Exact solutions and localized excitations of a (3+1)-dimensional Gross–Pitaevskii system*

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Periodic wave solutions and solitary wave solutions to a generalized (3+1)-dimensional Gross–Pitaevskii equation with time-modulated dispersion, nonlinearity, and potential are derived in terms of an improved homogeneous balance principle and a mapping approach. These exact solutions exist under certain conditions via imposing suitable constraints on the functions describing dispersion, nonlinearity, and potential. The dynamics of the derived solutions can be manipulated by prescribing specific time-modulated dispersions, nonlinearities, and potentials. The results show that the periodic waves and solitary waves with novel behaviors are similar to the similaritons reported in other nonlinear systems.

Keywords: Gross–Pitaevskii system, mapping approach, exact solution, localized excitation

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1. Introduction

As is well known, many dynamical problems in physics and other fields of natural science are usually characterized by nonlinear partial differential equations (NLPDEs), known as governing equations. The analytical exact solutions of partial differential equations can model natural phenomena, and also can facilitate the testing of numerical solvers as well as the stability analysis.[1,2] Traditional studies of exact solutions and related localized excitations for NLPDEs are mainly focused on those nonlinear evolution equations where time-dependent variables do not appear explicitly.[3] However, many nonlinear systems with time-dependent nonlinearities and dispersions in reality attract great interest both from physicists and mathematicians, and become more and more absorbing.[4−6] Just as Towers and Malomed have reported, the specific dependencies of the equation coefficients on time-dependent variables can enhance the stabilities of the solutions to nonlinear systems.[7] Moreover, time-modulated nonlinearities and/or dispersions can facilitate the manipulation of the soliton behaviors. These facts have greatly enlarged our knowledge on nonlinear excitations and gave an origin to some important concepts such as nonautonomous soliton, which was first proposed by Serkin, Hasegawa, and Belyaeva.[8] In the context of many physical fields, the Bose–Einstein condensates (BECs) and nonlinear optics provide excellent proving grounds for exploring nonlinear systems with distributed coefficients. For example, the well-known Feschbach resonance has been used to control the nonlinearities of matter waves by manipulating the scattering length either in time or space, and has led to the proposal of many novel nonlinear phenomena.[9] Dispersion management for atomic matter waves has also been proposed recently, inducing plenty of consequent studies.[10] Moreover, the evolution of matter waves in time-dependent traps has been addressed, and the modulation instability of a one-dimensional BEC system in a time-dependent harmonic potential has been investigated.[11] In nonlinear optics, nonlinear management and dispersion management have also been widely used both experimentally and theoretically with temporal or spatial optical solitons, soliton lasers, and ultrafast soliton switches.[8] Furthermore, some recent progress on inhomogeneous nonlinear media has yielded novel con-

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cepts such as the optical similariton.[12,13]]

In this paper, by means of an extended mapping approach, we focus on deriving the analytical exact solutions to a generalized (3+1)-dimension Gross–Pitaevskii (GP) system with variable coefficients and study the solution's dynamics by prescribing specific nonlinear management, dispersion management, and time-modulated potentials.

2. Brief review on the mapping approach

In recent decades, many powerful approaches have been devised, such as inverse scattering theory, Bäcklund transformation, Hirota's bilinear method, Darboux transformation, the hyperbola function method, the mixed exponential method, the homogeneous balance method, and the multi-linear variable separation approach. Besides these methods, one can also obtain solutions of an NLPDE by using the technique of establishing and making full advantage of a direct mapping relation between the given NLPDE and another NLPDE, or sometimes its known solutions. For example, using the deformation mapping method first proposed by Lou and his coworkers,[14] some scholars[15−17] obtained many localized wave solutions and periodic wave solutions of nonlinear models through finding the relations between the exact solutions of the given models and those of the cubic nonlinear Klein–Gordon (NKG) system that has been studied extensively.[14] The basic idea of the algorithm is as follows. For a general nonlinear physical system

$$\boldsymbol{P}(\boldsymbol{v}) \equiv \boldsymbol{P}(x_0 = t, x_1, x_2, \ldots, x_n, \boldsymbol{v}, v_{x_i}, v_{x_i x_j}, \ldots), \quad (1)$$

where $\boldsymbol{v} = (v_1, v_2, \ldots, v_q)^{\mathrm{T}}$, $\boldsymbol{P}(\boldsymbol{v}) = (P_1(\boldsymbol{v}), P_2(\boldsymbol{v}), \ldots, P_q(\boldsymbol{v}))^{\mathrm{T}}$ are the polynomials of v_i $(i = 1, 2, \ldots, q)$, and $P_i(v)$ are their derivatives (T indicates the transposition of a matrix). By using a travelling wave transformation based on the following ansatz: $v_i = v_i(\xi)$, $\xi = \sum_{k=0}^{n} \lambda_k x_k$, where λ_k are real arbitrary constants, and substituting the ansatz into Eq. (1), we obtain an ordinary differential equation system. Then, $v_i(\xi)$ is expanded into a polynomial of $\phi(\xi)$ as

$$v_i = \sum_{j=0}^{N} \alpha_{ij} \phi^j(\xi), \quad \xi \equiv \sum_{k=0}^{n} \lambda_k x_k, \quad i = 1, 2, \ldots, q, \quad (2)$$

where α_{ij} are constants to be determined and N is fixed by balancing the linear term of the highest order with the nonlinear term in Eq. (1). If we suppose $\phi(\xi) = \tanh(\xi)$, $\phi(\xi) = \mathrm{sech}(\xi)$, and $\phi(\xi) = \mathrm{sn}(\xi)$ or $\phi(\xi) = \mathrm{cn}(\xi)$, respectively, then the corresponding approaches are usually called the tanh-function method, the sech-function method, and the Jacobian-function method. Although the elliptic Jacobian-function method is improved compared with the tanh-function method and the sech-function method, calculations are often tedious since different $\phi(\xi)$ functions should be treated repeatedly. The idea of the deformation mapping approach is that, $\phi(\xi)$ is not assumed to be a specific function such as tanh, sech, sn, and cn, but a solution of the mapping equation, such as the Riccati equation $\phi' = \sigma + \phi^2$ (σ is a constant and ϕ' denotes the differentiation of ϕ with respect to ξ), the cubic nonlinear KG equation $\phi_\xi^2 = c + \lambda\phi^2 + \mu\phi^4$ (c, λ, and μ are all arbitrary constants), and the elliptic Jacobian equation. Using the mapping relation (2) and the solutions of the related mapping equations, one can obtain many explicit and exact travelling wave solutions of system (1). In this way, some scholars further assume that the mapping function $\phi(\xi)$ is an exact excitation of the generalized elliptic Jacobian equation $\phi_\xi^2 = \sum_{j=0}^{4} c_j \phi^j$, where c_i are all arbitrary constants. Because the generalized elliptic Jacobian equation possesses more exact solutions,[18] by using it one may derive more travelling wave solutions of system (1) than previous ones.

Recently, the above mapping transformation approach is further extended for finding novel localized excitations of a physical model.[19−21] With the help of the mapping transformation idea and based on the general reduction theory, the mapping algorithm is extended. For the system (1) we assume its solution in an extended symmetric form

$$v_i = \sum_{j=-N}^{N} \alpha_{ij}(x) \phi^j(\omega(x)),$$
$$x \equiv (t, x_1, x_2, \ldots, x_n), \quad i = 1, 2, \ldots, q, \quad (3)$$

where $\alpha_{ij}(x)$ and $\omega(x)$ are arbitrary functions to be determined, and ϕ is a solution of the Riccati equation or the Jacobian elliptic equation. The parameter N is also determined by balancing the highest nonlinear terms and the highest-order partial terms in the given nonlinear system. Substituting the ansatz (3) together with the related mapping equations into Eq. (1), collecting coefficients of polynomials of ϕ, then setting each coefficient to zero yields a set of partial differential equations of $\alpha_{ij}(x)$ and $\omega(x)$. Solving the system of partial differential equations to obtain $\alpha_{ij}(x)$ and $\omega(x)$, substituting the derived results and the solutions of the related mapping equations into Eq. (3),

one can derive many exact solutions to the given nonlinear system. Based on the derived exact solutions, one can obtain many significant localized excitations. In general, the former travelling wave solutions are some special cases of the present extended mapping solutions.

3. Exact solutions to the generalized Gross–Pitaevskii equation

In the following, using the above extended mapping approach, we present exact analytical solutions describing solitary matter/optical waves in full three-dimensional space and focus on the generalized (3+1)-dimensional Gross–Pitaevskii equation[22]

$$\mathrm{i}\psi_t + \frac{1}{2}\rho(t)\boldsymbol{\nabla}^2\psi - g(t)|\psi|^2\psi - V(t)r^2\psi = 0, \quad (4)$$

where $\boldsymbol{\nabla}^2 \equiv \partial_{xx} + \partial_{yy} + \partial_{zz}$, $r^2 = x^2 + y^2 + z^2$, $\rho(t)$ is the time-modulated dispersion of a complex wave function $\psi(x, y, z, t)$, $g(t)$ is the nonlinearity, and $V(t)$ is the external harmonic trap potential in BECs. $V(t)$ is usually absent in the case of nonlinear optical transmission. These coefficients are often assumed to be real. The GP equation (4) describes the propagation of electromagnetic waves in nonlinear optical waveguides and the ground-state wave functions of BECs, which is actually one of the fundamental dynamical nonlinear models.[23,24]

First, we define the complex wave function ψ in Eq. (4) in terms of its amplitude and phase as in Refs. [25] and [26]

$$\psi(x, y, z, t) = u(x, y, z, t)\exp\left[\mathrm{i}v(x, y, z, t)\right]. \quad (5)$$

where $u(x, y, z, t)$ and $v(x, y, z, t)$ are real functions of the indicated arguments to be determined.

Substituting $\psi(x, y, z, t)$ into Eq. (4), we find the following coupled equations:

$$\begin{aligned} &u_t + \frac{1}{2}\rho u(v_{xx} + v_{yy} + v_{zz}) \\ &+ \rho(u_x v_x + u_y v_y + u_z v_z) = 0, \end{aligned} \quad (6)$$

$$\begin{aligned} &- uv_t + \frac{1}{2}\rho(u_{xx} + u_{yy} + u_{zz}) \\ &- \frac{1}{2}\rho u(v_x^2 + v_y^2 + v_z^2) - gu^3 \\ &- Vu(x^2 + y^2 + z^2) = 0. \end{aligned} \quad (7)$$

Now, we seek the exact analytical solutions to Eqs. (6) and (7). Suppose that

$$u = f_0(t) + f_1(t)F(\theta) + f_2(t)F^{-1}(\theta), \quad (8)$$

$$v = a(t)(x^2 + y^2 + z^2) + b(t)(x + y + z) + e(t), \quad (9)$$

$$\theta = k(t)x + l(t)y + m(t)z + w(t), \quad (10)$$

where f_0, f_1, f_2, k, l, m, w, a, b, and e are the parameter functions to be determined. The function $F(\theta)$ is an elliptic Jacobian function satisfying the following general first-order and second-order nonlinear ordinary differential equations:

$$\left(\frac{\mathrm{d}F}{\mathrm{d}\theta}\right)^2 = c_0 + c_2F^2 + c_4F^4, \quad (11)$$

$$\frac{\mathrm{d}^2F}{\mathrm{d}\theta^2} = c_2F + 2c_4F^3, \quad (12)$$

where c_0, c_2, and c_4 are real constants related to the elliptic modulus of elliptic Jacobian function $F(\theta)$ (see Table 1). If x^qF^n, y^qF^n, z^qF^n ($q = 0, 1, 2$; $n = 0, 1, \ldots, 6$), and $\sqrt{c_0 + c_2F^2 + c_4F^4}$ are separately equal to zero, substituting Eqs. (8)–(10) into Eqs. (6) and (7) leads to a system of algebraic or the first-order ordinary differential equations for f_p, k, l, m, w, a, b, and e as follows:

$$\frac{\mathrm{d}f_p}{\mathrm{d}t} + 3\rho a f_p = 0, \quad (13)$$

$$f_j\left(\frac{\mathrm{d}k}{\mathrm{d}t} + 2\rho a k\right) = 0, \quad (14)$$

$$f_j\left(\frac{\mathrm{d}l}{\mathrm{d}t} + 2\rho a l\right) = 0, \quad (15)$$

$$f_j\left(\frac{\mathrm{d}m}{\mathrm{d}t} + 2\rho a m\right) = 0, \quad (16)$$

$$f_j\left(\frac{\mathrm{d}w}{\mathrm{d}t} + \rho(k + l + m)b\right) = 0, \quad (17)$$

$$f_j\left(\frac{\mathrm{d}a}{\mathrm{d}t} + V + 2\rho a^2\right) = 0, \quad (18)$$

$$-6gf_0f_j^2 = 0, \quad (19)$$

$$\begin{aligned} &f_1\left(\frac{\mathrm{d}e}{\mathrm{d}t} - \frac{1}{2}c_2\rho(k^2 + l^2 + m^2)\right. \\ &\left. + \frac{3}{2}\rho b^2 + 3gf_0^2 + 3gf_1f_2\right) = 0, \end{aligned} \quad (20)$$

$$f_1(gf_1^2 - c_4\rho(k^2 + l^2 + m^2)) = 0, \quad (21)$$

$$f_1(gf_2^2 - c_0\rho(k^2 + l^2 + m^2)) = 0, \quad (22)$$

where $p = 0, 1, 2$ and $j = 1, 2$. The constants c_0, c_2, and c_4 in Eqs. (20)–(22) are related to the square of the elliptic modulus M of elliptic Jacobian function (see Table 1). By solving Eqs. (13)–(22) self-consistently, one can obtain a set of conditions on the coefficients and parameters for the GP system (4) to have exact periodic wave solutions. Here, we consider the most general case, in which f_1 and f_2 are nonzero and $a(t)$ is an arbitrary function of time, then we find the following set of exact solutions:

$$f_0 = 0, \quad f_1 = f_{10}\alpha^{3/2}, \quad f_2 = \epsilon\sqrt{\frac{c_0}{c_4}}f_1, \tag{23}$$

$$k = k_0\alpha, \quad l = l_0\alpha, \quad m = m_0\alpha,$$

$$w = w_0 - b_0(k_0 + l_0 + m_0)\int_0^t \rho\alpha^2\,\mathrm{d}t, \tag{24}$$

$$b = b_0\alpha,$$

$$e = e_0 + \frac{1}{2}[(c_2 - 6\epsilon\sqrt{c_0c_4}) \times (k_0^2 + l_0^2 + m_0^2) - 3b_0^2]\int_0^t \rho\alpha^2\,\mathrm{d}t, \tag{25}$$

$$V = -2\rho a^2 - \frac{\mathrm{d}a}{\mathrm{d}t}, \tag{26}$$

$$g = \frac{c_4(k_0^2 + l_0^2 + m_0^2)}{f_{10}}\rho\alpha^{-1}, \tag{27}$$

where $\alpha = \exp\left(-\int_0^t 2\rho a\mathrm{d}t\right)$, $\epsilon = 0, -1, 1$, Eqs. (26) and (27) are specific constrains on the dispersion ρ, and V is the potential to construct exact solutions ψ.

Table 1. Jacobi elliptic functions.

Solution	c_0	c_2	c_4	F	$M=0$	$M=1$
1	1	$-(1+M^2)$	M^2	sn	sin	tanh
2	$1-M^2$	$2M^2-1$	$-M^2$	cn	cos	sech
3	M^2-1	$2-M^2$	-1	dn	1	sech
4	M^2	$-(1+M^2)$	1	ns	cosec	coth
5	$-M^2$	$2M^2-1$	$1-M^2$	nc	sec	cosh
6	-1	$2-M^2$	M^2-1	nd	1	cosh
7	1	$2-M^2$	$1-M^2$	sc	tan	sinh
8	$1-M^2$	$2-M^2$	1	cs	cot	cosech
9	1	$-(1+M^2)$	M^2	cd	cos	1
10	M^2	$-1+M^2$	1	dc	sec	1

Finally, incorporating these solutions into Eq. (5), we obtain the general periodic travelling wave solutions to the generalized (3+1)-dimensional Gross–Pitaevskii equation:

$$\psi = f_{10}\alpha^{3/2}\left[F(\theta) + \epsilon\sqrt{\frac{c_0}{c_4}}F^{-1}(\theta)\right] \times \exp\{\mathrm{i}[a(x^2+y^2+z^2) + b(x+y+z) + e]\}, \tag{28}$$

where $\theta = \alpha(k_0x + l_0y + m_0z) + w_0 - b_0(k_0 + l_0 + m_0)\int_0^t \rho\alpha^2\,\mathrm{d}t$, and $F(\theta)$ is presented in Table 1. Obviously, by selecting different $F(\theta)$ functions, one can derive a large number of exact wave solutions such as periodic wave solutions, double periodic wave solutions, localized wave solutions.

4. Propagation characteristics of periodic waves

For simplicity, in the following discussion, we only analyze the periodic wave solutions ψ expressed by Eq. (28) and rewrite it in a simple form ($\epsilon = 0,\ c_0 = M^2 - 1,\ c_2 = 2 - M^2,\ c_4 = -1$) as follows:

$$\psi = f_{10}\alpha^{3/2}F(\theta)\exp\{\mathrm{i}[a(x^2+y^2+z^2) + b(x+y+z) + e]\}, \tag{29}$$

$$A \equiv |\psi|^2 = f_{10}^2\alpha^3\,\mathrm{dn}(kx + ly + mz + w, M)^2, \tag{30}$$

where M is the modulus of elliptic Jacobi function and A is the density of the complex wave function ψ.

It can be seen that in order to construct exact solutions ψ, the dispersion ρ, nonlinearity g, and potential V are not arbitrary, but interrelated through relations (26) and (27). However, we still have a large freedom to choose those functions to obtain meaningful physical solutions.

As the first example, we focus on the case of $V(t) = V_0\cos t$ and $\rho(t) = \rho_0\cos t$, where V_0 and ρ_0 are constants. Actually, this special case gives periodically time-modulated potential and diffraction. Substituting the expressions for V and ρ to Eq. (26) and solving the differential equation, we can obtain

$$a(t) = -\frac{\sqrt{2V_0\rho_0}}{2\rho_0}\tan[\sqrt{2V_0\rho_0}(\sin t + a_0)], \tag{31}$$

where a_0 is the integral constant. Consequently, the function α can be given by

$$\alpha = \frac{1}{\cos[\sqrt{2V_0\rho_0}(\sin t + a_0)]}, \tag{32}$$

by integrating $\alpha = \exp\left(-\int_0^t 2\rho a\mathrm{d}t\right)$.

Substituting Eq. (32) and the expression for $\rho(t)$ into Eq. (27), we can obtain the time-modulated nonlinearity $g(t)$

$$g(t) = -\frac{\rho_0(k_0^2+l_0^2+m_0^2)}{f_{10}^2}\cos t \times \cos\left[\sqrt{2V_0\rho_0}(\sin t + a_0)\right]. \quad (33)$$

It is important to mention that the constants V_0, ρ_0, and f_{10} should be selected carefully to avoid some singularities of the tan-type function.

Figure 1 shows the curves of α and g as a function of t. The function α has an important role in modulating the amplitude of the solution A. It is seen in this situation that α periodically oscillates along the t axis. Consequently, the corresponding A will be explicit breathing solitary wave solution, and its density $|\psi|^2$ depends on the propagation distance X and time t, as shown in Fig. 2 where $X \equiv kx + ly + mz$.

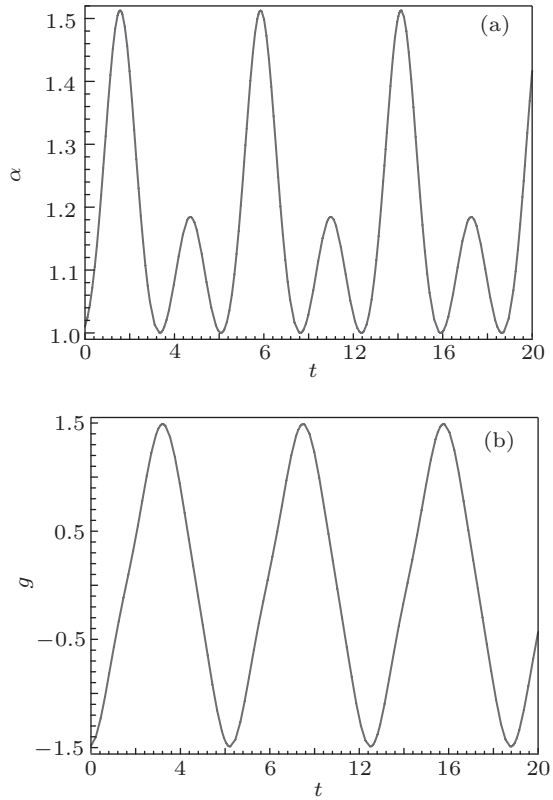


Fig. 1. (a) Function α (Eq. (32)) as a function of t. (b) Time-modulated nonlinearity g (Eq. (33)) as a function of t. $f_{10} = 0$, $k_0 = 1$, $l_0 = 1$, $m_0 = 1$, $\rho_0 = 1/2$, $V_0 = 1/2$, and $a_0 = 1/5$.

As the second example, we focus on the following case:

$$V(t) = -\frac{V_0(2\sin t + 2V_0 + V_0\cos^2 t)}{8(1+V_0^2+2V_0\sin t - V_0^2\cos^2 t)}, \quad \rho(t) = 1, \quad (34)$$

$$a(t) = -\frac{V_0\cos t}{4(1+V_0\sin t)}. \quad (35)$$

Substituting Eqs. (34) and (35) to α yields

$$\alpha = \sqrt{1+V_0\sin t}. \quad (36)$$

Then, we can obtain the nonlinear coefficient $g(t)$

$$g = -\frac{k_0^2+l_0^2+m_0^2}{f_{10}\sqrt{1+V_0\sin t}}. \quad (37)$$

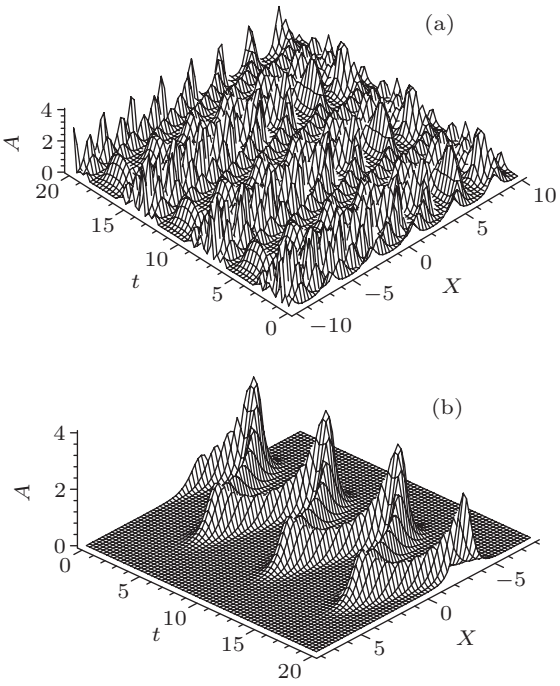


Fig. 2. Density A of ψ (Eq. (30)) as a function of the propagation distance X and t for (a) $M = 0.7$ and (b) $M \to 1$. $f_{10} = 0$, $k_0 = 1$, $l_0 = 1$, $m_0 = 1$, $w_0 = 0$, $\rho = (\cos t)/2$, $V = (\cos t)/2$, $b_0 = 1$, and $a_0 = 1/5$.

Figure 3 shows the profiles of α and g as a function of t. It is clearly seen that both functions are periodically time-modulated, and the nonlinearity g is always negative. The dynamics of the corresponding density $A \equiv |\psi|^2$ is shown in Fig. 4(a), from which we can see that the density of the localized wave has strong variations on its propagation and shows a snaking behavior, which reconfirms the previous results reported in Refs. [27] and [28] using other techniques. The variations are getting more and more significant as the wave propagates, and the amplitude of the density also oscillates. In Fig. 4(b), we plot the density $A \equiv |\psi|^2$ in a smaller time interval from 0 to 5 and can clearly see the snaking behavior of the density along the X axis and the amplitude's oscillation. The solutions can help understand the matter-wave dynamics in BECs

with the interaction strength and the external potential changing with time.

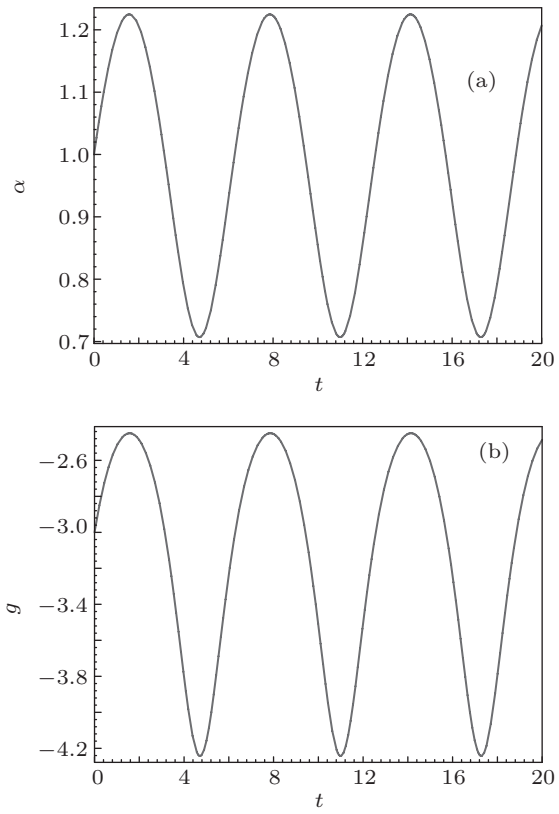


Fig. 3. (a) Function α (Eq. (36)) as a function of t. (b) Time-modulated nonlinearity g (Eq. (37)) as a function of t. $f_{10} = 0$, $k_0 = 1$, $l_0 = 1$, $m_0 = 1$, and $V_0 = 1/2$.

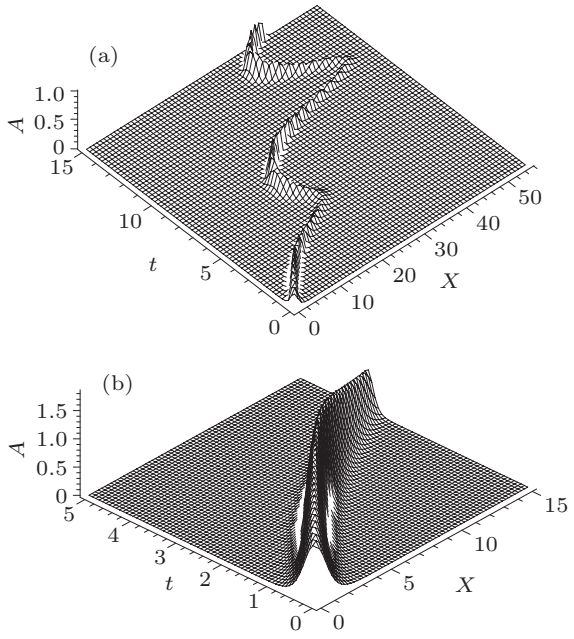


Fig. 4. (a) Density $A \equiv |\psi|^2$ (Eq. (30)) as a function of the propagation distance X and time t for $M \to 1$. (b) Drawing of partial enlargement of (a). $f_{10} = 0$, $k_0 = 1$, $l_0 = 1$, $m_0 = 1$, $V_0 = 1/2$, $b_0 = 1$, and $w_0 = 0$.

Naturally, by prescribing different $\rho(t)$ and $V(t)$, one can obtain different $a(t)$, $\alpha(t)$, and $g(t)$. Correspondingly, more novel dynamic behaviors of derived analytical solutions will be revealed.

5. Conclusion

In summary, an improved mapping technique is successfully applied to the generalized (3+1)-dimensional Gross–Pitaevskii equation with variable coefficients. Many exact analytical localized wave solutions are obtained. Such exact solutions exist under certain conditions by imposing constraints on the functions describing dispersion, nonlinearity, and potential function. The dynamics of the derived solutions can be manipulated by prescribing specific time-modulated dispersions, nonlinearities, and potentials. The results show that the periodic waves and localized waves have novel behaviors similar to the similaritons reported in other nonlinear systems. The present mapping approach provides a reliable technique that is more transparent and less tedious than the Lie point symmetry group analysis, variational methods, and other expansion approaches. The technique can also be applied to other multi-dimensional nonlinear partial differential equation systems.

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