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On the role of the uncertainty principle in superconductivity and superfluidity

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We discuss the general interplay between the uncertainty principle and the onset of dissipationless transport phenomena such as superconductivity and superfluidity. We argue that these phenomena are possible because of the robustness of many-body quantum states with respect to the external environment, which is directly related to the uncertainty principle as applied to coordinates and momenta of the carriers. In the case of superconductors, this implies relationships between macroscopic quantities such as critical temperature and critical magnetic field, and microscopic quantities such as the amount of spatial squeezing of a Cooper pair and its correlation time. In the case of ultracold atomic Fermi gases, this should be paralleled by a connection between the critical temperature for the onset of superfluidity and the corresponding critical velocity. Tests of this conjecture are finally sketched with particular regard to the understanding of the behaviour of superconductors under external pressures or mesoscopic superconductors, and the possibility to mimic these effects in ultracold atomic Fermi gases using Feshbach resonances and atomic squeezed states.

Keywords: superconductivity, superfluidity, uncertainty principle, squeezed states

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1. Introduction

Superconductivity and superfluidity phenomena are central to our understanding of many-body quantum systems and their ground state. Although they can be considered as long-range manifestations of quantum coherence, the paradigms used for their microscopic descriptions are quite different. In the case of superconductors, emphasis has been put on many-body, cooperative Cooper pairing assumed in BCS theory,[1] while in the case of superfluids the onset of Bose–Einstein condensation (BEC) in a real gas is considered as the key ingredient.[2,3] Due to the discovery of high-temperature superconductors[4] and the observation of Bose–Einstein condensation in dilute atomic gases,[5,6] there are now strong motivations to study the crossover between the BCS and the BEC descriptions of supertransport phenomena. High-temperature superconductors have shorter coherence lengths than the low-temperature ones, implying a more tight binding between individual electrons of the Cooper pairs supposed to be responsible for the current transport, and therefore closer to the Schafroth limit of a charged Bose–Einstein gas.[7] BEC of dilute gases is well described by the Bogoliubov theory, however the BEC of fermionic species may also occur in a regime quite close to the Schafroth limit, and the BEC–BCS crossover has been studied in ultracold Fermi gases.[8−12]

Although quantum-mechanical coherence has been proven crucial to understand the onset of superconductivity and the BEC phases, as experimentally shown by the Josephson effect in superconductors[13] and the interference fringes in atomic condensates,[14] there has been rare discussion, to our knowledge, over the role played by the quantum fluctuations in explaining the phase transition and more in general the spatial and temporal intrinsic robustness of many-body quantum phenomena. More specifically, the uncertainty principle has been applied in a macroscopic context to the superfluids[15] for a qualitative understanding of the third sound in thin helium films. The uncertainty principle has been also used[16] to provide an upper limit to the critical field in dirty-limit superconductors by relating the diffusivity and the slope of the upper critical magnetic field near the critical temperature. In Ref. [17], the uncertainty principle was applied to model residual superconducting effects in the normal state, while in Ref. [18] a phase-number uncertainty relationship was demonstrated for the super-

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current flowing through two Josephson tunnel junctions.

In this paper, we aim to discuss some semiqualitative implications of the uncertainty principle for both superconductors and dilute atomic gases. We develop a general picture of superconductors in which the key role is played by microscopic fluctuations in the center of mass position and the total momentum of the Cooper pairs. This allows us to better understand the rigidity of the many-body BCS state[19] in the position space. We then analyze the stability of the superconducting state in the presence of external magnetic fields or lattice disorder due to the finite environment temperature. This allows to introduce microscopic parameters directly related to the critical superconducting temperature and the critical magnetic field. The intentional modulation of microscopic fluctuations with external agents (e.g., external pressure) or mesoscopic surface effects leads to predictions which may also be tested unambiguously by means of ultracold atom gases, due to their pristine setting and easiness of controllability.

2. Position and momentum squeezing in superconductors

Let us first discuss the case of superconductors, in which the interaction responsible for the formation of Cooper pairs is always attractive, considering each electron of a Cooper pair with coordinates $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ as well as momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$. From the classical viewpoint, a system of charged particles will manifest time-reversal symmetry, i.e., reversible motions with no dissipation, if the motions are such that the magnetic field is zero in the entire region where the particles move.[20−22] This is for instance possible if a unique frame of reference exists, in which all Cooper pairs are at rest, with the only allowed motion internal to the Cooper pair with $\boldsymbol{p}_1 = -\boldsymbol{p}_2$, and any intrinsic magnetic field such as the one due to the spin, summing up to zero. Under Galileian invariance, this state should also be reversible in any other frames of reference. However, the Galileian invariance is an approximate one and Lorentz transformations give rise, in any frame of reference different from the rest frame, to the emergence of a magnetic field, whatever small is the velocity of the electric charges. Therefore, the only time-reversed state for a system of electric charges in relativistic classical physics is the one with zero overall velocity, i.e., with no associated electric current. In other words, a dissipationless, superconducting state cannot occur in classical physics because of the immediate appearance of magnetic fields. For non-zero velocities of the electric charges, the time-reversal invariance will be destroyed.

The situation becomes a bit more robust if we discuss the time-reversal invariance in quantum mechanics. Let us introduce commutation relationships for position and momentum operators as $[\hat{r}_1, \hat{p}_1] = [\hat{r}_2, \hat{p}_2] = \mathrm{i}\hbar$, the center of mass operator $\hat{R} = (\hat{r}_1 + \hat{r}_2)/2$, and the momentum operator $\hat{P} = \hat{p}_1 + \hat{p}_2$ of the Cooper pair. Their corresponding commutation relationship is simply written as

$$[\hat{R}, \hat{P}] = [\hat{r}_1 + \hat{r}_2, \hat{p}_1 + \hat{p}_2]/2 = \mathrm{i}\hbar. \tag{1}$$

According to the usual interpretation of the commutation relationship, the simultaneous definition of the center of mass and the total momentum of the Cooper pair is forbidden. Therefore, two Cooper pairs having centers of mass R_i and R_j fluctuating by an amount smaller that the amount allowed by the uncertainty principle should be considered at rest. This picture enforces the rigidity of the superconducting wavefunction in configurational space. Likewise, the magnetic fields generated by a Cooper pair will have to be considered as compatible with a zero value if the electron momenta fluctuate by an amount smaller than the one allowed by the uncertainty principle. Rigidity of the wavefunction and zero magnetic field enforcing time-reversal invariance define the superconducting state at the dynamical, microscopic level. External magnetic fields and lattice temperatures may actually destroy such a ground state. A finite and uncompensated external magnetic field directly attempts to frustrate time-reversal invariance, while the lattice temperature may affect the uncertainty in the relative motion of the Cooper pairs. This will forbid the existence of a unique frame of reference in which all the centers of mass of the Cooper pairs are at rest, thereby spoiling the rigidity of the macroscopic wavefunction.

This simple picture developed in terms of microscopic fluctuations naturally suggests that position or momentum uncertainties could be squeezed, for instance using phonon modulated couplings,[23] thereby originating a variety of phase diagrams. To develop a semiquantitative approach, let us suppose that τ is the correlation time between two Cooper pairs, each having mass $m = 2m_\mathrm{e}$ and electric charge $q = 2e$ (m_e and e are the mass and the electric charge of an electron). This timescale can be interpreted as the reciprocal of a characteristic frequency related to the interaction between Cooper pairs: $\tau \simeq \omega^{-1}$, or as the virtual phonon lifetime in the BCS model. In doing this we assume

that the coupling between two generic nearby Cooper pairs is of the same order of magnitude as the one binding together two electrons in a Cooper pair. Let us also imagine that the position uncertainty is generally squeezed by a factor ρ with respect to the natural fluctuation lengthscale $\Delta R = (\hbar/2m\omega)^{1/2}$. Thus, the relative uncertainty of the centers of mass ΔR will be of the order of $\Delta R = (\hbar\tau/2m)^{1/2}\rho$. Let us now introduce the effective internal temperature associated to the centers of mass of the Cooper pairs, in the limit of a large number of Cooper pairs, as the average kinetic energy due to the average position fluctuations occurring during a correlation time τ

$$T_{\rm eff} = \frac{m}{2k_{\rm B}}\frac{\Delta R^2}{\tau^2} = \frac{\hbar}{4k_{\rm B}\tau}\rho^2. \tag{2}$$

If the lattice temperature T satisfies $T \ll T_{\rm eff}$, its destructive effect on the superconducting state will be negligible. On the contrary, the Cooper pairs will no longer have a common motion and a unique rigid wavefunction, leading to the usual normal conducting state. In this framework, the critical temperature between the superconducting and the normal state is the borderline situation in which the lattice temperature almost equals the effective temperature of the Cooper pairs ($T \simeq T_{\rm eff}$). This suggests that under some external influence, the position uncertainty of the Cooper pairs increases, which allows for more tolerance on the external environment and also a larger critical temperature. In addition, the increase of the position uncertainty will decrease the momenta uncertainty of the electrons. This will consequently affect the magnetic field fluctuations in the pair, making the Cooper pairs more vulnerable to external magnetic fields, and thus inducing a lower critical magnetic field. Through purely-dimensional considerations, a momentum uncertainty $\Delta P = \hbar/2\Delta R$ will create an uncertainty of the magnetic vector potential

$$\Delta A = \frac{\Delta P}{q} \simeq \frac{1}{q}\left(\frac{\hbar m}{2\tau}\right)^{1/2}\frac{1}{\rho}, \tag{3}$$

and a magnetic field uncertainty

$$\Delta B \simeq \frac{\Delta A}{\Delta R} = \frac{m}{q\tau}\frac{1}{\rho^2}. \tag{4}$$

The critical temperature $T_{\rm c}$ and the critical magnetic field $B_{\rm c}$ are then seen as the upper bounds to the effective temperature and the magnetic field uncertainty, respectively, i.e. $T_{\rm eff} \leq T_{\rm c}, \Delta B \leq B_{\rm c}$.

The ratio between the magnetic field uncertainty and the effective temperature of the Cooper pairs is independent of the correlation time τ and only depends upon fundamental constants and ρ, i.e.,

$$\frac{\Delta B}{T_{\rm eff}} = \frac{4k_{\rm B}m_{\rm e}}{e\hbar}\frac{1}{\rho^4} = \frac{3.1}{\rho^4}\ \ ({\rm T/K})\ . \tag{5}$$

This equation resembles the limit set by the Pauli paramagnetism in the normal state firstly discussed by Chandrasekhar[24] who derived the relationship $B_{\rm c}/T_{\rm c} \simeq 2.6$ T/K and by Clogston[25] who discussed an analogous relationship $B_{\rm c}/T_{\rm c} = 1.84$ T/K. However, in our framework, equation (5) should be conceived as a relationship where both maximum and minimal values are allowed, depending upon the squeezing factor ρ. This is consistent with experimental observations that violate the Clogston limit.[26–29]

The product of the magnetic field uncertainty and the effective temperature

$$\Delta B\ T_{\rm eff} = \frac{\hbar m_{\rm e}}{4k_{\rm B}e}\frac{1}{\tau^2} = \frac{10^{-23}}{\tau^2}\ \ ({\rm TK}), \tag{6}$$

is instead independent of the squeezing factor and expresses the correlation time τ in terms of macroscopic quantities alone. This is similar to Eq. (5) for the squeezing factor.

3. Confrontation between the squeezing model and experimental data

In order to make comparison between our phenomenological scenario and existing experimental data, we have plotted in Fig. 1 the critical magnetic field versus the transition temperature for various superconductors ranging from W ($T_{\rm c} = 1.54 \times 10^{-2}$ K) to $Tl_2Ca_2Ba_2Cu_3O_{10}$ ($T_{\rm c} = 125$ K). Besides the experimental data, we have also plotted the estimate of $B_{\rm c}$ versus $T_{\rm c}$ from Eq. (5) as a benchmark, with $B_{\rm c} = \Delta B$, $T_{\rm c} = T_{\rm eff}$, and a squeezing factor $\rho = 1$. In spite of a broad diversity of superconducting materials, it is evident that critical temperatures and critical magnetic fields are linearly correlated. A different offset is apparent, with a transition in the range of about 4–40 K. Based on Eq. (5), we can evaluate the squeezing factor to satisfy the condition $\Delta B/T_{\rm eff} = B_{\rm c}/T_{\rm c}$ if ΔB and $T_{\rm eff}$ are replaced by $B_{\rm c}$ and $T_{\rm c}$. The results are shown in Fig. 2(a). Due to the dependence upon the fourth power, a moderate range of values of

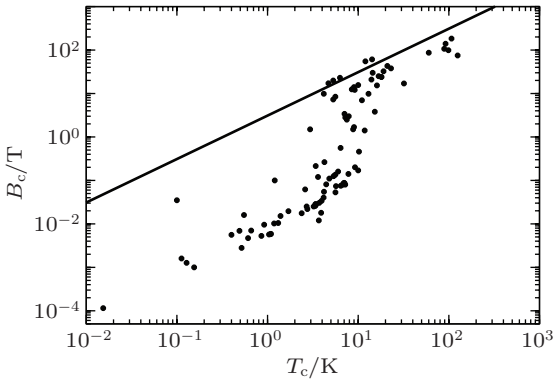


Fig. 1. Critical magnetic field versus transition temperature for various superconductors (dots), ranging from W ($T_c = 1.54\times10^{-2}$ K) to $Tl_2Ca_2Ba_2Cu_3O_10$ ($T_c = 125$ K). In the case of high-temperature superconductors, we have evaluated B_c as $B_c = (dB_c/dT)|_{T=T_c}T_c/2$. We also plot the expectation of Eq. (5) (line) with $B_c = \Delta B$, $T_c = T_{\rm eff}$, and no squeezing ($\rho = 1$).

the squeezing factor is required to fulfill the relationship (5). Even if the absolute value of ρ is uncertain due to the rough dimensional estimates, it is worth noting that it manifests a sort of sharp transition decreasing from an average value of $\rho \simeq 3$ for the superconductors with critical temperatures below 20 K to $\rho \simeq 1$ for superconductors with larger critical temperatures, confirming the trend in Fig. 1. The presence of regularities is also confirmed by evaluating the correlation time τ in Eq. (6) and then comparing it with the data, as shown in Fig. 2(b). We observe that high-temperature superconductors correspond to coupling strength $g \propto \omega \simeq \tau^{-1}$ larger than that of low-temperature superconductors.

If the energy gap for the superconductors is simply expressed in terms of the timescale τ as

$$E_{\rm g} = \frac{h}{\tau} = \frac{8\pi k_{\rm B} T_{\rm c}}{\rho^2}, \tag{7}$$

we can predict the zero-temperature superconducting energy gap for the same superconductors considered in Fig. 1, as shown in Fig. 3. Assuming an average value of $\rho \simeq 3$ for the low-temperature superconductors, we obtain $E_{\rm g} \simeq (8\pi/9)k_{\rm B}T_{\rm c} \simeq 2.8k_{\rm B}T_{\rm c}$, which is comparable to the BCS prediction $E_{\rm g} = 3.76k_{\rm B}T_{\rm c}$. For the high-temperature superconductors, the energy gap has to be scaled up by a factor of 9 with respect to the one of low-temperature superconductors, i.e., $E_{\rm g} \simeq 25k_{\rm B}T_{\rm c}$. The experimental values of the energy gap for high-temperature superconductors are anisotropic for values ranging from $4k_{\rm B}T_{\rm c}$ to $8k_{\rm B}T_{\rm c}$. The larger values predicted in our framework are not in contradiction with the experimental values. Due to

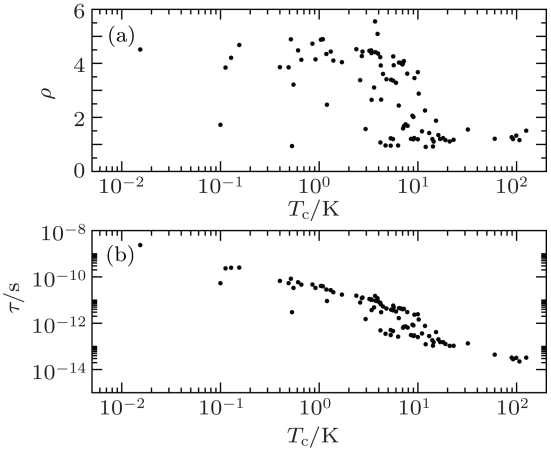


Fig. 2. (a) Squeezing factor ρ and (b) correlation time τ versus the critical temperature for the same superconductors considered in Fig. 1, obtained using Eqs. (5) and (6).

the Anderson effect,[30] the energy gap is highly sensitive to the presence of impurities. Therefore our estimate could be considered as an upper limit to the observed energy gaps in an impurity-free sample.

The proposed picture has also temporal implications by evaluating the ratio between the correlation time τ and the characteristic time of the thermal bath at the critical temperature $t_{\rm c} = \hbar/k_{\rm B}T_{\rm c}$ as $\xi = \tau/t_{\rm c} = \tau k_{\rm B}T_{\rm c}/\hbar = \rho^2/4$. According to the previous estimates, we have $\xi \simeq 2$ for low $T_{\rm c}$ and $\xi \simeq 1/4$ for high $T_{\rm c}$. This ratio provides an estimate to the question of how much the Cooper pairs live outside thermal equilibrium, since $t_{\rm c}$ represents the reaction time of the reservoir. From this perspective, it seems that the high-temperature superconductors live longer from thermal equilibrium than the low-temperature ones. If the Cooper pairs are forming and dissociating on the timescale τ this could be seen as a sort of many-body analogue of the dynamical suppression of

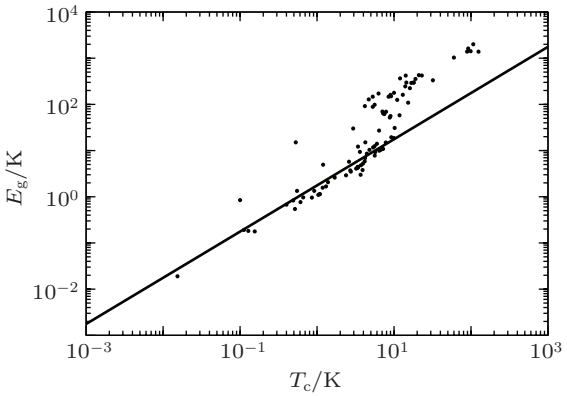


Fig. 3. Energy gap predicted by Eq. (7) (dots) versus the critical temperature for the same superconductors considered in Fig. 1. The solid line corresponds to the BCS prediction.

decoherence suggested in Refs. [31] and [32]. This also implies that non-equilibrium quantum statistical mechanics should play a more important role in the study of high-temperature superconductors than previously thought.

4. Modulating the phase diagram of superconductors and superfluids

The heuristic picture for superconductivity developed in the previous section lends itself to experimental confrontation due to the fact that the critical magnetic fields and critical temperatures are macroscopic parameters which are related to the amplitude of microscopic quantum fluctuations of the charge carriers.

In particular, squeezing of quantum fluctuations seems to be the key element to modulate the two macroscopic parameters determining the onset of superconductivity. We envisage systematic explorations of this idea in at least two different frameworks related to superconductivity. First, one may study the effect of external pressure on T_c and B_c. If the correlation time is not affected by the external pressure, we expect that the product $T_{\rm c}B_{\rm c}$ stays constant. Therefore one can trade a higher critical temperature with a lower critical magnetic field or viceversa. The current experimental situation is instead pointing to a different scenario, in which the external pressure is of hydrostatic nature, exerted uniformly in all directions. Since this changes in general the interaction coupling between the electrons, we instead expect that the squeezing factor remains constant and the correlation time changes, leading to a constant $B_{\rm c}/T_{\rm c}$ ratio. Measurements on aluminum, gallium, and thorium confirm this picture,[33] with agreement in the case of thorium extended up to pressures of 13 GPa.[34]. Second, we propose to study squeezing of quantum fluctuations in mesoscopic samples, to reinterpret the measurements of critical magnetic field and critical temperature in mesoscopic systems such as granular[35] and ultrathin[36] films, rings of various shapes,[37] and nanoparticles.[38,39] In a recent paper, the combination of the two effects, i.e., pressure and mesoscopic dependences of the critical superconducting parameters, has been evidenced by the discovery of high-pressure-induced superconductivity in boron nanowires.[40] A detailed analysis of all these experiments is beyond the reach of this paper. It is however worth noticing that all these mesoscopic systems share an extraordinary dependence of T_c and B_c on the surface-to-volume ratio, and their values strongly differ from the corresponding ones measured in the bulk.

In the case of a dilute Bose gas, the position uncertainty is easily discussed in an inhomogeneous trapping potential. For the non-interacting case, and assuming for simplicity harmonic and isotropic trapping, the position uncertainty is given by the variance of the Gaussian ground state, also called standard quantum limit (SQL):[41]

$$\Delta R^2 = \langle \hat{R}^2 \rangle - \langle \hat{R} \rangle^2 = \frac{\hbar}{m\omega} \equiv a_{\rm ho}^2, \tag{8}$$

where ω is the trapping angular frequency of the harmonic potential and m is the mass of a bosonic atom. In the interacting case, and in the Thomas–Fermi limit, the density distribution $n(r) = |\psi(r)|^2$, with $\psi(r)$ being the order parameter of the condensate, is expressed as:

$$n(r) = g^{-1}\left[\mu - \frac{1}{2}m\omega^2 r^2\right], \tag{9}$$

where $\mu = \hbar\omega/2(15Na/a_{\rm ho})^{2/5}$ is the chemical potential, N is the number of atoms, a is the s-wave scattering length (supposed to be positive) corresponding to a repulsive interaction, and g is a normalization factor. The calculation of the position uncertainty in this limit is straightforward

$$\Delta R^2 = \left(\frac{3}{5^{3/2}}\frac{Na}{a_{\rm ho}}\right)^{2/5} a_{\rm ho}^2. \tag{10}$$

Thus, the repulsive interaction leads to a position spreading larger than the one allowed by the confined potential for a non-interacting gas. This can be also interpreted by introducing a squeezing factor $\rho = (\Delta R^2/a_{\rm ho}^2)^{1/2}$ ($\rho > 1$), leading to a larger ground-state position uncertainty with respect to the so-called standard quantum limit $\Delta R^2 = a_{\rm ho}^2$, since the quantity Na/a_0 is larger than unity in the Thomas–Fermi regime. Note that a large position spread should lead to a narrower momentum distribution of the condensate according to the uncertainty principle. This has been recently observed by means of in-situ coherent Bragg velocimetry.[42] Moreover, working with atoms having negative elastic scattering lengths (either natural[43,44] or via a Feshbach resonance) should lead to a position uncertainty smaller than the standard quantum limit, while momentum squeezing for ultracold fermions has been recently achieved.[45]. Conversely, the momentum spreading in this case corresponds to the possibility of a non-zero critical velocity for the breakdown of superfluidity, which is analogous to the critical magnetic field in superconductors. Thus, we envisage two classes of possible experimental

tests. In the first one, the pairing correlation time between Cooper atoms may be changed isotropically by using magnetic Feshbach resonances. This maps a system of Fermi atoms onto the formerly analyzed case of superconductivity induced under high pressure in hydrostatic conditions at constant squeezing parameter ρ and variable correlation time τ. Then, we expect that the product of critical temperature and critical velocity will depend upon the amplitude of the scattering length near a Feshbach resonance.

Alternatively, in a second class of experiments one may induce anisotropies and variable squeezing parameter ρ at a constant correlation time. This is obtained changing the shape of the trapping potential experienced by the fermions. Most of the experiments also involve Bose species to allow for sympathetic cooling of the Fermi species, which may also provide an effective lattice useful for BCS-like coupling among the Fermi pairs. The experiments should allow for independent trapping of the Bose and Fermi species, which is obtainable through species-selective traps, as discussed in Refs. [46]–[48]. This will induce anisotropic critical temperatures and critical velocities for the Fermi superfluid. A proposal for the creation of anisotropic critical velocities in a Bose–Einstein condensate exploiting Raman-induced modulation of the spatial coupling has been discussed in Ref. [49].

5. Conclusions

In conclusion, we have investigated some consequences of the uncertainty principle at the level of phase space for the Cooper pairs in a superconductor, and discussed how the uncertainty principle offers a degree of protection from the environment.

This semiqualitative analysis may be considered as complementary to the well-known BCS view based on energy gap considerations, allowing to bridge fundamental microscopic quantum fluctuations to the existence of a region of stability for the superconducting state even in the presence of sources of disturbance due to the finite lattice temperature and external magnetic fields. This analysis may give further insights to the debated issue of the validity of the BCS approach and its universal applicability to all existing superconducting materials, also partially addressing the lack of intuitive criteria in the BCS model to understand qualitative trends of the critical temperature in materials.[50] Two microscopic parameters have been introduced to understand the phase diagram of a superconductor, the squeezing factor of quantum fluctuations and the correlation time between the centers of mass of the charge carriers. A smaller squeezing factor for the high-temperature superconductors is consistent with the observation that they have smaller coherence lengths than the low-temperature ones, therefore better approximating the Schafroth limit of individual pairs. Moreover, the smaller ratio between the lifetime of a Cooper pair and the characteristic timescale of the thermal bath for a high-temperature superconductor is evidence that they require a description in terms of non-equilibrium quantum statistical mechanics. We have highlighted implications both for the study of superconductors in the presence of high pressures or mesoscopic effects, and for experimental tests of these concepts at the level of ultracold Fermi atoms by creating squeezed states and observing the predicted simultaneous modulation of superfluid critical temperatures and velocities.

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