\alpha_2^*,\\
&\frac{\mathrm{d}\alpha_0}{\mathrm{d}\tau} = -\gamma_{\mathrm{r}}\alpha_0 + \frac{\bar{\varepsilon}}{\gamma_1} - \frac{\chi}{2\gamma_1}(\alpha_1^2 - \alpha_2^2),\\
&\bar{\varepsilon} = \bar{\varepsilon}_0 F,\\
&F = \frac{1}{2\Delta\omega}\int_{-\Delta\omega}^{\Delta\omega}\cos[\Delta\omega'\tau]\,\mathrm{d}\Delta\omega' = \frac{\sin[\Delta\omega\tau]}{\Delta\omega\tau},\\
&\Delta\omega' = \omega - \omega_0,
\end{aligned} \tag{1}$$

where $\tau = \gamma_1 t$ is the dimensionless time, χ is the cou-

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pling constant for the nonlinear coupling between the modes, γ_0 and γ_1 are the damping rates, the decay ratio is $\gamma_{\rm r} = \gamma_0/\gamma_1$, $\bar{\varepsilon}_0/\gamma_1 = \mu\sqrt{\gamma_{\rm r}/2\eta}$, $\bar{\varepsilon}/\gamma_1 = \bar{\varepsilon}_0 F/\gamma_1$, and F is the integral of the pumping around the central frequency ω_0 with half bandwidth $\Delta\omega$. Frequencies $\Delta\omega$, ω, and ω_0 are in the unit of damping rate γ_1. In the limit of $\Delta\omega \to 0$, $F \to 1$, $\bar{\varepsilon} = \bar{\varepsilon}_0$, it is just the case studied in our previous papers. Write each of the complex numbers α_1 and α_2 into the sum of real and imaginary parts, i.e., $\alpha_1 \to \alpha_1 + {\rm i}\tilde{\alpha}_1$, $\alpha_2 \to \alpha_2 + {\rm i}\tilde{\alpha}_2$. The numerical solutions of Eq. (1) yield the dependences of parametric functions α_1, $\tilde{\alpha}_1$, α_2, and $\tilde{\alpha}_2$ on time

$$\begin{aligned}
\alpha_1(t) &= \exp\left[-\int_0^t (k-\varepsilon)\,{\rm d}t'\right] = \tilde{\alpha}_2(t),\\
\tilde{\alpha}_1(t) &= \exp\left[-\int_0^t (k+\varepsilon)\,{\rm d}t'\right]\\
&= \exp\left[-2\int_0^t k\,{\rm d}t'\right]\alpha_1^{-1}(t) = \alpha_2(t), \quad (2)
\end{aligned}$$

where k is the loss, and ε is the gain.

Let Δx_1 and Δy_1 be the real and imaginary parts of the variance of the quantum fluctuation, we have

$$\begin{aligned}
\langle(\Delta x_1)^2\rangle &= \frac{1}{4} + (A_1 - A_{10}\alpha_1^2(t)),\\
\langle(\Delta y_1)^2\rangle &= \frac{1}{4} + (\tilde{A}_1 - \tilde{A}_{10}\tilde{\alpha}_1^2(t)), \quad (3)
\end{aligned}$$

where

$$\begin{aligned}
A_1 &= A_{10}\alpha_1^2(t) + \frac{\alpha_1^2(t)}{4\alpha_1^2(t-t_0)}\\
&\quad -\frac{1}{4} + \frac{\alpha_1^2(t)k}{2}\int_{t-t_0}^t \alpha_1^{-2}(t')\,{\rm d}t',\\
\tilde{A}_1 &= \tilde{A}_{10}\tilde{\alpha}_1^2(t) + \frac{\tilde{\alpha}_1^2(t)}{4\tilde{\alpha}_1^2(t-t_0)}\\
&\quad -\frac{1}{4} + \frac{\tilde{\alpha}_1^2(t)k}{2}\int_{t-t_0}^t \tilde{\alpha}_1^{-2}(t')\,{\rm d}t',\\
A_{10} &= \mu/4(1-\mu), \tilde{A}_{10} = \mu/4(1+\mu). \quad (4)
\end{aligned}$$

In Eq. (4), t_0 denotes the time needed by the parametric signal to pass through the amplifier, and the last integral term contains the noise accumulated in the diffusion process. Similarly, Δx_2 and Δy_2 for the variance of the quantum fluctuation read

$$\begin{aligned}
\langle(\Delta x_2)^2\rangle &= \frac{1}{4} + (A_2 - A_{20}\alpha_2^2(t)),\\
\langle(\Delta y_2)^2\rangle &= \frac{1}{4} + (\tilde{A}_2 - \tilde{A}_{20}\tilde{\alpha}_2^2(t)), \quad (5)
\end{aligned}$$

where

$$\begin{aligned}
A_2 &= A_{20}\alpha_2^2(t) + \frac{\alpha_2^2(t)}{4\alpha_2^2(t-t_0)}\\
&\quad -\frac{1}{4} + \frac{\alpha_2^2(t)k}{2}\int_{t-t_0}^t \alpha_2^{-2}(t')\,{\rm d}t',\\
\tilde{A}_2 &= \tilde{A}_{20}\tilde{\alpha}_2^2(t) + \frac{\tilde{\alpha}_2^2(t)}{4\tilde{\alpha}_2^2(t-t_0)}\\
&\quad -\frac{1}{4} + \frac{\tilde{\alpha}_2^2(t)k}{2}\int_{t-t_0}^t \tilde{\alpha}_2^{-2}(t')\,{\rm d}t',\\
A_{20} &= -\mu/4(1+\mu), \tilde{A}_{20} = -\mu/4(1-\mu). \quad (6)
\end{aligned}$$

Furthermore, the quantum fluctuation of the time-dependent linearly driven NOPA system is

$$\begin{aligned}
\langle(\Delta X_1)^2\rangle &= \frac{1}{2}(4\langle(\Delta x_1)^2\rangle + 4\langle(\Delta x_2)^2\rangle)\\
&= 1 + 2(A_1 - A_{10}\alpha_1^2(t)) + 2(A_2 - A_{20}\alpha_2^2(t)),\\
\langle(\Delta Y_2)^2\rangle &= \frac{1}{2}(4\langle(\Delta y_1)^2\rangle + 4\langle(\Delta y_2)^2\rangle)\\
&= 1 + 2(\tilde{A}_1 - \tilde{A}_{10}\tilde{\alpha}_1^2(t))\\
&\quad + 2(\tilde{A}_2 - \tilde{A}_{20}\tilde{\alpha}_2^2(t)), \quad (7)
\end{aligned}$$

where factor 1/2 is introduced to manifest the probabilities of the system, half in the in phase state and half in the out of phase state. Substituting Eqs. (4) and (6) into Eq. (7) yields the quantum fluctuation of the time-dependent linearly driven NOPA system

$$\begin{aligned}
\langle(\Delta X_1)^2\rangle &= \frac{\alpha_{1{\rm n}}^2(t)}{2} + \frac{\alpha_{2{\rm n}}^2(t)}{2}\\
&\quad + k\alpha_1^2(t)\int_{t-t_0}^t \alpha_1^{-2}(t')\,{\rm d}t'\\
&\quad + k\alpha_2^2(t)\int_{t-t_0}^t \alpha_2^{-2}(t')\,{\rm d}t',\\
\langle(\Delta Y_2)^2\rangle &= \frac{\tilde{\alpha}_{1{\rm n}}^2(t)}{2} + \frac{\tilde{\alpha}_{2{\rm n}}^2(t)}{2}\\
&\quad + k\tilde{\alpha}_1^2(t)\int_{t-t_0}^t \tilde{\alpha}_1^{-2}(t')\,{\rm d}t'\\
&\quad + k\tilde{\alpha}_2^2(t)\int_{t-t_0}^t \tilde{\alpha}_2^{-2}(t')\,{\rm d}t', \quad (8)
\end{aligned}$$

where, for simplification, we adopt the normalized parametric functions

$$\begin{aligned}
\alpha_{1{\rm n}}(t) &= \frac{\alpha_1(t)}{\alpha_1(t-t_0)}, \quad \tilde{\alpha}_{1{\rm n}}(t) = \frac{\tilde{\alpha}_1(t)}{\tilde{\alpha}_1(t-t_0)},\\
\alpha_{2{\rm n}}(t) &= \frac{\alpha_2(t)}{\alpha_2(t-t_0)}, \quad \tilde{\alpha}_{2{\rm n}}(t) = \frac{\tilde{\alpha}_2(t)}{\tilde{\alpha}_2(t-t_0)}. \quad (9)
\end{aligned}$$

The correlation between X_1 and Y_2 satisfies

$$\begin{aligned}
\langle\Delta X_1\Delta Y_2\rangle &= \frac{\alpha_{1{\rm n}}(t)\tilde{\alpha}_{2{\rm n}}(t) - \tilde{\alpha}_{1{\rm n}}(t)\alpha_{2{\rm n}}(t)}{2}\\
&\quad + k\alpha_1^2(t)\int_{t-t_0}^t \alpha_1^{-2}(t')\,{\rm d}t'\\
&\quad - k\tilde{\alpha}_1^2(t)\int_{t-t_0}^t \tilde{\alpha}_1^{-2}(t')\,{\rm d}t'. \quad (10)
\end{aligned}$$

Substituting the above results into the formula and evaluating variance V_1,[21] we have

$$V_1 = \langle(\Delta X_1)^2\rangle - \frac{\langle\Delta X_1 \Delta Y_2\rangle^2}{\langle(\Delta Y_2)^2\rangle}. \quad (11)$$

The above formula belongs to the frequency degenerate and polarization non-degenerate solution.

In the case of the frequency non-degenerate, the two modes with frequencies ω_1, ω_2 satisfy the condition $\omega_1 + \omega_2 = \omega_0$, where ω_0 is the central frequency of the pump wave. We assume the signal parametric wave with detuning $\Delta\omega$ relative to frequency $\omega_0/2$, namely, $\omega_1 = \omega_0/2 + \Delta\omega$ and $\omega_2 = \omega_0/2 - \Delta\omega$, then variance $\bar{V}_{1\mathrm{A}}$ for the frequency non-degenerate optical parametric amplifier is[17−19,22]

$$\bar{V}_{1\mathrm{A}} = \frac{V_1}{1 - \sin^2(2\varphi)\dfrac{\langle\Delta X_1 \Delta Y_2\rangle^2}{\langle(\Delta X_1)^2\rangle^2}}, \quad \varphi = \Delta\omega\tau. \quad (12)$$

3. Squeezing

The quantum fluctuation of the time-dependent linearly driven degenerate optical parametric amplification (DOPA) system is[17−19]

$$\langle(\Delta y_1)^2\rangle = \frac{1}{4} + (\tilde{A}_1 - \tilde{A}_{10}\tilde{\alpha}_1^2), \quad (13)$$

where

$$\tilde{A}_1 = \left(\tilde{A}_{10} + \frac{1}{4}\right)\tilde{\alpha}_1^2(t) - \frac{1}{4} + \frac{\tilde{\alpha}_1^2(t)k}{2}\int_0^t \tilde{\alpha}_1^{-2}(t')\mathrm{d}t'.$$

Figure 1 depicts squeezing $4\langle(\Delta y_1)^2\rangle$ as a function of time τ, where the parameters are $\mu = 2$, $\eta = 1/1000$, $\gamma_{\mathrm{r}} = 10$, $\Delta\omega = 2$, $\tau_0 = kt_0 = 3$, and the initial values are $\alpha_{00} = 0$, $\alpha_{10} = \alpha_{20} = 1$. Above the threshold, the squeezing curve is made of two phases, the short-time behavior is close to the linearization approximation $1/(1+\mu)$, whereas with the increase of τ, the long-time behavior is close to the vacuum fluctuation 1, which is quite different from the linear theorem.

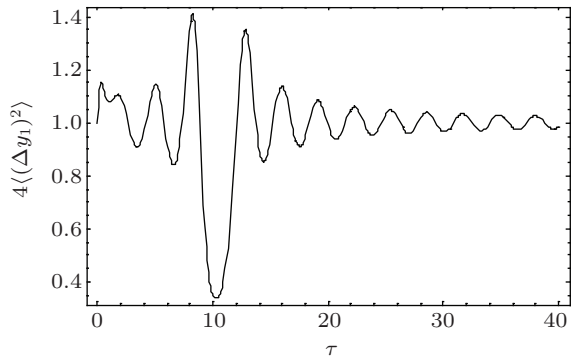


Fig. 1. Squeezing $4\langle(\Delta y_1)^2\rangle$ versus τ.

4. Quantum fluctuation variances

In order to evaluate quantum fluctuation variances V_1 and $\bar{V}_{1\mathrm{A}}$ numerically, we write the bandwidth function defined in Eq. (1) as $F = \sin[\Delta\omega(\tau - 10)]/(\Delta\omega(\tau - 10))$, namely, the origin of τ has been shifted from 0 to 10. Figures 2–4 show the results of variances V_1 and $\bar{V}_{1\mathrm{A}}$ from below the threshold to above the threshold respectively. In Figs. 2–4, panels (a) and (b) show variances V_1 and $\bar{V}_{1\mathrm{A}}$ versus τ, respectively. The bandwidths of the four curves in each diagram are $\Delta\omega = 0.2$ (dashed-dotted curve), $\Delta\omega = 0.5$ (wide-dashed curve), $\Delta\omega = 1$ (narrow-dashed curve), and $\Delta\omega = 2$ (solid curve).

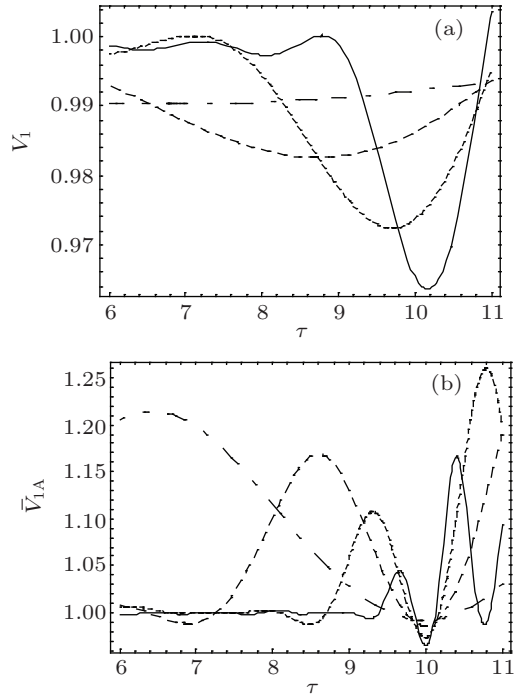


Fig. 2. Variances (a) V_1 and (b) $\bar{V}_{1\mathrm{A}}$ versus τ, with $\mu = 0.5$.

We now investigate the influence of the bandwidth of the pump beam on the variances. When the frequency non-degenerate is neglected, the broadband width has the effect of dropping on the minimum variance V_1 compared with the vaccum fluctuation variance $V_{\mathrm{vac}} = 1$, as shown in Fig. 2(a), where the minimun variances $V_1 = 0.99, 0.982, 0.972, 0.964$ correspond to $\Delta\omega = 0.2, 0.5, 1, 2$, respectively. The low variance data shrink to a narrow region. When the frequency non-degenerate is included, the minimum variance V_1 reduces, at the same time, the low variance data shrink to a narrow region, and the region is enhanced enormously, as shown in Fig. 2(b). With these points in mind, we observe the curves in Figs. 3 and

4 for $\mu = 1, 2$. Although the tendencies are the same, the minimum variance V_1 reduces more and more, the low variance data shrinking to a narrow region occurs more frequently, and much more variance data lie above the vaccum fluctuation variance $V_{\rm vac} = 1$. This effect shows that in the case of above the threshold and broad bandwidth pump, the frequency non-degenerate deteriorates seriously the entanglement between the signal and the idler beams.

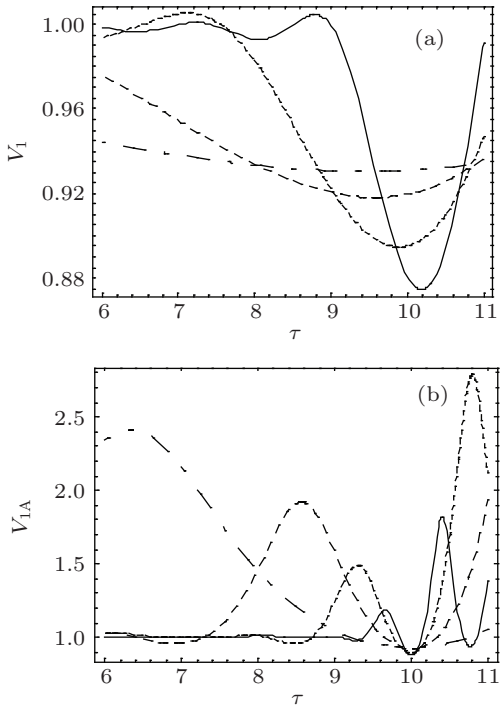


Fig. 3. Variances (a) V_1 and (b) $\bar{V}_{1A}$ versus τ, with $\mu = 1$.

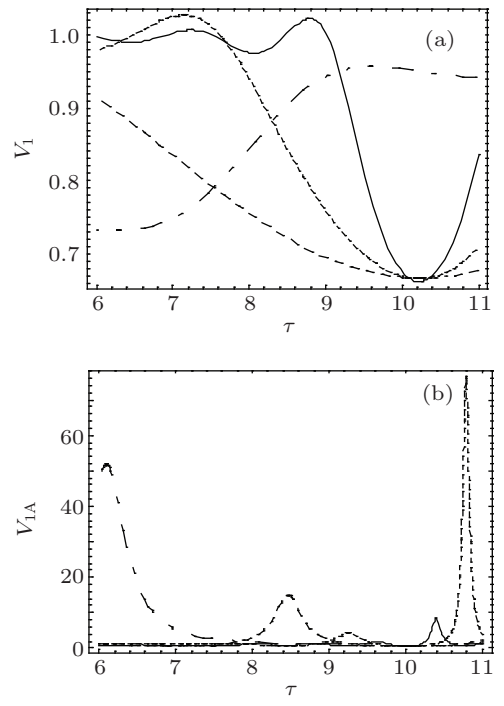


Fig. 4. Variances (a) V_1 and (b) $\bar{V}_{1A}$ versus τ, with $\mu = 2$.

5. Discussion and conclusion

From the analysis and numerical evaluation given in Sections 2–4, we realize that in experiments, the pumping beam's bandwidth should be controlled in a very narrow region, and the pump level should be kept below or near the threshold $\mu \leq 1$. Thus the frequency non-degenerate does not deteriorate seriously the bipartite entanglement. For example, we set $\gamma_1 = 10^6$ rad, $\Delta\omega = 0.2\gamma_1 = 2 \times 10^5$ rad, the bandwidth is $\Delta\omega/2\pi = 31$, which can be compared with $\Delta\omega = 100$ kHz in Ref. [23]. Figure 3(b) yields $V_1 \approx \bar{V}_{1A} = 0.94 < 1$.

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