A protocol to transform Svetlichny's genuine multipartite correlations into standard isotropic form*

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The standard isotropic correlations are widely used in the research of no-locality of quantum physics. We prove that any multipartite no-signaling correlation can be transformed into standard isotropic form through a randomization procedure, which does not change Svetlichny's genuine multipartite correlation. For the tripartite correlations, each part with two inputs and two outcomes, we explicitly give the protocol and the proof of its validity. We then generalize the protocol to deal with the case of an N-partite.

Keywords: Svetlichny's genuine multipartite correlation, isotropic form

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1. Introduction

Any classical correlation can be seen as a local hidden variable correlation, which means that the classical correlations come from a shared local hidden variable. Bell proved that all classical correlations cannot violate his inequality,[1] a more general version of Bell inequality for two qubits was given by Clauser, Horne, Shimony, and Holt[2,3] (CHSH inequality). Tsirelson[4] first proved that there exists an upper bound on the violation of Bell inequality for quantum correlation, which means that the amount of correlation allowed by quantum mechanics is limited. The Tsirelson bound of a Bell inequality violation raises an intriguing problem: what determines this upper bound? One may think that Tsirelson's bound is a consequence of the law of causation, but Popescu and Rohrlich[5] proved that quantum correlation is not the only no-signaling correlation that is stronger than classical correlations, and there exists a broad class of no-signaling correlations which even can breach Tsirelson's bound. For the convenience of investigation, one usually models all correlations as no-signaling boxes (NS-boxes),[6] so the measurement setting can be modeled as an input into the box and the measurement value can be modeled as an outcome from the box. An example of hypothetical no-signaling correlations is the Popescu and Rohrlich boxes (PR-boxes).[5] One pair of PR-boxes not only can violate the CHSH inequality to the algebraic maximum, but also has many intriguing properties of quantum theory, such as no cloning,[7,8] no broadcasting,[9] monogamy of correlations,[7,10] information-disturbance trade-offs,[11] and key distribution security.[12−15] Therefore, there is a strong demand to find some principles at the very root of quantum theory and use it to distinguish quantum correlation from all hypothetical no-signaling correlations. A lot of deep research has been done about this issue. In Refs. [16]–[19], the authors proved that some hypothetical no-signaling correlations can make distributed computational tasks implausibly simple. However, these distributed computational tasks are not trivial in quantum physics and it is believed that these tasks should not be trivial in the real physical world. Recently, Barnum *et al.*[20,21] showed that there is subtle relation between quantum correlation and quantum measurement assumption.

Pawłowski *et al.*[22] suggested an appealing information theoretical principle, information causality (IC), which states that communication of n classical bits causes information gain of at most n bits. When $n = 0$, IC is just the no-signaling principle. Pawłowski *et al.* proved that IC is obeyed in both the quantum and classical worlds, but all hypothetical correlations which can breach Tsirelson's bound would violate it. They even derived Tsirelson's bound from

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IC. It must be noted that IC cannot completely determine the boundary between quantum correlations and all hypothetical correlations. Recent research showed that there are some stronger-than-quantum correlations which are not known to violate IC.[23,24] By using IC, our recent work[25] extended the research of understanding quantum correlation to genuine multipartite correlations. Multipartite correlations have much richer structures than that of bipartite correlations.[6,26] In our work[25] we dealt only with Svetlichny genuine multipartite correlation, which is relevant to Svetlichny's inequality.[27,28] We proved that IC determines the quantum bound of Svetlichny genuine multipartite correlation. There has been some progress regarding using Svetlichny's inequality to detect genuine multipartite correlation.[29–34] In particular, in Refs. [30]–[32] the authors generalized Svetlichny's inequality to deal with N-partite d-dimensional systems (N-qudit).

However, as far as we know, almost all research on this issue only deals with correlations with standard isotropic form: the local outcomes are made uniformly random and the strength of correlations is independent of inputs. For the bipartite correlations, Masanes *et al.*[35] proved that any bipartite correlations can be brought to the standard isotropic form by using a suitable local randomization procedure which does not change the bipartite correlations, they provided the protocol. In the present work, we provide a local randomization procedure which can transform any multipartite correlations into standard isotropic form. We prove that this protocol does not change Svetlichny's genuine multipartite correlation.

2. A protocol to transform tripartite correlations into standard isotropic form

Consider three observers, Alice, Bob, and Carol, each of the three observers can choose to measure one of the two dichotomous observables. We denote $x \in \{0,1\}$ and $a \in \{0,1\}$ as Alice's measurement choice and outcome respectively, and similarly y and b (z and c) for Bob's (Carol's). The joint probability distribution for the outcomes, in the condition that the observables chosen by the three parties is $P(a,b,c|x,y,z)$. We request that the joint probability distribution satisfies the normalization conditions and no-signaling conditions. The Svetlichny inequality[25,27] can be expressed as

$$\frac{1}{8}\sum_{x,y,z} P(a\oplus b\oplus c = xy\oplus yz\oplus xz|x,y,z) \le \frac{3}{4}, \quad (1)$$

where $P(a\oplus b\oplus c = xy\oplus yz\oplus xz|x,y,z)$ is the probability that the sum (in this work all additions are addition modulo 2) of the outcomes of the three observers equal $xy\oplus yz\oplus xz$ in the condition that the observables chosen by the three parties are x,y,z, and it can be written in terms of $P(a,b,c|x,y,z)$ as

$$\begin{aligned} &P(a\oplus b\oplus c = xy\oplus yz\oplus xz|x,y,z) \\ &= \sum_{a\oplus b\oplus c = xy\oplus yz\oplus xz} P(a,b,c|x,y,z), \end{aligned} \quad (2)$$

where $\sum_{a\oplus b\oplus c = xy\oplus yz\oplus xz}$ is the sum of all different a, b, c in the condition $a\oplus b\oplus c = xy\oplus yz\oplus xz$. If all outcomes a,b,c are unbiased, and all entries of $P(a\oplus b\oplus c = xy\oplus yz\oplus xz|x,y,z)$ are equal, i.e., the tripartite no-signaling correlation can be written in a simple form: $P(a\oplus b\oplus c = xy\oplus yz\oplus xz|x,y,z) = (1+E)/2$ $(0 \le E \le 1)$, we call the tripartite no-signaling correlation standard isotropic correlation. There exists a randomization procedure which can be used to take every single tripartite no-signaling correlation to standard isotropic form and does not change the value of the Svetlichny operator (or say the value of $\sum_{x,y,z} P(a\oplus b\oplus c = xy\oplus yz\oplus xz|x,y,z)$). We explicitly give the randomization procedure and the proof of its validity in the following text.

In the tripartite case, we list the protocol as follows.

(I) Alice and Bob perform with probability 1/2 one of the following two operations: (i) nothing, and (ii) flip a and b.

(II) Alice and Carol perform with probability 1/2 one of the following two operations: (i) nothing, and (ii) flip a and c.

(III) Bob and Carol perform with probability 1/2 one of the following two operations: (i) nothing, and (ii) flip b and c.

(IV) With probability 1/8 all three parties perform one of the following eight operations: (i) nothing, (ii) flip x, $b_{y=1}$, and $c_{z=1}$, (iii) flip y, $a_{x=1}$, and $c_{z=1}$, (iv) flip z, $a_{x=1}$, and $b_{y=1}$, (v) flip x, y, $a_{x=0}$, and $b_{y=1}$, (vi) flip x, z, $a_{x=1}$, and $c_{z=0}$, (vii) flip y, z, $b_{y=0}$, and $c_{z=1}$, (viii) flip x, y, z, a, b, and c. Here, flipping $a_{x=1}$ means that only when $x=1$ Alice flips a, and it is just $a \to a\oplus x$; flipping $a_{x=0}$ is $a \to a\oplus x\oplus 1$.

Proof We assume the initial joint probability distribution for the outcomes, on the condition that the observables chosen by the three parties is

$P(a,b,c|x,y,z)$, after the first three steps this joint probability changes to $P'(a,b,c|x,y,z)$,

$$\begin{aligned}
&P'(a,b,c|x,y,z)\\
&=\frac{1}{4}[P(a,b,c|x,y,z)+P(a\oplus 1,b\oplus 1,c|x,y,z)\\
&\quad+P(a\oplus 1,b,c\oplus 1|x,y,z)\\
&\quad+P(a,b\oplus 1,c\oplus 1|x,y,z)]. \qquad (3)
\end{aligned}$$

From Eq. (3) we find that after the first three steps the outcomes are locally unbiased. After the fourth step the probability $P'(a,b,c|x,y,z)$ changes to $P''(a,b,c|x,y,z)$

$$\begin{aligned}
&P''(a,b,c|x,y,z)\\
&=\frac{1}{8}[P'(a,b,c|x,y,z)+P'(a,b\oplus y,c\oplus z|x\oplus 1,y,z)\\
&\quad+P'(a\oplus x,b,c\oplus z|x,y\oplus 1,z)\\
&\quad+P'(a\oplus x,b\oplus y,c|x,y,z\oplus 1)\\
&\quad+P'(a\oplus x\oplus 1,b\oplus y,c|x\oplus 1,y\oplus 1,z)\\
&\quad+P'(a\oplus x,b,c\oplus z\oplus 1|x\oplus 1,y,z\oplus 1)\\
&\quad+P'(a,b\oplus y\oplus 1,c\oplus z|x,y\oplus 1,z\oplus 1)\\
&\quad+P'(a\oplus 1,b\oplus 1,c\oplus 1|x\oplus 1,y\oplus 1,z\oplus 1)]. \qquad (4)
\end{aligned}$$

The final tripartite no-signaling correlation corresponding to $P''(a,b,c|x,y,z)$ is

$$\begin{aligned}
&P''(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)\\
&=\sum_{a\oplus b\oplus c=xy\oplus yz\oplus xz}P''(a,b,c|x,y,z)\\
&=\frac{1}{8}\sum_{a\oplus b\oplus c=xy\oplus yz\oplus xz}[P'(a,b,c|x,y,z)\\
&\quad+P'(a,b\oplus y,c\oplus z|x\oplus 1,y,z)\\
&\quad+P'(a\oplus x,b,c\oplus z|x,y\oplus 1,z)\\
&\quad+P'(a\oplus x,b\oplus y,c|x,y,z\oplus 1)\\
&\quad+P'(a\oplus x\oplus 1,b\oplus y,c|x\oplus 1,y\oplus 1,z)\\
&\quad+P'(a\oplus x,b,c\oplus z\oplus 1|x\oplus 1,y,z\oplus 1)\\
&\quad+P'(a,b\oplus y\oplus 1,c\oplus z|x,y\oplus 1,z\oplus 1)\\
&\quad+P'(a\oplus 1,b\oplus 1,c\oplus 1|x\oplus 1,y\oplus 1,z\oplus 1)]\\
&=\frac{1}{8}[P'(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)\\
&\quad+P'(a\oplus b\oplus c\\
&=(x\oplus 1)y\oplus yz\oplus(x\oplus 1)z|x\oplus 1,y,z)\\
&\quad+P'(a\oplus b\oplus c\\
&=x(y\oplus 1)\oplus(y\oplus 1)z\oplus xz|x,y\oplus 1,z)\\
&\quad+P'(a\oplus b\oplus c\\
&=xy\oplus y(z\oplus 1)\oplus x(z\oplus 1)|x,y,z\oplus 1)\\
&\quad+P'(a\oplus b\oplus c\\
&=(x\oplus 1)(y\oplus 1)\oplus(y\oplus 1)z\\
&\quad\oplus(x\oplus 1)z|x\oplus 1,y\oplus 1,z)+P'(a\oplus b\oplus c\\
&=(x\oplus 1)y\oplus y(z\oplus 1)\oplus(x\oplus 1)\\
&\quad\times(z\oplus 1)|x\oplus 1,y,z\oplus 1)+P'(a\oplus b\oplus c\\
&=x(y\oplus 1)\oplus(y\oplus 1)(z\oplus 1)\\
&\quad\oplus x(z\oplus 1)|x,y\oplus 1,z\oplus 1)+P'(a\oplus b\oplus c\\
&=(x\oplus 1)(y\oplus 1)\oplus(y\oplus 1)(z\oplus 1)\\
&\quad\oplus(x\oplus 1)(z\oplus 1)|x\oplus 1,y\oplus 1,z\oplus 1)], \qquad (5)
\end{aligned}$$

where $P'(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)$ is the tripartite no-signaling correlation corresponding to $P'(a,b,c|x,y,z)$, i.e., $P'(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)=\sum_{a\oplus b\oplus c=xy\oplus yz\oplus xz}P'(a,b,c|x,y,z)$. In the third equality in Eq. (5), we used

$$\begin{aligned}
&\sum_{a\oplus b\oplus c=xy\oplus yz\oplus xz}P'(a,b\oplus y,c\oplus z|x\oplus 1,y,z)\\
&=\sum_{a'\oplus b'\oplus c'=(x\oplus 1)y\oplus yz\oplus(x\oplus 1)z}P'(a',b',c'|x\oplus 1,y,z)\\
&=P'(a'\oplus b'\oplus c'=(x\oplus 1)y\oplus yz\\
&\quad\oplus(x\oplus 1)z|x\oplus 1,y,z)\\
&=P'(a\oplus b\oplus c=(x\oplus 1)y\oplus yz\\
&\quad\oplus(x\oplus 1)z|x\oplus 1,y,z), \qquad (6)
\end{aligned}$$

and similar for $P'(a\oplus b\oplus c=x(y\oplus 1)\oplus(y\oplus 1)z\oplus xz|x,y\oplus 1,z)$.

From Eq. (5) we find that the final tripartite no-signaling correlation $P''(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)$ is unbiased for x,y,z, and we also note that $P'(a,b,c|x,y,z)$ is unbiased for a,b,c, so we obtain a standard isotropic tripartite no-signaling correlation $P''(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)$.

We can also prove that all steps in the randomization procedure do not change the value of $\sum_{x,y,z}P(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)$.

$$\begin{aligned}
&\sum_{x,y,z}P''(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)\\
&=\sum_{x,y,z}\frac{1}{8}[P'(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)\\
&\quad+P'(a\oplus b\oplus c\\
&=(x\oplus 1)y\oplus yz\oplus(x\oplus 1)z|x\oplus 1,y,z)\\
&\quad+P'(a\oplus b\oplus c=x(y\oplus 1)\\
&\quad\oplus(y\oplus 1)z\oplus xz|x,y\oplus 1,z)\\
&\quad+P'(a\oplus b\oplus c=xy\oplus y(z\oplus 1)\\
&\quad\oplus x(z\oplus 1)|x,y,z\oplus 1)\\
&\quad+P'(a\oplus b\oplus c=(x\oplus 1)(y\oplus 1)\oplus(y\oplus 1)z\\
&\quad\oplus(x\oplus 1)z|x\oplus 1,y\oplus 1,z)
\end{aligned}$$

$$
\begin{aligned}
&+P'(a\oplus b\oplus c=(x\oplus 1)y\oplus y(z\oplus 1)\\
&\oplus(x\oplus 1)(z\oplus 1)|x\oplus 1,y,z\oplus 1)\\
&+P'(a\oplus b\oplus c=x(y\oplus 1)\oplus(y\oplus 1)(z\oplus 1)\\
&\oplus x(z\oplus 1)|x,y\oplus 1,z\oplus 1)\\
&+P'(a\oplus b\oplus c=(x\oplus 1)(y\oplus 1)\oplus(y\oplus 1)(z\oplus 1)\\
&\oplus(x\oplus 1)(z\oplus 1)|x\oplus 1,y\oplus 1,z\oplus 1)]\\
=&\sum_{x,y,z}P'(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z)\\
=&\sum_{x,y,z}\sum_{a\oplus b\oplus c=xy\oplus yz\oplus xz}\frac{1}{4}[P(a,b,c|x,y,z)\\
&+P(a\oplus 1,b\oplus 1,c|x,y,z)\\
&+P(a\oplus 1,b,c\oplus 1|x,y,z)\\
&+P(a,b\oplus 1,c\oplus 1|x,y,z)]\\
=&\sum_{x,y,z}P(a\oplus b\oplus c=xy\oplus yz\oplus xz|x,y,z). \qquad (7)
\end{aligned}
$$

Hence, the proof is completed. □

3. A protocol to transform N-partite correlations into standard isotropic form

It is obvious that the above protocol can be generalized to deal with N-partite correlations. Suppose there are N players who shared N particles; each of them performs dichotomous measurements on each of the N particles. The measurement settings are represented by x_1, x_2, ..., x_N respectively, with possible values $0, 1$. The measurement results are represented by a_1, a_2, ..., a_N respectively, and also with possible values $0, 1$. Then the N-particle Svetlichny inequality can be written as[25,28]

$$
\begin{aligned}
&\frac{1}{2^N}\sum_{\{x_i\}}P\Big(\sum_i^N a_i=\sum_{i<j\le N}x_ix_j|x_1,x_2,\ldots,x_N\Big)\\
\le&\ \frac{3}{4}, \qquad (8)
\end{aligned}
$$

where $\{x_i\}$ stands for an N-tuple $x_1,\ldots,x_N$, $\sum_i^N$ and $\sum_{i<j\le N}$ both denote summation modula 2, and P is the probability that $\sum_i^N a_i=\sum_{i<j\le N}x_ix_j$ with given $x_1,x_2,\ldots,x_N$. If all entries of $P\Big(\sum_i^N a_i=\sum_{i<j\le N}x_ix_j|x_1,x_2,\ldots,x_N\Big)$ are equal, and all outcomes a_1, a_2, ..., a_N are unbiased, we call the N-partite no-signaling correlation standard isotropic correlation.

In order to present the protocol which changes N-partite correlations into standard isotropic form, we first define two kinds of operations. All operations of the first kind can be marked as (i,j), where $i=1,2,\ldots,N$, $j=1,2,\ldots,N$ and $i\neq j$. The operation of (i,j) is that the i-th and the j-th players perform with probability $\frac{1}{2}$ one of the following two operations: (i) nothing, and (ii) flip a_i and a_j. The number of the operations of the first kind is $\binom{N}{2}$. All operations of the second kind can be marked as N-tuple $(k_1,\ldots,k_N)$, where $k_i\in\{0,1\}$ and its value determines whether the i-th player flip his input x_i. The operation of the N-tuple $(k_1,\ldots,k_N)$ is: if any k_i is equal to 1, the i-th player flips his input x_i, and the N players operate (flip or not flip) their outcomes a_1, a_2, ..., a_N locally in order that the operated inputs $\{x_i'\}$ and outcomes $\{a_i'\}$ fulfill the condition

$$
\sum_{i<j\le N}x_i'x_j'-\sum_{i<j\le N}x_ix_j=\sum_{i=1}^N a_i'-\sum_{i=1}^N a_i. \qquad (9)
$$

For any N-tuple $(k_1,\ldots,k_N)$ we can always find the locally operation on the outcomes $\{a_i\}$ such that the condition of Eq. (9) can be fulfilled, such as that in the step IV in the protocol for tripartite correlations. The number of the operations of the second kind is 2^N.

Now we present the protocol for N-partite correlations as follows.

(I) Perform all operations of the first kind in turn, there are altogether $\binom{N}{2}$ steps.

(II) Perform with probability $1/2^N$ one of the operations of the second kind, it is the last step.

We find that after the first $\binom{N}{2}$ steps the outcomes are locally unbiased, and the last step makes all entries of $P\Big(\sum_i^N a_i=\sum_{i<j\le N}x_ix_j|x_1,x_2,\ldots,x_N\Big)$ equal. At the same time, all the $\binom{N}{2}+1$ steps are intrinsically local operations and do not change the value of Svetlichny operator

$$
\sum_{\{x_i\}}P\Big(\sum_i^N a_i=\sum_{i<j\le N}x_ix_j|x_1,x_2,\ldots,x_N\Big).
$$

Indeed there exists a hidden variable model which can implement the protocol. Consider that every pair among the N players share one hidden variable which takes the value of 0 or 1 with probability 1/2, respectively, there are $\binom{N}{2}$ hidden variables of this kind which can be used to help perform the first $\binom{N}{2}$ steps. There still exists one hidden variable, which is shared by all N players, the hidden variable is just the N-tuple $(k_1,\ldots,k_N)$ and can be used to help performing the last step.

4. Conclusion

The standard isotropic correlations are widely used in the research of no-locality of quantum physics. We prove that any multipartite no-signaling correlation can be transformed into standard isotropic form through a randomization procedure, which does not change Svetlichny's genuine multipartite correlation. For the tripartite correlation, every part with two inputs and two outcomes, we explicitly give the protocol and the proof of its validity. We then generalize the protocol to deal with the case of N-partite. This protocol expands the range of validity of the results of our recent work.[25] We think that this protocol can also be generalized to deal with the N-qudit case, and this may be an interesting problem for future work.

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