Effects of information asymmetry on cooperation in the prisoners' dilemma game*

Wang Xi-Peng(王锡朋)[a], Jiang Luo-Luo(姜罗罗)[b)†], and Wang Bing-Hong(汪秉宏)[a)‡]

[a)] *Department of Modern Physics, University of Science and Technology of China, Hefei* 230026, *China*

[b)] *College of Physics and Electronic Information Engineering, Wenzhou University, Wenzhou* 325035, *China*

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Effects of information asymmetry on cooperation in the prisoners' dilemma game are investigated. The amplitude A is introduced to describe the degree of information asymmetry. It is found that there exists an optimal value of amplitude $A_{\rm opt}$ at which the fraction of cooperation reaches its maximal value. The reason lies in that cooperators on the two-dimensional grid form large clusters at $A_{\rm opt}$. In addition, the theoretical analysis in terms of the mean-field theory is used to understand this kind of phenomenon. It is confirmed that the information asymmetry plays an important role in the dynamics of the dilemma games of spatial prisoners.

Keywords: information asymmetry, cooperation, strategy

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1. Introduction

Evolutionary games have become a powerful tool to understand the emergence of cooperative behaviors among selfish individuals in natural and social systems.[1−4] The prisoners' dilemma game (PDG)[3] is one of the most widely-used games. In this game, two participants must simultaneously choose cooperation or defection, and receive a reward (R) if both choose cooperation and a punishment (P) if both choose defection. However, one cooperator will receive an S when encountering a defector, who conversely receives a T. The ranking of payoff for four cases is $T > P > R > S$. Obviously, to defect is the best choice regardless of the opponent's selection if participants aspire to maximize their payoff. This results in the disappearance of cooperators in the absence of supporting mechanisms. In many previous studies, several mechanisms that support the evolution of cooperation, such as repeated interactions,[5] spatial extensions,[6] memory effects,[7] social diversity,[8] partially random contacts,[9] strategic complexity,[10] success-driven migration,[11,12] reducing the heterogeneity of payoff,[13] effect of fluctuations,[14] diverse inertia,[15] heterogeneous aspirations,[16] aspiration-induced reconnection,[17] and impact of link deletions,[18] have been identified.

Besides the mechanisms mentioned above, Rasmusen[19] introduced information as the elementary factor in game theory. The information of participants in this paper refers to their strategy and payoff. Therefore, it is necessary to obtain neighbors' information when a participant determines to learn their behaviors. However, in real life, participants may not acquire their neighbors' actual information for various reasons. For example, some participants deliberately conceal or exaggerate their information, which is called information asymmetry.[20] Then, a question arises: how does the information asymmetry affect the cooperation? To answer this question, we introduce the strength of information asymmetry in the PDG to quantitatively study the effects of information asymmetry on cooperation. It is found that there exists an optimal asymmetry strength that can maximize the cooperation. We also present the explanation to the critical phenomenon. Furthermore, we find that the size of cooperation clusters plays a key role in the promotion of cooperation, and the average number of cooperation neighbors around cooperators (who work as defenders of cooperation clusters to avoid the invasion of defectors) is nearly constant as the asymmetry strength varies. In addition, the mean-field theory is applied to calculate the fraction

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†Corresponding author. E-mail: jiangluoluo@gmail.com

‡Corresponding author. E-mail: bhwang@ustc.edu.cn

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of cooperation.

2. Model

We investigate the PDG with participants located on the vertices of a two-dimensional square lattice with a size of $L \times L$ and periodic boundary conditions. Each individual is only allowed to interact with its four nearest neighbors, and self-interactions are excluded. As mentioned above, each participant can adopt two simple strategies: cooperation and defection, which are described as

$$\boldsymbol{\phi} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1}$$

Following Ref. [6], the elements of payoff matrix can be readjusted: $R = 1$, $P = S = 0$, and $T = b$ $(1 < b < 2)$, and can be expressed by a matrix

$$\boldsymbol{\psi} = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{2}$$

where the parameter b characterizes the temptation of defection against cooperation. The total payoff of the participant i is the sum payoff after the participant i interacts with its four neighbors, which can be written as

$$p_i = \sum_{j \in \Omega_i} \boldsymbol{\phi}_i^{\mathrm{T}} \boldsymbol{\psi} \boldsymbol{\phi}_j, \tag{3}$$

where Ω_i represents the four neighbors of participant i. In the classical PDG, a participant updates its strategy according to the following rules: the participant i plays the PDG with its four nearest neighbors, randomly selects a neighbor participant j, and makes its strategy with a probability of

$$G_{i \to j} = \frac{1}{1 + \exp[(p_i - p_j)/K]}, \tag{4}$$

where K characterizes the stochastic noise. For $K = 0$, the participant always adopt the best strategy determinately, while irrational changes are allowed for $K > 0$. As reported in previous study that a small K does not affect the qualitative results,[21] without loss of generality, the value K is fixed to be 0.1 in this paper.[22] From the probability depicted by Eq. (4), we can conclude that if the selected neighbor participant j possesses more payoff than the participant i, the participant i will adopt a strategy with a larger probability. Conversely, if the participant j possesses less payoff than the participant i, the probability will be smaller. It is worth noting that the parameter b imposes a significant effect on the cooperation process and as the parameter b increases, more and more participants will be defectors.

In Ref. [20], the author considers that the "corporate insider" who acquires more information about trading than others that leads to information asymmetry can result in "insider gains". Insider gains have a great influence on market economy. However, in real life, individuals of different statuses cannot obtain equal information. Therefore, information asymmetry is introduced in our model. Without loss of generality, we use the simple distribution (uniform distribution) of asymmetry strength to investigate the effect of information asymmetry. The uniform distribution of asymmetry strength are introduced as follows:

$$\xi = A\chi, \tag{5}$$

where χ is a uniformly distributed random number in the interval $[-1, 1]$, and $\int_{-1}^{1} \xi \mathrm{d}\chi = 0$ ensures that the average of ξ over all the groups is zero. Some participants have negative asymmetry strength and the others have positive asymmetry strength. The parameter A determines the amplitude of asymmetry strength, and a larger A means a larger asymmetry strength. If $A = 0$, the system is reduced to the origin PDG.

When the participant i updates its strategy according to Eq. (4), because of the information asymmetry, the participant i usually cannot know the real payoff of selected neighbors. Therefore, the p_i and p_j can be rescaled as follows:

$$W_i = p_i, \tag{6}$$

$$W_j = p_j(1 + \xi_j). \tag{7}$$

Therefore, the probability that participant i adopts the strategy adopted by participant j can be rescaled as follows:

$$G_{i \to j} = \frac{1}{1 + \exp[(W_i - W_j)/K]}. \tag{8}$$

3. Simulations and analysis

First of all, we would like to study the motion of cooperation level through computer simulations, which are carried out on 100×100 participants according to the synchronous updating rule. Initially, the two strategies are distributed over all participants with equal probability, and the distribution of ξ is determined at the beginning of the game. The quantity

$\rho_{\rm C}$ characterizing the cooperative behavior of the system is defined as follows:

$$\rho_{\rm C} = \frac{1}{L^2}\sum_{i=1}^{L^2} \phi_i^{\rm T} \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (9)$$

In all the simulations, the $\rho_{\rm C}$ is obtained from the average over 10^4 different realizations after the system reaches steady states.

As shown in Fig. 1, with the increase of temptation to defect (b), the $\rho_{\rm C}$ decreases monotonically for all values of A. Moreover, as shown in Fig. 2, for a fixed b, different values of A can lead to different final cooperation levels, and there exists an optimal value $A_{\rm opt}$, which induces the highest cooperation level. That is, when A increases from 0 to $A_{\rm opt}$, the value of $\rho_{\rm C}$ will increase monotonously, while for $A > A_{\rm opt}$, the value of $\rho_{\rm C}$ decreases with A. It is worth noting that the effect of information asymmetry is greater for larger temptation b. Take b=1.01 as example, for the origin PDG ($A = 0$), the fraction of cooperation is $\rho_{\rm C} = 0.386$, and for the best case ($A \simeq 0.4$), $\rho_{\rm C} = 0.642$. Thus, the increment of $\rho_{\rm C}$ is 0.256. However, for a larger temptation $b = 1.03$, the increment is 0.543.

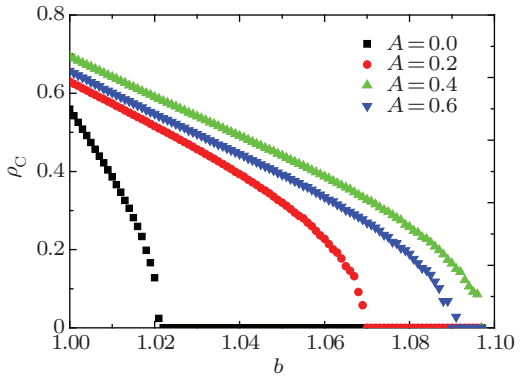


Fig. 1. (colour online) Fraction of cooperation $\rho_{\rm C}$ as a function of b for different values of A.

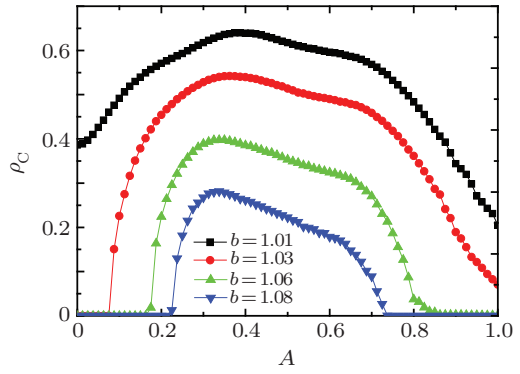


Fig. 2. (colour online) Fraction of cooperation $\rho_{\rm C}$ as a function of A for different values of b.

Since all the above simulation results agree with our estimation, we will explain the phenomenon and work out the critical and maximal points of the curves in the analysis. To understand the behaviors induced by the information asymmetry, we also apply the mean-field method to investigate the dynamics of $\rho_{\rm C}$.

As reported in Ref. [23], cooperation clusters play a crucial role in sustaining cooperation. As depicted in Fig. 3, most cooperators are distributed in clusters when the asymmetry strength is very small (e.g., $A = 0$) or very large (e.g., $A = 1$), and there are only a few cooperation clusters in the system. While at a moderate value of $A = 0.4$, there are so many cooperation clusters that the cooperation is greatly promoted. From the simulation results, as shown in Fig. 4, we find that the number of cooperation clusters has no remarkable change for the whole region of A. However, the curves describing the maximal and average number of cooperation clusters have a similar shape. The number of cooperation clusters will increase greatly in some region of A, reaching its maximal value at $A = 0.4$, which is consistent with $A_{\rm opt}$. Thus, a possible reason is that the size of clusters formed by cooperators play a decisive role in the frequency of cooperation rather than the size of cooperation clusters. The reason is that larger cooperation clusters make more cooperative participants immune from the invasion of defectors due to the fact that there exist more cooperators with cooperative neighbors in large cooperation clusters than in small ones. For the sake of the defending effects of cooperation clusters, the cooperation participants along the boundary are worth investigating. As shown in Fig. 5, the average cooperation neighbors along the boundary is 2.4, but the average cooperation neighbors of defectors along the boundary is 1.6. Hence, the average payoff of cooperation participants along the boundary is $P_{\rm C} = 2.4$, and the payoff of a defector is $1.6b$. The probability that a cooperator on the boundary changes into a defector is

$$W_{\rm C\to D} = \frac{1}{1+\exp[(2.4-1.6b(1+Ax_{\rm D}))/K]}. \quad (10)$$

The probability that a defector changes into a cooperator is

$$W_{\rm D\to C} = \frac{1}{1+\exp[(1.6b-2.4(1+Ax_{\rm C}))/K]}. \quad (11)$$

If $W_{\rm C\to D} < W_{\rm D\to C}$, the cooperation participants along the boundary are stronger than the defectors along the boundary. We define $[2.4-1.6b(1+Ax_{\rm D})]$ as

$\Delta_{\rm C}$ and $[1.6b-2.4(1+Ax_{\rm C})]$ as $\Delta_{\rm D}$. Since $W_{\rm C\to D}$ and $W_{\rm D\to C}$ are determined separately by $\Delta_{\rm C}$ and $\Delta_{\rm D}$, the relation $W_{\rm C\to D} < W_{\rm D\to C}$ is equivalent to $\Delta_{\rm C} > \Delta_{\rm D}$. If $\Delta_{\rm C} < \Delta_{\rm D}$, the defectors who are in position of strength can intrude cooperation clusters. We define $\Delta = \Delta_{\rm D} - \Delta_{\rm C}$, i.e.,

$$\Delta = (1.6bx_{\rm D} - 2.4x_{\rm C})A + 3.2\,b - 4.8. \qquad (12)$$

From the above equation, when $\Delta > 0$, one can find that the defector along the boundary are in position of strength, and will invade the cooperation clusters. Consequently, a large cooperation cluster is reduced into small cooperation clusters. As shown in Fig. 3, when $b > 1.5$, the fraction of cooperation is zero. Therefore, we are interested in the region of $0 < b < 1.5$. However, when $3.2\,b - 4.8 < 0$ ($b < 1.5$), or when $(1.6bx_{\rm D} - 2.4x_{\rm C}) < 0$, then $\Delta < 0$. The defectors along the boundary are weaker than the cooperators. However, when $1.6bx_{\rm D} - 2.4x_{\rm C} > 0$, with the increase of A, more defectors become stronger than some cooperators. For $\Delta > 0$, we can obtain

$$A > \frac{4.8 - 3.2\,b}{1.6bx_{\rm D} - 2.4x_{\rm C}}. \qquad (13)$$

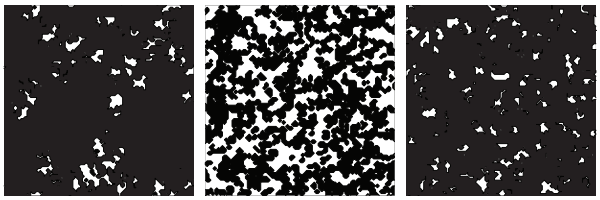


Fig. 3. Typical snapshots of distributions of cooperators (light gray) and defectors (black) on a square 100×100 lattice at $b = 1.01$ for (a) $A = 0$, (b) 0.4, and (c) 1.0.

Therefore, we can conclude that $(4.8 - 3.2\,b)/(1.6bx_{\rm D} - 2.4x_{\rm C}) > (4.8 - 3.2\,b)/(1.6b + 2.4)$. With the increase of A, more defectors with a large asymmetry strength will become stronger than the cooperators with a small asymmetry strength, and invade the cooperation clusters. More large clusters will be reduced into small clusters which disapprove cooperation. However, when $A < (4.8 - 3.2\,b)/(1.6b + 2.4)$, all the cooperators are stronger than all the defectors. With the increase of A, the cooperators satisfying $1.6bx_{\rm D} - 2.4x_{\rm C} < 0$ become stronger, and the cooperation clusters become larger. From the above analysis, one can find that $A = (4.8 - 3.2\,b)/(1.6b + 2.4)$ is the optimal value to promote the cooperation, and a smaller or larger A will lead to the decrease of cooperation. At $b = 1.01$, 1.03, 1.06, and 1.08, the optimal value of A is 0.4, 0.37, 0.34, and 0.33, respectively, which agrees with the simulation results.

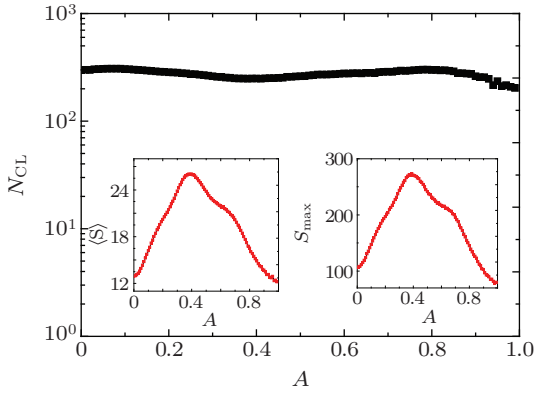


Fig. 4. (colour online) Number of clusters ($N_{\rm CL}$) formed by cooperators as a function of A for $b = 1.01$. The left inset shows the average number $\langle S\rangle$ of clusters, the right inset shows the number of the largest clusters $S_{\rm max}$ as a function of A.

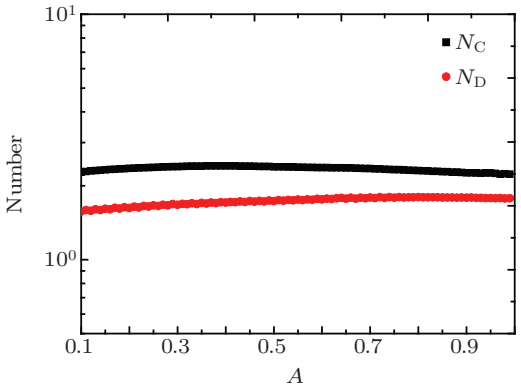


Fig. 5. (colour online) $N_{\rm C}$ and $N_{\rm D}$ as functions of A for $b = 1.01$. Here, the $N_{\rm C}$ denotes the average number of cooperative neighbors of cooperators, and the $N_{\rm D}$ denotes the average number of cooperative neighbors of defectors.

We also investigate the dynamics of the fraction of cooperators $\rho_{\rm C}$ via the mean-field method. The $G_{\rm C\to D}$ denotes the transition probability that the cooperators change into defectors and $G_{\rm D\to C}$ denotes the transition probability that the defectors change into cooperators. As a result, we obtain the following equation:

$$\frac{\partial \rho_{\rm C}}{\partial t} = (1 - \rho_{\rm C})G_{\rm D\to C} - \rho_{\rm C}G_{\rm C\to D}. \qquad (14)$$

When the system is in the steady state, i.e., $\partial\rho_{\rm C}/\partial t = 0$, we can obtain the following equation:

$$\rho_{\rm C} = \frac{1}{1 + G_{\rm C\to D}/G_{\rm D\to C}}. \qquad (15)$$

The simulation results of $G_{\rm C\to D}$ and $G_{\rm D\to C}$ are shown in Fig. 6. Then, we can calculate the value of cooperation fraction $\rho_{\rm C}$ according to Eq. (15). In Fig. 6, we

observe that the transition probability $G_{C\to D}$ changes more sharply than $G_{D\to C}$, the $G_{D\to C}$ increases until it reaches its maximum at a critical value of A and then decreases. However, the $G_{C\to D}$ behaves oppositely. As a result, the ratio $G_{C\to D}/G_{D\to C}$ varies as shown in the inset of Fig. 6. The explanation agrees with the general mechanism of cooperation in the prisoner's game.[13,24]

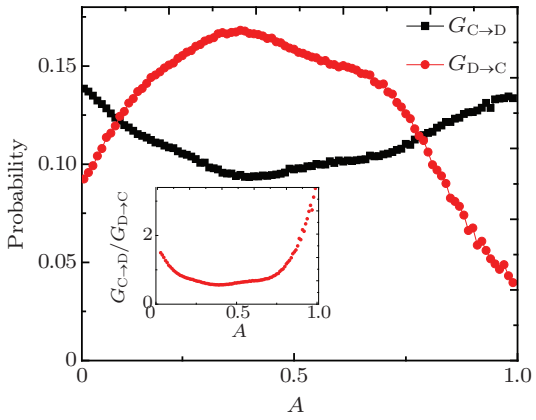


Fig. 6. (colour online) Average transition probability as a function of A for $b = 1.01$. The inset shows the value of $G_{C\to D}/G_{D\to C}$ as a function of A.

From the above simulations and analysis, we can conclude that, with the increases of A, the fraction of cooperation ρ_C increases and the cooperation clusters become larger and larger until A reaches A_{opt}. At the same time, the fraction of cooperation and the average size of cooperation clusters reach maximal values. However, with further increase of A, the cooperation is suppressed again, more and more individuals become defectors. In addition, it is found that the appearance of larger cooperation clusters leads to the promotion of cooperation, and the number of cooperation clusters will not vary but the size of them will be remarkably enlarged when the fraction of cooperation is promoted. Furthermore, the average number of cooperation neighbors of a cooperator or a defector along the boundary keeps constant for different values of A. Our findings agree with previous studies on the point that clustering cooperators play a crucial role in the promotion of cooperation. The cooperators along the boundary, which resist defectors, can be reinforced, such as heterogeneous structure.[25–28]

4. Conclusion

In conclusion, we have studied the PDG on a square lattice by introducing the information asymmetry, which is described by the uniform distribution of asymmetry strength. There exists a moderate value A_{opt}, at which the cooperation is promoted greatly. With the increase of A, the fraction of cooperation varies in accordance to the average and maximal sizes of cooperation clusters, and they reach their maximal values at A_{opt}. The number of cooperation clusters is nearly constant for all values of A. It is confirmed that a larger size of cooperation clusters rather than the number of cooperation clusters can promote the cooperation. In addition, cooperators along the boundary of cooperation clusters play an important role in maintaining cooperation clusters.[14] In addition, the mean-field theory is also applied to calculate the fraction of cooperation, which agrees with our simulation results. It is worth mentioning that our results is also confirmed by the study of inhomogeneous activities of teaching in Ref. [29], where enhanced teaching activity is found to induce the increase of cooperativity. We believe our study can be useful to understand the the effect of asymmetry payoff,[30] effect of diversity,[21,31,32] aging,[33] collaboration networks,[34] and epidemic immunization.[35]

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