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Novel delay-dependent stability analysis of Takagi–Sugeno fuzzy uncertain neural networks with time varying delays

M. Syed Ali†

Department of Mathematics, Thiruvalluvar University, Vellore 632 106*, Tamilnadu, India*

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This paper presents the stability analysis for a class of neural networks with time varying delays that are represented by the Takagi–Sugeno (T–S) model. The main results given here focus on the stability criteria using a new Lyapunov functional. New relaxed conditions and new linear matrix inequality-based designs are proposed that outperform the previous results found in the literature. Numerical examples are provided to show that the achieved conditions are less conservative than the existing ones in the literature.

Keywords: neutral neural networks, linear matrix inequality, Lyapunov stability, time varying delays

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1. Introduction

In recent years, dynamical neural networks have attracted considerable attention because of their potential applications in various signal processing problems, such as optimization, image processing, and associative memory design. However, time delays, both constant and time varying, are often encountered due to the finite switching speeds of the amplifiers in electronics systems or the finite signal propagation time. In order to study the neural networks, a delay parameter must be introduced into the equations of the neural network systems. The existence of the time delay may lead to some complex dynamical behaviors, such as chaos, oscillation, and instability. Therefore, the stability analysis of the neural networks with time delays has been extensively studied in the past few years. Various sufficient conditions have been presented to ensure the global stability of the neural networks with time delays. One may refer to Refs. [1]–[15] and the references therein for various stability results for different neural network models. Moreover, in the hardware implementation of the neural networks, the deviations in the network parameters due to the tolerances of the electronic components might have an impact on the equilibrium and the stability properties of the neural networks. Therefore, the stability analysis of the neural networks with parameter uncertainties is also important. In this case, one has to guarantee the robust stability for the neural networks. Some global robust stability results for delayed neural networks have been reported in Refs. [16]–[21]. Due to the complicated dynamical properties of the neural cells in the real world, the existing neural network models in many cases cannot characterize the properties of the neural reaction process precisely. The neural network systems should contain some information about the derivative of the past state to model the dynamics for such complex neural reactions. Thus, the stability analysis of the neural networks of the neutral type have been received considerable attention in the recent years (see, for example, Refs. [22]–[28]).

Fuzzy systems in the form of the Takagi–Sugeno (T–S) model[29] have attracted rapidly growing interest in recent years.[30,31] The T–S fuzzy systems are nonlinear systems described by a set of if-then rules. Some nonlinear dynamic systems can be approximated by the overall fuzzy linear T–S models for the purpose of stability analysis.[30,31] Originally, Tanaka and his colleagues have provided a sufficient condition for the quadratic stability of the T–S fuzzy systems in the sense of Lyapunov in a series of papers[32−34] by considering a Lyapunov function of sub-fuzzy systems of the T–S fuzzy systems. Motivated by Refs. [32]–[34], several authors derived the stability criteria for the T–S fuzzy neural networks by using the Lyapunov functional.[35−40]

†Corresponding author. E-mail: syedgru@gmail.com

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Based on the above discussion, we shall generalize the ordinary T–S fuzzy models to study a class of neutral neural networks with time varying delays. The main purpose of this paper is to study the global stability of the T–S fuzzy neural networks of the uncertain neutral type with time varying delays (TSFNNNs) in terms of linear matrix inequalities (LMIs). The main advantage of the LMI based approaches is that the LMI stability conditions can be solved numerically using the MATLAB LMI toolbox,[41] which implements state-of-the-art interior-point algorithms.[42] We also provide numerical examples to demonstrate the effectiveness of the proposed stability results.

2. System description and preliminaries

Consider the following neutral neural networks with time-varying delays:

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\big[\boldsymbol{x}(t)-(\boldsymbol{D}+\Delta\boldsymbol{D}(t))\boldsymbol{x}(t-\tau(t))\big]\\ =\;&-(\boldsymbol{C}+\Delta\boldsymbol{C}(t))\boldsymbol{x}(t)+(\boldsymbol{A}+\Delta\boldsymbol{A}(t))\boldsymbol{f}(\boldsymbol{x}(t))\\ &+(\boldsymbol{B}+\Delta\boldsymbol{B}(t))\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))),\end{aligned}\tag{1}$$

where $\boldsymbol{x}(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^{\mathrm{T}}\in\mathbf{R}^n$ is the neural state vector, $\boldsymbol{C}=\mathrm{diag}\{c_1,c_2,\ldots,c_n\}$ is a positive diagonal matrix, matrices $\boldsymbol{A}=(a_{ij})_{n\times n}$, $\boldsymbol{B}=(b_{ij})_{n\times n}$, and $\boldsymbol{D}=(d_{ij})_{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons,

$$\boldsymbol{f}(\boldsymbol{x}(t)))=[f_1(x_1(t)),\ f_2(x_2(t)),\ldots,f_n(x_n(t))]^{\mathrm{T}}\in\mathbf{R}^n$$

is the neuron activation function with $\boldsymbol{f}(0)=0$, and $\tau(t)$ is the time-varying delay and assumed to satisfy $0\leq\tau(t)\leq\bar{\tau}$, $\dot{\tau}(t)\leq d<1$. The initial condition associated with model (1) is $x_i(s)=\phi_i(s)$, $i=1,2,\ldots,n$, where $\phi_i(s)$ is bounded and continuously differentiable on $[-\rho,0]$, $\rho=\max\{h,\tau\}$.

Throughout this paper, we make the following assumption. There exist positive numbers $L_q>0$ such that

$$|\boldsymbol{f}(\boldsymbol{x})-\boldsymbol{f}(y)|\leq L_q|\boldsymbol{x}-\boldsymbol{y}|,\quad q=1,2,\ldots,n$$

for all $x,y\in\mathbf{R}$, $x\neq y$. And we denote

$$\boldsymbol{L}=\mathrm{diag}\{L_1,L_2,\ldots,L_n\}.$$

The type of activation function is clearly more general than both the usual sigmoid activation function and the piecewise linear function $f(x)=\frac{1}{2}(|x+1|-|x-1|)$ used in the cellular neural networks.

In this section, we will consider a neutral neural network with time varying delays, which is represented by a T–S fuzzy model composed of a set of fuzzy implications. Each of the implications is expressed as a linear system model.

The continuous fuzzy system is proposed to represent a nonlinear system.[29] The system dynamics can be captured by a set of fuzzy rules that characterize the local correlation in the state space. Each local dynamic described by the fuzzy if–then rule has the property of linear input–output relation. Based on the T–S fuzzy model concept, a general class of T–S fuzzy neutral neural networks with time-varying delays are considered here. The model of the T–S fuzzy neutral neural networks with time-varying delays is described as follows.

Plant rule k If $\{\theta_1(t)$ is $\boldsymbol{M}_{k1}\}$ and, $\cdots$, and $\{\theta_r(t)$ is $\boldsymbol{M}_{kr}\}$, then

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\big[\boldsymbol{x}(t)-(\boldsymbol{D}_k+\Delta\boldsymbol{D}_k(t))\boldsymbol{x}(t-\tau(t))\big]\\ =\;&-(\boldsymbol{C}_k+\Delta\boldsymbol{C}_k(t))\boldsymbol{x}(t)+(\boldsymbol{A}_k+\Delta\boldsymbol{A}_k(t))\boldsymbol{f}(\boldsymbol{x}(t))\\ &+(\boldsymbol{B}_k+\Delta\boldsymbol{B}_k(t))\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))),\end{aligned}$$

where $\theta_i(t)$ $(i=1,2,\ldots,r)$ are known variables, $\boldsymbol{M}_{kl}$ $(k\in\{1,2,\ldots,m\},\ l\in\{1,2,\ldots,r\})$ is the fuzzy set, and m is the number of model rules.

By inferring from the fuzzy models, the final output of the TSFNNNs is obtained as

$$\begin{aligned}\dot{\boldsymbol{x}}(t)=&\sum_{k=1}^{m}\omega_k(\theta)(t)\{-(\boldsymbol{C}_k+\Delta\boldsymbol{C}_k(t))\boldsymbol{x}(t)\\ &+(\boldsymbol{A}_k+\Delta\boldsymbol{A}_k(t))\boldsymbol{f}(\boldsymbol{x}(t))\\ &+(\boldsymbol{B}_k+\Delta\boldsymbol{B}_k(t))\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))\\ &+(\boldsymbol{D}_k+\Delta\boldsymbol{D}_k(t))\dot{\boldsymbol{x}}(t-\tau(t))\}\\ =&\sum_{k=1}^{m}\eta_k(\theta(t))\{-\bar{\boldsymbol{C}}_k\boldsymbol{x}(t)+\bar{\boldsymbol{A}}_k\boldsymbol{f}(\boldsymbol{x}(t))\\ &+\bar{\boldsymbol{B}}_k\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))+\bar{\boldsymbol{D}}_k\dot{\boldsymbol{x}}(t-\tau(t))\},\end{aligned}\tag{2}$$

where $\bar{\boldsymbol{C}}_k=(\boldsymbol{C}_k+\Delta\boldsymbol{C}_k(t))$, $\bar{\boldsymbol{A}}_k=(\boldsymbol{A}_k+\Delta\boldsymbol{A}_k(t))$, $\bar{\boldsymbol{B}}_k=(\boldsymbol{B}_k+\Delta\boldsymbol{B}_k(t))$, and $\bar{\boldsymbol{D}}_k=(\boldsymbol{D}_k+\Delta\boldsymbol{D}_k(t))$.

The weight and the averaged weight of each fuzzy rule are respectively denoted by

$$\omega_k(\theta(t))=\prod_{l=1}^{t}\boldsymbol{M}_{kl}(\theta(t)),$$

and

$$\eta_k(\theta(t))=\omega_k(\theta(t))\Big/\sum_{k=1}^{m}\omega_k(\theta(t)),$$

where $\boldsymbol{M}_{kl}(\theta(t))$ denotes the grade membership of $\theta_l(t)$ in $\boldsymbol{M}_{kl}$. We assume that for all $t \geq 0$,

$$\omega_k(\theta(t)) \geq 0,\ k \in \{1, 2, \ldots, m\},$$

and

$$\sum_{k=1}^{m} \eta_k(\theta(t)) = 1.$$

The parameter uncertainties $\Delta\boldsymbol{C}_k(t)$, $\Delta\boldsymbol{A}_k(t)$, $\Delta\boldsymbol{B}_k(t)$, and $\Delta\boldsymbol{D}_k(t)$ are time varying matrices with appropriate dimensions, which are defined as

$$\begin{aligned}&\Big[\Delta\boldsymbol{C}_k(t), \quad \Delta\boldsymbol{A}_k(t), \quad \Delta\boldsymbol{B}_k(t), \quad \Delta\boldsymbol{D}_k(t)\Big]\\ &= \boldsymbol{E}_k\boldsymbol{F}(t)\Big[\boldsymbol{G}_{1k}, \quad \boldsymbol{G}_{2k}, \quad \boldsymbol{G}_{3k}, \quad \boldsymbol{G}_{4k}\Big], \end{aligned} \tag{3}$$

where $\boldsymbol{G}_{1k}$, $\boldsymbol{G}_{2k}$, $\boldsymbol{G}_{3k}$, $\boldsymbol{G}_{4k}$, and $\boldsymbol{E}_k$ are known constant matrices of appropriate dimensions, and $\boldsymbol{F}(t)$ is a known time-varying matrix with the Lebegue measurable elements bounded by

$$\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t) \leq \boldsymbol{I},$$

with $\boldsymbol{I}$ being the identity matrix with appropriate dimensions.

Now we consider the fuzzy neutral neural networks without uncertainties. It is given by

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = \sum_{k=1}^{m} \eta_k(\theta(t))\Big\{ &-\boldsymbol{C}_k\boldsymbol{x}(t) + \boldsymbol{A}_k\boldsymbol{f}(\boldsymbol{x}(t))\\ &+ \boldsymbol{B}_k\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) + \boldsymbol{D}_k\dot{\boldsymbol{x}}(t-\tau(t))\Big\}. \end{aligned} \tag{4}$$

Lemma 1 (Schur complement[42]) The LMI

$$\begin{bmatrix} \boldsymbol{Q}(\boldsymbol{y}) & \boldsymbol{S}(\boldsymbol{y}) \\ \boldsymbol{S}(\boldsymbol{y})^{\mathrm{T}} & \boldsymbol{R}(\boldsymbol{y}) \end{bmatrix} < 0$$

is equivalent to

$$\boldsymbol{R}(\boldsymbol{y}) < 0, \quad \boldsymbol{Q}(\boldsymbol{y}) - \boldsymbol{S}(\boldsymbol{y})\boldsymbol{R}(\boldsymbol{y})^{-1}\boldsymbol{S}(\boldsymbol{y})^{\mathrm{T}} < 0,$$

where $\boldsymbol{Q}(\boldsymbol{y}) = \boldsymbol{Q}(\boldsymbol{y})^{\mathrm{T}}$, $\boldsymbol{R}(\boldsymbol{y}) = \boldsymbol{R}(\boldsymbol{y})^{\mathrm{T}}$, and $\boldsymbol{S}(\boldsymbol{y})$ depend affinely on $\boldsymbol{y}$.

Lemma 2[43] For matrices $\boldsymbol{P} \in \mathbf{R}^{n\times n}$, $\boldsymbol{M} \in \mathbf{R}^{n\times k}$, $\boldsymbol{N} \in \mathbf{R}^{l\times n}$, and $\boldsymbol{F} \in \mathbf{R}^{k\times l}$ with $\boldsymbol{P} > 0$, $\|\boldsymbol{F}\| \leq 1$ and scalar $\epsilon > 0$, we have

$$(\boldsymbol{MFN})^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{MFN}) \leq \epsilon\boldsymbol{PMM}^{\mathrm{T}}\boldsymbol{P} + \epsilon^{-1}\boldsymbol{N}^{\mathrm{T}}\boldsymbol{N}.$$

3. Global stability results

In this section, we obtain a stability criterion for the fuzzy neural networks of the neutral type with time-varying delays.

Theorem 1 Given scalars $\bar{\tau} \geq 0$ and $d > 0$, the equilibrium point of system (4) is globally asymptotically stable if there exist positive definite matrices $\boldsymbol{P}_1 > 0$, $\boldsymbol{P}_2 > 0$, $\boldsymbol{P}_3 > 0$, $\boldsymbol{Q}_1 > 0$, $\boldsymbol{Q}_2 > 0$, $\boldsymbol{R}_1 > 0$, $\boldsymbol{R}_2 > 0$, $\boldsymbol{S} > 0$, symmetric matrices $\boldsymbol{X}$, $\boldsymbol{Y}$, any matrices $\boldsymbol{N}_i$, $\boldsymbol{M}_i$ $(i = 1, \ldots, 6)$ such that the following conditions hold for $k = 1, 2, \ldots, m$:

$$\boldsymbol{\Pi}^k = \begin{bmatrix} \boldsymbol{\Pi}_{11}^k & \boldsymbol{\Pi}_{12}^k & \boldsymbol{\Pi}_{13}^k & \boldsymbol{\Pi}_{14}^k & \boldsymbol{\Pi}_{15}^k & \boldsymbol{\Pi}_{16}^k & \boldsymbol{\Pi}_{17}^k & \boldsymbol{\Pi}_{18}^k \\ * & \boldsymbol{\Pi}_{22}^k & \boldsymbol{\Pi}_{23}^k & \boldsymbol{\Pi}_{24}^k & \boldsymbol{\Pi}_{25}^k & \boldsymbol{\Pi}_{26}^k & \boldsymbol{\Pi}_{27}^k & \boldsymbol{\Pi}_{28}^k \\ * & * & \boldsymbol{\Pi}_{33}^k & \boldsymbol{\Pi}_{34}^k & \boldsymbol{\Pi}_{35}^k & \boldsymbol{\Pi}_{36}^k & \boldsymbol{\Pi}_{37}^k & \boldsymbol{\Pi}_{38}^k \\ * & * & * & \boldsymbol{\Pi}_{44}^k & \boldsymbol{\Pi}_{45}^k & \boldsymbol{\Pi}_{46}^k & \boldsymbol{\Pi}_{47}^k & \boldsymbol{\Pi}_{48}^k \\ * & * & * & * & \boldsymbol{\Pi}_{55}^k & \boldsymbol{\Pi}_{56}^k & \boldsymbol{\Pi}_{57}^k & \boldsymbol{\Pi}_{58}^k \\ * & * & * & * & * & \boldsymbol{\Pi}_{66}^k & \boldsymbol{\Pi}_{67}^k & \boldsymbol{\Pi}_{68}^k \\ * & * & * & * & * & * & \boldsymbol{\Pi}_{77}^k & \boldsymbol{\Pi}_{78}^k \\ * & * & * & * & * & * & * & \boldsymbol{\Pi}_{88}^k \end{bmatrix} < 0, \tag{5}$$

where

$$\begin{aligned}&\boldsymbol{\Pi}_{11}^k = \boldsymbol{P}_2 + \boldsymbol{P}_2^{\mathrm{T}} + \boldsymbol{Q}_1 + \tau\boldsymbol{S} + \boldsymbol{\Phi}_{11}^k + \boldsymbol{L}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{L} + 2\boldsymbol{Y}\boldsymbol{L},\\ &\boldsymbol{\Pi}_{12}^k = -\boldsymbol{P}_2 + \boldsymbol{\Phi}_{12}^k,\\ &\boldsymbol{\Pi}_{13}^k = \boldsymbol{\Phi}_{13}^k - \boldsymbol{Y}, \quad \boldsymbol{\Pi}_{14}^k = \boldsymbol{\Phi}_{14}^k,\\ &\boldsymbol{\Pi}_{15}^k = \boldsymbol{P}_1 + \boldsymbol{\Phi}_{15}^k, \quad \boldsymbol{\Pi}_{16}^k = -\boldsymbol{P}_1\boldsymbol{D}_k + \boldsymbol{\Phi}_{16}^k,\\ &\boldsymbol{\Pi}_{17}^k = \boldsymbol{P}_3, \quad \boldsymbol{\Pi}_{18}^k = \boldsymbol{\Phi}_{17}^k,\end{aligned}$$

$$
\begin{aligned}
&\boldsymbol{\Pi}_{22}^k = -(1-d)\boldsymbol{Q}_1 + \boldsymbol{\Phi}_{22}^k + \boldsymbol{P}_2 + \boldsymbol{P}_2^{\mathrm{T}}, \quad \boldsymbol{\Pi}_{23}^k = \boldsymbol{\Phi}_{23}^k,\\
&\boldsymbol{\Pi}_{24}^k = \boldsymbol{\Phi}_{24}^k, \quad \boldsymbol{\Pi}_{25}^k = -\boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{P}_1 + \boldsymbol{\Phi}_{25}^k,\\
&\boldsymbol{\Pi}_{26}^k = \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{D}_k + \boldsymbol{\Phi}_{26}^k, \quad \boldsymbol{\Pi}_{27}^k = -\boldsymbol{P}_3,\\
&\boldsymbol{\Pi}_{28}^k = \boldsymbol{\Phi}_{27}^k, \quad \boldsymbol{\Pi}_{33}^k = \boldsymbol{Q}_2 + \boldsymbol{\Phi}_{33}^k - \boldsymbol{X},\\
&\boldsymbol{\Pi}_{34}^k = \boldsymbol{\Phi}_{34}^k, \ \boldsymbol{\Pi}_{35}^k = \boldsymbol{\Phi}_{35}^k, \ \boldsymbol{\Pi}_{36}^k = \boldsymbol{\Phi}_{36}^k, \ \boldsymbol{\Pi}_{37}^k = 0,\\
&\boldsymbol{\Pi}_{38}^k = \boldsymbol{\Phi}_{37}^k, \boldsymbol{\Pi}_{44}^k = -(1-d)\boldsymbol{Q}_2 + \boldsymbol{\Phi}_{44}^k,\\
&\boldsymbol{\Pi}_{45}^k = \boldsymbol{\Phi}_{45}^k, \quad \boldsymbol{\Pi}_{46}^k = \boldsymbol{\Phi}_{46}^k, \ \boldsymbol{\Pi}_{47}^k = 0, \quad \boldsymbol{\Pi}_{48}^k = \boldsymbol{\Phi}_{47}^k,\\
&\boldsymbol{\Pi}_{55}^k = \boldsymbol{R}_1 + \tau\boldsymbol{R}_2 + \boldsymbol{\Phi}_{55}^k, \quad \boldsymbol{\Pi}_{56}^k = \boldsymbol{\Phi}_{56}^k,\\
&\boldsymbol{\Pi}_{57}^k = \boldsymbol{P}_2, \quad \boldsymbol{\Pi}_{58}^k = \boldsymbol{\Phi}_{57}^k, \quad \boldsymbol{\Pi}_{66}^k = -(1-d)\boldsymbol{R}_1 + \boldsymbol{\Phi}_{66}^k,\\
&\boldsymbol{\Pi}_{67}^k = -\boldsymbol{P}_2, \quad \boldsymbol{\Pi}_{68}^k = \boldsymbol{\Phi}_{67}^k, \ \boldsymbol{\Pi}_{77}^k = -\tau S,\\
&\boldsymbol{\Pi}_{78}^k = 0, \quad \boldsymbol{\Pi}_{88}^k = \boldsymbol{\Phi}_{77}^k - \tau\boldsymbol{R}_2, \quad \boldsymbol{\Phi}_{77}^k = 0,\\
&\boldsymbol{\Phi}_{11}^k = \boldsymbol{N}_1 + \boldsymbol{N}_1^{\mathrm{T}} - \boldsymbol{M}_1\boldsymbol{C}_k - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_1,\\
&\boldsymbol{\Phi}_{12}^k = -\boldsymbol{N}_1 + \boldsymbol{N}_2^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_2^{\mathrm{T}},\\
&\boldsymbol{\Phi}_{13}^k = \boldsymbol{N}_3^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_3^{\mathrm{T}} + \boldsymbol{M}_1\boldsymbol{A}_k,\\
&\boldsymbol{\Phi}_{14}^k = \boldsymbol{N}_4^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}} + \boldsymbol{M}_1\boldsymbol{B}_k,\\
&\boldsymbol{\Phi}_{15}^k = \boldsymbol{N}_5^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}} - \boldsymbol{M}_1,\\
&\boldsymbol{\Phi}_{16}^k = \boldsymbol{N}_6^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}} + \boldsymbol{M}_1\boldsymbol{D}_k, \ \boldsymbol{\Phi}_{17}^k = -\tau\boldsymbol{N}_1,\\
&\boldsymbol{\Phi}_{22}^k = -\boldsymbol{N}_2 - \boldsymbol{N}_2^{\mathrm{T}}, \quad \boldsymbol{\Phi}_{23}^k = -\boldsymbol{N}_3^{\mathrm{T}} + \boldsymbol{M}_2\boldsymbol{A}_k,\\
&\boldsymbol{\Phi}_{24}^k = -\boldsymbol{N}_4^{\mathrm{T}} + \boldsymbol{M}_2\boldsymbol{B}_k, \quad \boldsymbol{\Phi}_{25}^k = -\boldsymbol{N}_5^{\mathrm{T}} - \boldsymbol{M}_2,\\
&\boldsymbol{\Phi}_{26}^k = -\boldsymbol{N}_6^{\mathrm{T}} + \boldsymbol{M}_2\boldsymbol{D}_k, \quad \boldsymbol{\Phi}_{27}^k = -\tau\boldsymbol{N}_2,\\
&\boldsymbol{\Phi}_{33}^k = \boldsymbol{M}_3\boldsymbol{A}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_3, \ \boldsymbol{\Phi}_{34}^k = \boldsymbol{M}_3\boldsymbol{B}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}},\\
&\boldsymbol{\Phi}_{35}^k = -\boldsymbol{M}_3 + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}}, \ \boldsymbol{\Phi}_{36}^k = \boldsymbol{M}_3\boldsymbol{D}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}},\\
&\boldsymbol{\Phi}_{37}^k = -\tau\boldsymbol{N}_3, \quad \boldsymbol{\Phi}_{44}^k = \boldsymbol{M}_4\boldsymbol{B}_k + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}},\\
&\boldsymbol{\Phi}_{45}^k = -\boldsymbol{M}_4 + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}},\\
&\boldsymbol{\Phi}_{46}^k = \boldsymbol{M}_4\boldsymbol{D}_k + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}}, \quad \boldsymbol{\Phi}_{47}^k = -\tau\boldsymbol{N}_4,\\
&\boldsymbol{\Phi}_{55}^k = -\boldsymbol{M}_5 - \boldsymbol{M}_5^{\mathrm{T}}, \quad \boldsymbol{\Phi}_{56}^k = \boldsymbol{M}_5\boldsymbol{D}_k - \boldsymbol{M}_6^{\mathrm{T}},\\
&\boldsymbol{\Phi}_{57}^k = -\tau\boldsymbol{N}_5, \quad \boldsymbol{\Phi}_{66}^k = \boldsymbol{M}_6\boldsymbol{D}_k + \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}},\\
&\boldsymbol{\Phi}_{67}^k = -\tau\boldsymbol{N}_6.
\end{aligned}
$$

Proof We use the following Lyapunov functional to derive the stability result:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t),$$

where

$$
\begin{aligned}
V_1(t) &= \sum_{k=1}^{m}\{[\boldsymbol{x}(t) - \boldsymbol{D}_k\boldsymbol{x}(t-\tau(t))]^{\mathrm{T}}\boldsymbol{P}_1[\boldsymbol{x}(t)\\
&\quad - \boldsymbol{D}_k\boldsymbol{x}(t-\tau(t))]\},\\
V_2(t) &= 2[\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau(t))]^{\mathrm{T}}\boldsymbol{P}_2\int_{t-\tau(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s,\\
V_3(t) &= \Big[\int_{t-\tau(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s\Big]^{\mathrm{T}}\boldsymbol{P}_3\int_{t-\tau(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s,\\
V_4(t) &= \int_{t-\tau(t)}^{t}\big[\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{x}(s) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{Q}_2\boldsymbol{f}(\boldsymbol{x}(s))\\
&\quad + \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\big]\mathrm{d}s,\\
V_5(t) &= \int_{-\tau}^{0}\int_{t+\theta}^{t}\big[\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{S}\boldsymbol{x}(s) + \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{x}}(s)\big]\mathrm{d}s\mathrm{d}\theta.
\end{aligned}
$$

We can calculate the derivative of $V(t)$ along the trajectories of system (4). Then we have

$$
\begin{aligned}
\dot{V}(t) \le \sum_{k=1}^{m}\eta_k(\boldsymbol{\theta}(t))\Big\{&2[\boldsymbol{x}(t) - \boldsymbol{D}_k\boldsymbol{x}(t-\tau(t))]^{\mathrm{T}}\boldsymbol{P}_1[\dot{\boldsymbol{x}}(t) - \boldsymbol{D}_k\dot{\boldsymbol{x}}(t-\tau(t))] + 2[\dot{\boldsymbol{x}}(t) - \dot{\boldsymbol{x}}(t-\tau(t))]^{\mathrm{T}}\\
&\times\boldsymbol{P}_2\int_{t-\tau(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s + 2[\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau(t))]^{\mathrm{T}}\boldsymbol{P}_2[\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau(t))]\\
&+ 2[\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau(t))]^{\mathrm{T}}\boldsymbol{P}_3\int_{t-\tau(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s + \boldsymbol{x}^{\mathrm{T}}(t)[\boldsymbol{Q}_1 + \tau\boldsymbol{S}]\boldsymbol{x}(t) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{Q}_2\boldsymbol{f}(\boldsymbol{x}(t))\\
&+ \dot{\boldsymbol{x}}^{\mathrm{T}}(t)[\boldsymbol{R}_1 + \tau\boldsymbol{R}_2]\dot{\boldsymbol{x}}(t) - (1-d)[\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}_1\boldsymbol{x}(t-\tau(t)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))\boldsymbol{Q}_2\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))\\
&+ \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{R}_1\dot{\boldsymbol{x}}(t-\tau(t))] - \int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s - \int_{t-\tau(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{S}\boldsymbol{x}(s)\mathrm{d}s\\
&+ 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Y}\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{X}\boldsymbol{f}(\boldsymbol{x}(t)) - 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Y}\boldsymbol{f}(\boldsymbol{x}(t)) - \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{X}\boldsymbol{f}(\boldsymbol{x}(t))\Big\}.
\end{aligned}
$$

According to the Leibniz–Newton formula, for any matrices $\boldsymbol{N}_i$, $\boldsymbol{M}_i$ $(i = 1, \ldots, 4)$, the following equations hold:

$$
\begin{aligned}
&2\Big[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{N}_1 + \boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\ \boldsymbol{N}_2 + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{N}_3\\
&+ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t))\boldsymbol{N}_4 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{N}_5 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{N}_6\Big]\\
&\times\Big[\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s\Big] = 0, \qquad (6)\\
&2\Big[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{M}_1 + \boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\ \boldsymbol{M}_2 + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{M}_3\\
&+ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))\boldsymbol{M}_4 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{M}_5 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{M}_6\Big]\\
&\times\sum_{k=1}^{m}\eta_k(\theta(t))\Big\{-\dot{\boldsymbol{x}}(t) - \boldsymbol{C}_k\boldsymbol{x}(t) + \boldsymbol{A}_k\boldsymbol{f}(\boldsymbol{x}(t))
\end{aligned}
$$

$$+\boldsymbol{B}_k\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))+\boldsymbol{D}_k\dot{\boldsymbol{x}}(t-\tau(t))\Big\}=0. \quad (7)$$

From the assumption, for any symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, we can have

$$\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Y}\boldsymbol{f}(\boldsymbol{x}(t))\leq\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Y}\boldsymbol{L}\boldsymbol{x}(t), \quad (8)$$

$$\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{X}\boldsymbol{f}(\boldsymbol{x}(t))\leq\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{L}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{L}\boldsymbol{x}(t). \quad (9)$$

From expressions (5) and (6), we have

$$\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}\boldsymbol{\xi}_1^{\mathrm{T}}(t,s)\boldsymbol{\Phi}^k\boldsymbol{\xi}_1(t,s)\,\mathrm{d}s=0, \quad (10)$$

where

$$\boldsymbol{\Phi}^k=\begin{bmatrix}\boldsymbol{\Phi}_{11}^k&\boldsymbol{\Phi}_{12}^k&\boldsymbol{\Phi}_{13}^k&\boldsymbol{\Phi}_{14}^k&\boldsymbol{\Phi}_{15}^k&\boldsymbol{\Phi}_{16}^k&\boldsymbol{\Phi}_{17}^k\\ *&\boldsymbol{\Phi}_{22}^k&\boldsymbol{\Phi}_{23}^k&\boldsymbol{\Phi}_{24}^k&\boldsymbol{\Phi}_{25}^k&\boldsymbol{\Phi}_{26}^k&\boldsymbol{\Phi}_{27}^k\\ *&*&\boldsymbol{\Phi}_{33}^k&\boldsymbol{\Phi}_{34}^k&\boldsymbol{\Phi}_{35}^k&\boldsymbol{\Phi}_{36}^k&\boldsymbol{\Phi}_{37}^k\\ *&*&*&\boldsymbol{\Phi}_{44}^k&\boldsymbol{\Phi}_{45}^k&\boldsymbol{\Phi}_{46}^k&\boldsymbol{\Phi}_{47}^k\\ *&*&*&*&\boldsymbol{\Phi}_{55}^k&\boldsymbol{\Phi}_{56}^k&\boldsymbol{\Phi}_{57}^k\\ *&*&*&*&*&\boldsymbol{\Phi}_{66}^k&\boldsymbol{\Phi}_{67}^k\\ *&*&*&*&*&*&\boldsymbol{\Phi}_{77}^k\end{bmatrix},$$

and $\boldsymbol{\xi}_1(t,s)=[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\ \ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\ \ \boldsymbol{f}^{\mathrm{T}}\boldsymbol{x}(t-\tau(t))\ \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\ \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\ \dot{\boldsymbol{x}}^{\mathrm{T}}(s)]^{\mathrm{T}}$. Adding terms from the left-hand side of Eq. (10) to $\dot{V}(t)$ and using inequalities (8) and(9), we can express $\dot{V}(t)$ as

$$\dot{V}(t)=\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}\boldsymbol{\xi}^{\mathrm{T}}(t,s)\boldsymbol{\Pi}^k\boldsymbol{\xi}(t,s)\,\mathrm{d}s,$$

where $\boldsymbol{\xi}(t,s)=[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\ \ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\ \ \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t)-\tau(t))\ \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\ \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{x}^{\mathrm{T}}(s)\ \dot{\boldsymbol{x}}^{\mathrm{T}}(s)]^{\mathrm{T}}$, and $\boldsymbol{\Pi}^k$ are defined in Eq. (4). If $\boldsymbol{\Pi}^k<0$, then $\dot{V}(t)<0$, which indicates from the Lyapunov stability theory that the dynamic of the fuzzy delayed neural network (3) is globally and asymptotically stable. This completes the proof.

If we choose $\boldsymbol{P}_2=0$, $\boldsymbol{P}_3=0$, and $\boldsymbol{S}=0$, we have the following criterion. Note that these values will lead a different Lyapunov functional.

Corollary 1 Given scalars $\bar{\tau}\geq 0$ and $d>0$, the equilibrium point of system (4) is globally asymptotically stable if there exist positive definite matrices $\boldsymbol{P}_1>0$, $\boldsymbol{Q}_1>0$, $\boldsymbol{Q}_2>0$, $\boldsymbol{R}_1>0$, $\boldsymbol{R}_2>0$, symmetric matrices $\boldsymbol{X}$, $\boldsymbol{Y}$, any matrices $\boldsymbol{N}_i$, $\boldsymbol{M}_i$ $(i=1,\ldots,6)$ such that the following conditions hold for $k=1,2,\ldots,m$:

$$\boldsymbol{\Omega}^k=\begin{bmatrix}\boldsymbol{\Omega}_{11}^k&\boldsymbol{\Omega}_{12}^k&\boldsymbol{\Omega}_{13}^k&\boldsymbol{\Omega}_{14}^k&\boldsymbol{\Omega}_{15}^k&\boldsymbol{\Omega}_{16}^k&\boldsymbol{\Omega}_{17}^k\\ *&\boldsymbol{\Omega}_{22}^k&\boldsymbol{\Omega}_{23}^k&\boldsymbol{\Omega}_{24}^k&\boldsymbol{\Omega}_{25}^k&\boldsymbol{\Omega}_{26}^k&\boldsymbol{\Omega}_{27}^k\\ *&*&\boldsymbol{\Omega}_{33}^k&\boldsymbol{\Omega}_{34}^k&\boldsymbol{\Omega}_{35}^k&\boldsymbol{\Omega}_{36}^k&\boldsymbol{\Omega}_{37}^k\\ *&*&*&\boldsymbol{\Omega}_{44}^k&\boldsymbol{\Omega}_{45}^k&\boldsymbol{\Omega}_{46}^k&\boldsymbol{\Omega}_{47}^k\\ *&*&*&*&\boldsymbol{\Omega}_{55}^k&\boldsymbol{\Omega}_{56}^k&\boldsymbol{\Omega}_{57}^k\\ *&*&*&*&*&\boldsymbol{\Omega}_{66}^k&\boldsymbol{\Omega}_{67}^k\\ *&*&*&*&*&*&\boldsymbol{\Omega}_{77}^k\end{bmatrix}<0,$$

where

$$
\begin{aligned}
\boldsymbol{\Omega}_{11}^k &= \boldsymbol{Q}_1 + \boldsymbol{N}_1 + \boldsymbol{N}_1^{\mathrm{T}} - \boldsymbol{M}_1\boldsymbol{C}_k \\
&\quad - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_1 + \boldsymbol{L}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{L} + 2\boldsymbol{Y}\boldsymbol{L}, \\
\boldsymbol{\Omega}_{12}^k &= -\boldsymbol{N}_1 + \boldsymbol{N}_2^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_2^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{13}^k &= \boldsymbol{N}_3^{\mathrm{T}} + \boldsymbol{M}_1\boldsymbol{A}_k - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_3^{\mathrm{T}} - \boldsymbol{Y}, \\
\boldsymbol{\Omega}_{14}^k &= \boldsymbol{N}_4^{\mathrm{T}} + \boldsymbol{M}_1\boldsymbol{B}_k - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{15}^k &= \boldsymbol{P}_1 + \boldsymbol{N}_5^{\mathrm{T}} - \boldsymbol{M}_1 - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{16}^k &= -\boldsymbol{P}_1\boldsymbol{D}_k + \boldsymbol{N}_6^{\mathrm{T}} + \boldsymbol{M}_1\boldsymbol{D}_k - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{17}^k &= -\tau\boldsymbol{N}_1, \\
\boldsymbol{\Omega}_{22}^k &= -(1-d)\boldsymbol{Q}_1 - \boldsymbol{N}_2 - \boldsymbol{N}_2^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{23}^k &= -\boldsymbol{N}_3^{\mathrm{T}} + \boldsymbol{M}_2\boldsymbol{A}_k,\ \boldsymbol{\Omega}_{24}^k = -\boldsymbol{N}_4^{\mathrm{T}} + \boldsymbol{M}_2\boldsymbol{B}_k, \\
\boldsymbol{\Omega}_{25}^k &= -\boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{P}_1 - \boldsymbol{N}_5^{\mathrm{T}} - \boldsymbol{M}_2, \\
\boldsymbol{\Omega}_{26}^k &= \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{D}_k - \boldsymbol{N}_6^{\mathrm{T}} + \boldsymbol{M}_2\boldsymbol{D}_k, \\
\boldsymbol{\Omega}_{27}^k &= -\tau\boldsymbol{N}_2,\ \boldsymbol{\Omega}_{33}^k = \boldsymbol{Q}_2 + \boldsymbol{M}_3\boldsymbol{A}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_3 - X, \\
\boldsymbol{\Omega}_{34}^k &= \boldsymbol{M}_3\boldsymbol{B}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{35}^k &= -\boldsymbol{M}_3 + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{36}^k &= \boldsymbol{M}_3\boldsymbol{D}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}},\ \boldsymbol{\Omega}_{37}^k = -\tau\boldsymbol{N}_3, \\
\boldsymbol{\Omega}_{44}^k &= -(1-d)\boldsymbol{Q}_2 + \boldsymbol{M}_4\boldsymbol{B}_k + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_4, \\
\boldsymbol{\Omega}_{45}^k &= -\boldsymbol{M}_4 + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}},\ \boldsymbol{\Omega}_{46}^k = \boldsymbol{M}_4\boldsymbol{D}_k + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{47}^k &= -\tau\boldsymbol{N}_4,\ \boldsymbol{\Omega}_{55}^k = \boldsymbol{R}_1 + \tau\boldsymbol{R}_2 - \boldsymbol{M}_5 - \boldsymbol{M}_5^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{56}^k &= \boldsymbol{M}_5\boldsymbol{D}_k - \boldsymbol{M}_6^{\mathrm{T}},\ \boldsymbol{\Omega}_{57}^k = -\tau\boldsymbol{N}_5, \\
\boldsymbol{\Omega}_{66}^k &= -(1-d)\boldsymbol{R}_1 + \boldsymbol{M}_6\boldsymbol{D}_k + \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}}, \\
\boldsymbol{\Omega}_{67}^k &= -\tau\boldsymbol{N}_6,\ \boldsymbol{\Omega}_{77}^k = -\tau\boldsymbol{R}_2.
\end{aligned}
$$

Remark 1 In Theorem 1 and Corollary 2, the free-weighting matrices $\boldsymbol{N}_i$ $(i = 1, \ldots, 6)$ in inequality (5) express the relationship between items $\boldsymbol{x}(t), \boldsymbol{x}(t - \tau(t))$ and $\int_{t-\tau(t)}^{t} \dot{\boldsymbol{x}}(s)\mathrm{d}s$ and are obtained by solving the LMI. In fact, Corollary 1 can be obtained by using the free-weighting matrix method. Thus, matrices $\boldsymbol{P}_1, \boldsymbol{P}_2$, and $\boldsymbol{S}$ in Theorem 1, which are obtained by solving the LMI, provide the extra freedom.

On the other hand, if we choose $\boldsymbol{R}_2 = 0$ and $\boldsymbol{N}_i = 0$ $(i = 1, 2, \ldots, 6)$, another criterion can also be derived.

Corollary 2 Given scalars $\bar{\tau} \geq 0$, and $d > 0$, the equilibrium point of system (4) is globally asymptotically stable if there exist positive definite matrices $\boldsymbol{P}_1 > 0,\ \boldsymbol{P}_2 > 0,\ \boldsymbol{P}_3 > 0,\ \boldsymbol{Q}_1 > 0, \boldsymbol{Q}_2 > 0,\ \boldsymbol{R}_1 > 0,\ \boldsymbol{S} > 0$, symmetric matrices $\boldsymbol{X},\ \boldsymbol{Y}$, and any matrices $\boldsymbol{M}_i$ $(i = 1, \ldots, 6)$, such that the following conditions hold for $k = 1, 2, \ldots, m$:

$$
\boldsymbol{\Psi}^k = \begin{bmatrix}
\boldsymbol{\Psi}_{11}^k & \boldsymbol{\Psi}_{12}^k & \boldsymbol{\Psi}_{13}^k & \boldsymbol{\Psi}_{14}^k & \boldsymbol{\Psi}_{15}^k & \boldsymbol{\Psi}_{16}^k & \boldsymbol{\Psi}_{17}^k \\
* & \boldsymbol{\Psi}_{22}^k & \boldsymbol{\Psi}_{23}^k & \boldsymbol{\Psi}_{24}^k & \boldsymbol{\Psi}_{25}^k & \boldsymbol{\Psi}_{26}^k & \boldsymbol{\Psi}_{27}^k \\
* & * & \boldsymbol{\Psi}_{33}^k & \boldsymbol{\Psi}_{34}^k & \boldsymbol{\Psi}_{35}^k & \boldsymbol{\Psi}_{36}^k & \boldsymbol{\Psi}_{37}^k \\
* & * & * & \boldsymbol{\Psi}_{44}^k & \boldsymbol{\Psi}_{45}^k & \boldsymbol{\Psi}_{46}^k & \boldsymbol{\Psi}_{47}^k \\
* & * & * & * & \boldsymbol{\Psi}_{55}^k & \boldsymbol{\Psi}_{56}^k & \boldsymbol{\Psi}_{57}^k \\
* & * & * & * & * & \boldsymbol{\Psi}_{66}^k & \boldsymbol{\Psi}_{67}^k \\
* & * & * & * & * & * & \boldsymbol{\Psi}_{77}^k
\end{bmatrix} < 0,
$$

where

$$
\begin{aligned}
\boldsymbol{\Psi}_{11}^k &= \boldsymbol{P}_2 + \boldsymbol{P}_2^{\mathrm{T}} + \boldsymbol{Q}_1 + \tau\boldsymbol{S} - \boldsymbol{M}_1\boldsymbol{C}_k \\
&\quad - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_1 + \boldsymbol{L}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{L} + 2\boldsymbol{Y}\boldsymbol{L}, \\
\boldsymbol{\Psi}_{12}^k &= -\boldsymbol{P}_2 - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_2^{\mathrm{T}}, \\
\boldsymbol{\Psi}_{13}^k &= \boldsymbol{M}_1 A - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_3^{\mathrm{T}} - \boldsymbol{Y}, \\
\boldsymbol{\Psi}_{14}^k &= \boldsymbol{M}_1\boldsymbol{B}_k - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}}, \\
\boldsymbol{\Psi}_{15}^k &= \boldsymbol{P}_1 - \boldsymbol{M}_1 - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}}, \\
\boldsymbol{\Psi}_{16}^k &= -\boldsymbol{P}_1\boldsymbol{D} + \boldsymbol{M}_1\boldsymbol{D}_k - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}},\ \boldsymbol{\Psi}_{17}^k = \boldsymbol{P}_3, \\
\boldsymbol{\Psi}_{22}^k &= -(1-d)\boldsymbol{Q}_1 + \boldsymbol{P}_2 + \boldsymbol{P}_2^{\mathrm{T}}, \\
\boldsymbol{\Psi}_{23}^k &= \boldsymbol{M}_2\boldsymbol{A}_k,\ \boldsymbol{\Psi}_{24}^k = \boldsymbol{M}_2\boldsymbol{B}_k,\ \boldsymbol{\Psi}_{25}^k = -\boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{P}_1 - \boldsymbol{M}_2, \\
\boldsymbol{\Psi}_{26}^k &= \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{D}_k + \boldsymbol{M}_2\boldsymbol{D}_k,\ \boldsymbol{\Psi}_{27}^k = -\boldsymbol{P}_3, \\
\boldsymbol{\Psi}_{33}^k &= \boldsymbol{Q}_2 + \boldsymbol{M}_3 A_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_3 - \boldsymbol{X}, \\
\boldsymbol{\Psi}_{34}^k &= \boldsymbol{M}_3\boldsymbol{B}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}},\ \boldsymbol{\Psi}_{35}^k = -\boldsymbol{M}_3 + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}}, \\
\boldsymbol{\Psi}_{36}^k &= \boldsymbol{M}_3\boldsymbol{D}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}},\ \boldsymbol{\Psi}_{37}^k = 0, \\
\boldsymbol{\Psi}_{44}^k &= -(1-d)\boldsymbol{Q}_2 + \boldsymbol{M}_4\boldsymbol{B}_k + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_4, \\
\boldsymbol{\Psi}_{45}^k &= -\boldsymbol{M}_4 + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}}, \\
\boldsymbol{\Psi}_{46}^k &= \boldsymbol{M}_4\boldsymbol{D}_k + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}},\ \boldsymbol{\Psi}_{47}^k = 0, \\
\boldsymbol{\Psi}_{55}^k &= \boldsymbol{R}_1 - \boldsymbol{M}_5 - \boldsymbol{M}_5^{\mathrm{T}},
\end{aligned}
$$

$$\boldsymbol{\Psi}_{56}^k = \boldsymbol{M}_5\boldsymbol{D}_k - \boldsymbol{M}_6^{\mathrm{T}},\ \boldsymbol{\Psi}_{57}^k = \boldsymbol{P}_2,$$
$$\boldsymbol{\Psi}_{66}^k = -(1-d)\boldsymbol{R}_1 + \boldsymbol{M}_6\boldsymbol{D}_k + \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}},$$
$$\boldsymbol{\Psi}_{67}^k = -\boldsymbol{P}_2,\ \boldsymbol{\Psi}_{77}^k = -\tau\boldsymbol{S}.$$

4. Robust stability results

In the previous section, we developed a stability analysis approach for the fuzzy delayed neural network of the neutral type with time varying delays based on the LMI. In this section, we obtain a stability criterion for the fuzzy delayed neural network of the neutral type with uncertainties.

Theorem 2 Given scalars $\bar{\tau} \geq 0$ and $d > 0$, the equilibrium point of system (2) is globally and robustly stable if there exist matrices $\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{12}^{\mathrm{T}} & \boldsymbol{P}_{22} \end{bmatrix} > 0$, $\boldsymbol{Q}_1 = \boldsymbol{Q}_1^{\mathrm{T}} > 0, \boldsymbol{Q}_2 = \boldsymbol{Q}_2^{\mathrm{T}} > 0$, $\boldsymbol{R}_1 = \boldsymbol{R}_1^{\mathrm{T}} > 0$, $\boldsymbol{R}_2 = \boldsymbol{R}_2^{\mathrm{T}} > 0$, $\boldsymbol{S} = \boldsymbol{S}^{\mathrm{T}} > 0$, any matrices $\boldsymbol{N}_i$, $\boldsymbol{M}_i$ $(i = 1, \ldots, 6)$ such that the following conditions hold for $k = 1, 2, \ldots, m$:

$$\boldsymbol{\Xi}^k = \begin{bmatrix} \boldsymbol{\Xi}_{11}^k & \boldsymbol{\Xi}_{12}^k & \boldsymbol{\Xi}_{13}^k & \boldsymbol{\Xi}_{14}^k & \boldsymbol{\Xi}_{15}^k & \boldsymbol{\Xi}_{16}^k & \boldsymbol{\Xi}_{17}^k & \boldsymbol{\Xi}_{18}^k \\ * & \boldsymbol{\Xi}_{22}^k & \boldsymbol{\Xi}_{23}^k & \boldsymbol{\Xi}_{24}^k & \boldsymbol{\Xi}_{25}^k & \boldsymbol{\Xi}_{26}^k & \boldsymbol{\Xi}_{27}^k & \boldsymbol{\Xi}_{28}^k \\ * & * & \boldsymbol{\Xi}_{33}^k & \boldsymbol{\Xi}_{34}^k & \boldsymbol{\Xi}_{35}^k & \boldsymbol{\Xi}_{36}^k & \boldsymbol{\Xi}_{37}^k & \boldsymbol{\Xi}_{38}^k \\ * & * & * & \boldsymbol{\Xi}_{44}^k & \boldsymbol{\Xi}_{45}^k & \boldsymbol{\Xi}_{46}^k & \boldsymbol{\Xi}_{47}^k & \boldsymbol{\Xi}_{48}^k \\ * & * & * & * & \boldsymbol{\Xi}_{55}^k & \boldsymbol{\Xi}_{56}^k & \boldsymbol{\Xi}_{57}^k & \boldsymbol{\Xi}_{58}^k \\ * & * & * & * & * & \boldsymbol{\Xi}_{66}^k & \boldsymbol{\Xi}_{67}^k & \boldsymbol{\Xi}_{68}^k \\ * & * & * & * & * & * & \boldsymbol{\Xi}_{77}^k & \boldsymbol{\Xi}_{78}^k \\ * & * & * & * & * & * & * & \boldsymbol{\Xi}_{88}^k \end{bmatrix} < 0, \tag{11}$$

where

$$\boldsymbol{\Xi}_{11}^k = \boldsymbol{Q}_1 + \boldsymbol{\Gamma}_{11}^k + \boldsymbol{L}^{\mathrm{T}}\boldsymbol{X}\boldsymbol{L} + 2\boldsymbol{Y}\boldsymbol{L},$$
$$\boldsymbol{\Xi}_{12}^k = -\boldsymbol{P}_{12} + \boldsymbol{\Gamma}_{12}^k,\ \boldsymbol{\Xi}_{13}^k = \boldsymbol{\Gamma}_{13}^k - \boldsymbol{Y},$$
$$\boldsymbol{\Xi}_{14}^k = \boldsymbol{\Gamma}_{14}^k,\ \boldsymbol{\Xi}_{15}^k = \boldsymbol{P}_{11} + \boldsymbol{\Gamma}_{15}^k,$$
$$\boldsymbol{\Xi}_{16}^k = \boldsymbol{P}_{12} + \boldsymbol{\Gamma}_{16}^k,\ \boldsymbol{\Xi}_{17}^k = \boldsymbol{\Gamma}_{17}^k,$$
$$\boldsymbol{\Xi}_{22}^k = -(1-d)\boldsymbol{Q}_1 + \boldsymbol{\Gamma}_{22}^k,\ \boldsymbol{\Xi}_{23}^k = \boldsymbol{\Gamma}_{23}^k,$$
$$\boldsymbol{\Xi}_{24}^k = \boldsymbol{\Gamma}_{24}^k,\ \boldsymbol{\Xi}_{25}^k = \boldsymbol{P}_{12}^{\mathrm{T}} + \boldsymbol{\Gamma}_{25}^k,$$
$$\boldsymbol{\Xi}_{26}^k = \boldsymbol{P}_{22} + \boldsymbol{\Gamma}_{26}^k,\ \boldsymbol{\Xi}_{27}^k = \boldsymbol{\Gamma}_{27}^k,$$
$$\boldsymbol{\Xi}_{33}^k = \boldsymbol{Q}_2 + \boldsymbol{\Gamma}_{33}^k - \boldsymbol{X},\ \boldsymbol{\Xi}_{34}^k = \boldsymbol{\Gamma}_{34}^k,\ \boldsymbol{\Xi}_{35}^k = \boldsymbol{\Gamma}_{35}^k,$$
$$\boldsymbol{\Xi}_{36}^k = \boldsymbol{\Gamma}_{36}^k,\ \boldsymbol{\Xi}_{37}^k = \boldsymbol{\Gamma}_{37}^k,\ \boldsymbol{\Xi}_{44}^k = -(1-d)\boldsymbol{Q}_2 + \boldsymbol{\Gamma}_{44}^k,$$
$$\boldsymbol{\Xi}_{45}^k = \boldsymbol{\Gamma}_{45}^k,\ \boldsymbol{\Xi}_{46}^k = \boldsymbol{\Gamma}_{46}^k,$$
$$\boldsymbol{\Xi}_{47}^k = \boldsymbol{\Gamma}_{47}^k,\ \boldsymbol{\Xi}_{55}^k = \boldsymbol{R}_1 + \tau\boldsymbol{R}_2 + \boldsymbol{\Gamma}_{55}^k,$$
$$\boldsymbol{\Xi}_{56}^k = \boldsymbol{\Gamma}_{56}^k, \boldsymbol{\Xi}_{57}^k = \boldsymbol{\Gamma}_{57}^k,\ \boldsymbol{\Xi}_{66}^k = -(1-d)\boldsymbol{R}_1 + \boldsymbol{\Gamma}_{66}^k,$$
$$\boldsymbol{\Xi}_{67}^k = \boldsymbol{\Gamma}_{67}^k,\ \boldsymbol{\Xi}_{77}^k = \boldsymbol{\Gamma}_{77}^k - \tau\boldsymbol{R}_2,$$
$$\boldsymbol{\Xi}_{28}^k = \boldsymbol{\Xi}_{38}^k = \boldsymbol{\Xi}_{48}^k = \boldsymbol{\Xi}_{58}^k = \boldsymbol{\Xi}_{68}^k = \boldsymbol{\Xi}_{78}^k = 0,$$
$$\boldsymbol{\Xi}_{18}^k = \mathrm{diag}\{\boldsymbol{M}_1\boldsymbol{E}_k, \boldsymbol{M}_2\boldsymbol{E}_k, \boldsymbol{M}_3\boldsymbol{E}_k, \boldsymbol{M}_4\boldsymbol{E}_k, \boldsymbol{M}_5\boldsymbol{E}_k, \boldsymbol{M}_6\boldsymbol{E}_k\},$$
$$\boldsymbol{\Xi}_{88}^k = \mathrm{diag}\{-\epsilon_1\boldsymbol{I},\ -\epsilon_2\boldsymbol{I},\ -\epsilon_3\boldsymbol{I},\ -\epsilon_4\boldsymbol{I},\ -\epsilon_5\boldsymbol{I},\ -\epsilon_6\boldsymbol{I}\},$$

$$\boldsymbol{\Gamma}_{11}^k = \boldsymbol{N}_1 + \boldsymbol{N}_1^{\mathrm{T}} - \boldsymbol{M}_1\boldsymbol{C}_k - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_1 + \epsilon\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{G}_1,$$
$$\boldsymbol{\Gamma}_{12}^k = -\boldsymbol{N}_1 + \boldsymbol{N}_2^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_2^{\mathrm{T}},$$
$$\boldsymbol{\Gamma}_{13}^k = \boldsymbol{N}_3^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_3^{\mathrm{T}} + \boldsymbol{M}_1\boldsymbol{A}_k,$$
$$\boldsymbol{\Gamma}_{14}^k = \boldsymbol{N}_4^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}} + \boldsymbol{M}_1\boldsymbol{B}_k,$$
$$\boldsymbol{\Gamma}_{15}^k = \boldsymbol{N}_5^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}} - \boldsymbol{M}_1,$$
$$\boldsymbol{\Gamma}_{16}^k = \boldsymbol{N}_6^{\mathrm{T}} - \boldsymbol{C}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}} + \boldsymbol{M}_1\boldsymbol{D}_k,$$
$$\boldsymbol{\Gamma}_{17}^k = -\tau\boldsymbol{N}_1,\ \boldsymbol{\Gamma}_{22}^k = -\boldsymbol{N}_2 - \boldsymbol{N}_2^{\mathrm{T}},$$
$$\boldsymbol{\Gamma}_{23}^k = -\boldsymbol{N}_3^{\mathrm{T}} + \boldsymbol{M}_2\boldsymbol{A}_k,\ \boldsymbol{\Gamma}_{24}^k = -\boldsymbol{N}_4^{\mathrm{T}} + \boldsymbol{M}_2\boldsymbol{B}_k,$$
$$\boldsymbol{\Gamma}_{25}^k = -\boldsymbol{N}_5^{\mathrm{T}} - \boldsymbol{M}_2,\ \boldsymbol{\Gamma}_{26}^k = -\boldsymbol{N}_6^{\mathrm{T}} + \boldsymbol{M}_2\boldsymbol{D}_k,$$
$$\boldsymbol{\Gamma}_{27}^k = -\tau\boldsymbol{N}_2,\ \boldsymbol{\Gamma}_{33}^k = \boldsymbol{M}_3A_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_3 + \epsilon\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{G}_2,$$
$$\boldsymbol{\Gamma}_{34}^k = \boldsymbol{M}_3\boldsymbol{B}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}},$$
$$\boldsymbol{\Gamma}_{35}^k = -\boldsymbol{M}_3 + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}},\ \boldsymbol{\Gamma}_{36}^k = \boldsymbol{M}_3\boldsymbol{D}_k + \boldsymbol{A}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}},$$
$$\boldsymbol{\Gamma}_{37}^k = -\tau\boldsymbol{N}_3,\ \boldsymbol{\Gamma}_{47}^k = -\tau\boldsymbol{N}_4,$$
$$\boldsymbol{\Gamma}_{44}^k = \boldsymbol{M}_4\boldsymbol{B}_k + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}} + \epsilon\boldsymbol{G}_3^{\mathrm{T}}\boldsymbol{G}_3,$$
$$\boldsymbol{\Gamma}_{45}^k = -\boldsymbol{M}_4 + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}},$$
$$\boldsymbol{\Gamma}_{46}^k = \boldsymbol{M}_4\boldsymbol{D}_k + \boldsymbol{B}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}},\ \boldsymbol{\Gamma}_{55}^k = -\boldsymbol{M}_5 - \boldsymbol{M}_5^{\mathrm{T}},$$
$$\boldsymbol{\Gamma}_{56}^k = \boldsymbol{M}_5\boldsymbol{D}_k - \boldsymbol{M}_6^{\mathrm{T}},$$
$$\boldsymbol{\Gamma}_{57}^k = -\tau\boldsymbol{N}_5,\ \boldsymbol{\Gamma}_{66}^k = \boldsymbol{M}_6\boldsymbol{D}_k + \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}} + \epsilon\boldsymbol{G}_4^{\mathrm{T}}\boldsymbol{G}_4,$$
$$\boldsymbol{\Gamma}_{67}^k = -\tau\boldsymbol{N}_6,\ \boldsymbol{\Gamma}_{77}^k = 0.$$

Proof We use the following Lyapunov functional to derive the stability result:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),$$

where

$$\begin{aligned} V_1(t) &= \bar{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{x}}(t), \\ V_2(t) &= \int_{t-\tau(t)}^{t} \big[\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{x}(s) \\ &\quad + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s))\boldsymbol{Q}_2\boldsymbol{f}(\boldsymbol{x}(s))\big]\,\mathrm{d}s, \\ V_3(t) &= \int_{t-\tau(t)}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{\boldsymbol{x}}(s)\,\mathrm{d}s, \\ V_4(t) &= \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{x}}(s)\,\mathrm{d}s\,\mathrm{d}\theta, \end{aligned}$$

and $\bar{\boldsymbol{x}}(t) = [\boldsymbol{x}^{\mathrm{T}}(t)\ \boldsymbol{x}^{\mathrm{T}}(t-\tau(t))]^{\mathrm{T}}$. We can calculate the derivative of $V(t)$ along the trajectories of system (2). We have

$$\begin{aligned} \dot{V}(t) \le \sum_{k=1}^{m} \eta_k(\theta(t))\Big\{ & 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_{11}\dot{\boldsymbol{x}}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) \\ & \times \boldsymbol{P}_{12}^{\mathrm{T}}\dot{\boldsymbol{x}}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_{12}\dot{\boldsymbol{x}}(t-\tau(t)) \\ & + 2\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{P}_{22}\dot{\boldsymbol{x}}(t-\tau(t)) + \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}_1\boldsymbol{x}(t) \\ & + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{Q}_2\boldsymbol{f}(\boldsymbol{x}(t)) + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)[\boldsymbol{R}_1 + \tau\boldsymbol{R}_2]\dot{\boldsymbol{x}}(t) \\ & - (1-d)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}_1\boldsymbol{x}(t-\tau(t)) \\ & - (1-d)\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))\boldsymbol{Q}_2\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) \\ & - (1-d)\dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{R}_1\dot{\boldsymbol{x}}(t-\tau(t)) \\ & - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{\boldsymbol{x}}(s)\,\mathrm{d}s + 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Y}\boldsymbol{f}(\boldsymbol{x}(t)) \\ & + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{X}\boldsymbol{f}(\boldsymbol{x}(t)) - 2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Y}\boldsymbol{f}(\boldsymbol{x}(t)) \\ & - f^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{X}\boldsymbol{f}(\boldsymbol{x}(t))\Big\}. \end{aligned}$$

According to the Leibniz–Newton formula, for any matrices $\boldsymbol{N}_i$ $(i = 1, \ldots, 4)$, the following equations hold:

$$\begin{aligned} & 2\Big[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{N}_1 + \boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{N}_2 + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{N}_3 \\ & + f^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))\boldsymbol{N}_4 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{N}_5 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{N}_6\Big] \\ & \times \Big[\boldsymbol{x}(t) - \boldsymbol{x}(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{x}}(s)\,\mathrm{d}s\Big] = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} & 2\Big[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{M}_1 + \boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{M}_2 + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{M}_3 \\ & + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))\boldsymbol{M}_4 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{M}_5 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{M}_6\Big] \\ & \times \sum_{k=1}^{m} \eta_k(\theta(t))\Big\{ - \dot{\boldsymbol{x}}(t) - \bar{\boldsymbol{C}}_k\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_k\boldsymbol{f}(\boldsymbol{x}(t)) \\ & + \bar{\boldsymbol{B}}_k\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) + \bar{\boldsymbol{D}}_k\dot{\boldsymbol{x}}(t-\tau(t))\Big\} = 0. \end{aligned} \tag{13}$$

It follows from Lemma 2 and Eq. (3) that

$$\begin{aligned} & 2\Big[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{M}_1 + \boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{M}_2 \\ & + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{M}_3 + f^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))\boldsymbol{M}_4 \\ & + \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{M}_5 + \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{M}_6\Big] \\ & \times \Big[- \Delta\boldsymbol{C}_k(t)\boldsymbol{x}(t) + \Delta\boldsymbol{A}_k(t)\boldsymbol{f}(\boldsymbol{x}(t)) \\ & + \Delta\boldsymbol{B}_k(t)\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) + \Delta\boldsymbol{D}_k(t)\dot{\boldsymbol{x}}(t-\tau(t))\Big] \\ & \le \epsilon_1^{-1}\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{M}_1\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_1\boldsymbol{x}(t) + \epsilon_2^{-1}\boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) \\ & \times \boldsymbol{M}_2\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_2\boldsymbol{x}(t-\tau(t)) + \epsilon_3^{-1}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{M}_3E_k \\ & \times \boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_3\boldsymbol{f}(\boldsymbol{x}(t)) + \epsilon_4^{-1}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t))) \\ & \times \boldsymbol{M}_4\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_4\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) \\ & + \epsilon_5^{-1}\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{M}_5\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_5\dot{\boldsymbol{x}}(t) \\ & + \epsilon_6^{-1}\dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{M}_6E_kE_k^{\mathrm{T}}\boldsymbol{M}_6\dot{\boldsymbol{x}}(t-\tau(t)) \\ & + \epsilon\Big[\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{G}_{1k}^{\mathrm{T}}\boldsymbol{G}_{1k}\boldsymbol{x}(t) + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t))\boldsymbol{G}_{2k}^{\mathrm{T}}\boldsymbol{G}_{2k}\boldsymbol{f}(\boldsymbol{x}(t)) \\ & + \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))\boldsymbol{G}_{3k}^{\mathrm{T}}\boldsymbol{G}_{3k}\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) \\ & + \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\boldsymbol{G}_{4k}^{\mathrm{T}}\boldsymbol{G}_{4k}\dot{\boldsymbol{x}}(t-\tau(t))\Big]. \end{aligned} \tag{14}$$

From expresses (12)–(14), we have

$$\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \boldsymbol{\xi}^{\mathrm{T}}(t,s)\boldsymbol{\Gamma}^k\boldsymbol{\xi}(t,s)\,\mathrm{d}s = 0, \tag{15}$$

where

$$\boldsymbol{\Gamma}^k = \begin{bmatrix} \boldsymbol{\Gamma}_{10}^k & \boldsymbol{\Gamma}_{12}^k & \boldsymbol{\Gamma}_{13}^k & \boldsymbol{\Gamma}_{14}^k & \boldsymbol{\Gamma}_{15}^k & \boldsymbol{\Gamma}_{16}^k & \boldsymbol{\Gamma}_{17}^k \\ * & \boldsymbol{\Gamma}_{20}^k & \boldsymbol{\Gamma}_{23}^k & \boldsymbol{\Gamma}_{24}^k & \boldsymbol{\Gamma}_{25}^k & \boldsymbol{\Gamma}_{26}^k & \boldsymbol{\Gamma}_{27}^k \\ * & * & \boldsymbol{\Gamma}_{30}^k & \boldsymbol{\Gamma}_{34}^k & \boldsymbol{\Gamma}_{35}^k & \boldsymbol{\Gamma}_{36}^k & \boldsymbol{\Gamma}_{37}^k \\ * & * & * & \boldsymbol{\Gamma}_{40}^k & \boldsymbol{\Gamma}_{45}^k & \boldsymbol{\Gamma}_{46}^k & \boldsymbol{\Gamma}_{47}^k \\ * & * & * & * & \boldsymbol{\Gamma}_{50}^k & \boldsymbol{\Gamma}_{56}^k & \boldsymbol{\Gamma}_{57}^k \\ * & * & * & * & * & \boldsymbol{\Gamma}_{60}^k & \boldsymbol{\Gamma}_{67}^k \\ * & * & * & * & * & * & \boldsymbol{\Gamma}_{77}^k \end{bmatrix},$$

$$
\begin{aligned}
\boldsymbol{\Gamma}_{10}^{k} &= \boldsymbol{\Gamma}_{11}^{k} + \epsilon_1^{-1}\boldsymbol{M}_1\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_1^{\mathrm{T}} + \epsilon\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{G}_1,\\
\boldsymbol{\Gamma}_{20}^{k} &= \boldsymbol{\Gamma}_{22}^{k} + \epsilon_2^{-1}\boldsymbol{M}_2\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_2^{\mathrm{T}},\\
\boldsymbol{\Gamma}_{30}^{k} &= \boldsymbol{\Gamma}_{33}^{k} + \epsilon_3^{-1}\boldsymbol{M}_3\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_3^{\mathrm{T}} + \epsilon\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{G}_2,\\
\boldsymbol{\Gamma}_{40}^{k} &= \boldsymbol{\Gamma}_{44}^{k} + \epsilon_4^{-1}\boldsymbol{M}_4\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_4^{\mathrm{T}} + \epsilon\boldsymbol{G}_3^{\mathrm{T}}\boldsymbol{G}_3,\\
\boldsymbol{\Gamma}_{50}^{k} &= \boldsymbol{\Gamma}_{55}^{k} + \epsilon_5^{-1}\boldsymbol{M}_5\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_5^{\mathrm{T}},\\
\boldsymbol{\Gamma}_{60}^{k} &= \boldsymbol{\Gamma}_{66}^{k} + \epsilon_6^{-1}\boldsymbol{M}_6\boldsymbol{E}_k\boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{M}_6^{\mathrm{T}} + \epsilon\boldsymbol{G}_4^{\mathrm{T}}\boldsymbol{G}_4,\\
\epsilon &= \epsilon_1 + \epsilon_2 + \epsilon_3 + \epsilon_4 + \epsilon_5 + \epsilon_6,
\end{aligned}
$$

and $\boldsymbol{\xi}(t,s) = [\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) \;\; \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t)) \;\; \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(t-\tau(t)))\; \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\; \dot{\boldsymbol{x}}^{\mathrm{T}}(t-\tau(t))\; \dot{\boldsymbol{x}}^{\mathrm{T}}(s)]^{\mathrm{T}}$. Adding terms from the left-hand side of Eq. (15) to $\dot{V}(t)$, we can express $\dot{V}(t)$ as

$$
\dot{V}(\boldsymbol{x}(t)) = \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \boldsymbol{\xi}^{\mathrm{T}}(t,s)\boldsymbol{\Xi}^{k}\boldsymbol{\xi}(t,s)\mathrm{d}s,
$$

where $\boldsymbol{\Xi}^k$ are defined in inequality (11). If $\boldsymbol{\Xi}^k < 0$, then $\dot{V}(t) < 0$, which indicates from the Lyapunov stability theory that the dynamics of the fuzzy delayed neural network (2) is globally and robustly stable. This completes the proof.

Remark 2 When $r = 1$, system (3) is simplified to the general delayed neural network of the neutral type with time-varying delays and uncertainties. Recently, the delayed neural networks of the neutral type with time-varying delays were studied in Refs. [22]–[28]. Thus, our results are more general than those results already existing in the literature.

5. Numerical examples

Example 1 Consider the T–S fuzzy model of the delayed neural network of the neutral type with uncertainties in the following form (plant rules).

Rule 1 If $\{\theta_1(t) \text{ is } \eta^1\}$, then

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) = &-\bar{\boldsymbol{C}}_1\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_1\boldsymbol{f}(\boldsymbol{x}(t))\\
&+ \bar{\boldsymbol{B}}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))) + \bar{\boldsymbol{D}}_1\dot{\boldsymbol{x}}(t-\tau(t)).
\end{aligned}
$$

Rule 2 If $\{\theta_2(t) \text{ is } \eta^2\}$, then

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) = &-\bar{\boldsymbol{C}}_2\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_2\boldsymbol{f}(\boldsymbol{x}(t))\\
&+ \bar{\boldsymbol{B}}_2\boldsymbol{f}(\boldsymbol{x}(t-\tau(t)))\bar{\boldsymbol{D}}_2\dot{\boldsymbol{x}}(t-\tau(t)).
\end{aligned}
$$

Here

$$
\boldsymbol{C}_1 = \begin{bmatrix} 7 & 0 \\ 0 & 4 \end{bmatrix}, \quad \boldsymbol{A}_1 = \begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix},
$$

$$
\boldsymbol{B}_1 = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}, \quad \boldsymbol{D}_1 = \begin{bmatrix} 0.3 & 0.2 \\ -0.5 & 0.4 \end{bmatrix},
$$

$$
\boldsymbol{C}_2 = \begin{bmatrix} 3 & 0 \\ 0 & 8 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix},
$$

$$
\boldsymbol{B}_2 = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}, \quad \boldsymbol{D}_2 = \begin{bmatrix} 0.3 & -0.1 \\ 0.2 & 0.4 \end{bmatrix},
$$

$$
\boldsymbol{F}(t) = \begin{bmatrix} \sin(t) & -\cos(t) \\ -\cos(t) & -\sin(t) \end{bmatrix},
$$

$$
\boldsymbol{G}_{1k} = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix}, \quad \boldsymbol{G}_{2k} = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix}, \quad \boldsymbol{G}_{3k} = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix},
$$

$$
\boldsymbol{G}_{4k} = \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix}, \quad \boldsymbol{E}_k = [0.4\ 0.4],
$$

and $\boldsymbol{f}(\boldsymbol{x}) = \tanh(\boldsymbol{x})/5$. Obviously, the assumption is satisfied, $\boldsymbol{L} = \mathrm{diag}(0.2, 0.2)$. The membership functions for Rules 1 and 2 are $\eta^1 = 1/\mathrm{e}^{-2u_1(t)}$ and $\eta^2 = 1 - \eta^1$, respectively. By using the Matlab LMI toolbox, we solve the LMI (11) for $\epsilon > 0$, $d = 0.5$, and $\bar{\tau} = 1.5$. The feasible solutions are

$$
\boldsymbol{P}_{11} = \begin{bmatrix} 560.1649 & -25.5611 \\ -25.5611 & 298.8833 \end{bmatrix},
$$

$$
\boldsymbol{P}_{12} = \begin{bmatrix} 10.7802 & -1.1502 \\ 5.9033 & 6.1644 \end{bmatrix},
$$

$$
\boldsymbol{P}_{22} = \begin{bmatrix} 11.4812 & -0.5073 \\ -1.4712 & 4.6603 \end{bmatrix},
$$

$$
\boldsymbol{Q}_1 = \begin{bmatrix} 461.2914 & -97.6182 \\ -97.6182 & 401.3473 \end{bmatrix},
$$

$$
\boldsymbol{Q}_2 = 10^3\begin{bmatrix} 7.3293 & 0.0413 \\ 0.0413 & 5.9340 \end{bmatrix},
$$

$$
\boldsymbol{R}_1 = \begin{bmatrix} 71.0076 & -4.2793 \\ -4.2793 & 40.3323 \end{bmatrix},
$$

$$
\boldsymbol{R}_2 = \begin{bmatrix} 5.0884 & 0.0841 \\ 0.0841 & 1.4921 \end{bmatrix}.
$$

Therefore, the concerned neural networks with time varying delays are asymptotically stable.

Example 2 Consider the neural network of the neutral type with the following parameters:[23,27]

$$
\boldsymbol{C} = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix} \alpha & 0.1 \\ 0.1 & \alpha \end{bmatrix},
$$

$$
\boldsymbol{B} = \begin{bmatrix} 0.1 & 0.16 \\ 0.05 & 0.1 \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},
$$

with $\boldsymbol{f}(\boldsymbol{x}) = \tanh(\boldsymbol{x})$. By using the Matlab LMI toolbox, we solve the LMI (5) with $k = 1$. The feasible solutions for $\bar{\tau} = 1,\ d = 0$, and $\alpha = 1.3958$ are

$$\boldsymbol{P}_1 = \begin{bmatrix} 18.9231 & -0.4202 \\ -0.4202 & 19.3427 \end{bmatrix},$$

$$\boldsymbol{P}_2 = \begin{bmatrix} 0.9190 & -0.1798 \\ -0.1798 & 0.9583 \end{bmatrix},$$

$$\boldsymbol{P}_3 = \begin{bmatrix} 2.0030 & -0.0532 \\ -0.0532 & 2.0109 \end{bmatrix},$$

$$\boldsymbol{Q}_1 = \begin{bmatrix} 6.1459 & -0.4591 \\ -0.4591 & 6.2378 \end{bmatrix},$$

$$\boldsymbol{Q}_2 = \begin{bmatrix} 5.0582 & -0.0753 \\ -0.0753 & 5.3001 \end{bmatrix},$$

$$\boldsymbol{R}_1 = \begin{bmatrix} 5.2741 & -0.0200 \\ -0.0200 & 5.3659 \end{bmatrix},$$

$$\boldsymbol{R}_2 = \begin{bmatrix} 5.1311 & 0.0369 \\ 0.0369 & 5.2113 \end{bmatrix},$$

$$\boldsymbol{S} = \begin{bmatrix} 5.3831 & -0.3245 \\ -0.3245 & 5.4380 \end{bmatrix}.$$

Therefore, the concerned neural networks with time delays are asymptotically stable.

By applying the Theorem from Ref. [27] to the system, the maximum bound of α is found to be 1.198, and the maximum bound of α according to the criteria in Ref. [23] is 1.202. However, by using Theorem 1, we obtain the maximum bound of α to be 1.3958, which shows that our criterion is less conservative than that obtained in Refs. [23] and [27].

Example 3 Consider the neural network of the neutral type with the following parameters:[21,26]

$$\boldsymbol{C} = \begin{bmatrix} 0.9516 & 0 & 0 \\ 0 & 2.6649 & 0 \\ 0 & 0 & 0.1629 \end{bmatrix},$$

$$\boldsymbol{A} = \begin{bmatrix} -2.5362 & -0.6918 & -1.5937 \\ 0.7143 & -0.3420 & -1.4410 \\ 1.6236 & 0.3273 & -0.6289 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0.0877 & 1.1892 & 0.1746 \\ -1.1465 & -0.2376 & -0.1867 \\ 1.1909 & 0.3273 & 0.5258] \end{bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} -0.1999 & 0.3560 & 0.5954 \\ 0.3450 & 0.6451 & -0.6012 \\ 0.4078 & 0.3343 & -0.0099 \end{bmatrix},$$

$$\boldsymbol{L} = \begin{bmatrix} 0.2037 & 0 & 0 \\ 0 & 0.8020 & 0 \\ 0 & 0 & 0.1287 \end{bmatrix}.$$

By using the Matlab LMI toolbox, we solve the LMI (5) with $k = 1$. The feasible solutions for $\bar{\tau} = 2.0990$ and $d = 0.5$ are

$$\boldsymbol{P}_1 = \begin{bmatrix} 0.1493 & 0.0752 & 0.0711 \\ 0.0752 & 0.2816 & -0.1194 \\ 0.0711 & -0.1194 & 0.3511 \end{bmatrix},$$

$$\boldsymbol{P}_2 = \begin{bmatrix} 0.0007 & 0.0003 & 0.0002 \\ 0.0003 & 0.0018 & -0.0001 \\ 0.0002 & -0.0001 & 0.0007 \end{bmatrix},$$

$$\boldsymbol{P}_3 = \begin{bmatrix} 0.0043 & 0.0014 & 0.0009 \\ 0.0014 & 0.0229 & -0.0017 \\ 0.0009 & -0.0017 & 0.0028 \end{bmatrix},$$

$$\boldsymbol{Q}_1 = \begin{bmatrix} 0.10610.1206 - 0.0094 \\ 0.12060.9292 - 0.0319 \\ -0.0094 - 0.03190.0362 \end{bmatrix},$$

$$\boldsymbol{Q}_2 = \begin{bmatrix} 3.5996 & 0.6466 & 1.3448 \\ 0.6466 & 0.4184 & 0.1295 \\ 1.3448 & 0.1295 & 1.5278 \end{bmatrix},$$

$$\boldsymbol{R}_1 = \begin{bmatrix} 0.0030 & -0.0011 & -0.0005 \\ -0.0011 & 0.0047 & 0.0100 \\ -0.0005 & 0.0100 & 0.0337 \end{bmatrix},$$

$$\boldsymbol{R}_2 = \begin{bmatrix} 0.0100 & 0.0017 & 0.0248 \\ 0.0017 & 0.0019 & 0.0029 \\ 0.0248 & 0.0029 & 0.0663 \end{bmatrix},$$

$$\boldsymbol{S} = \begin{bmatrix} 0.0069 & 0.0010 & -0.0004 \\ 0.0010 & 0.0153 & -0.0009 \\ -0.0004 & -0.0009 & 0.0029 \end{bmatrix}.$$

Therefore, the concerned neural networks with time delays are asymptotically stable.

The equilibrium point of the system is asymptotically stable if $\tau = 0.2321$ according to the criteria in Ref. [21]. And the equilibrium point of the system is asymptotically stable if $\tau = 1.845$ according to the criteria in Ref. [26]. However, by using Theorem 1, we can obtain the maximum bound of the delay, which is

$\bar{\tau} = 2.0990$. Therefore, our result presents a less conservative result than those obtained in Refs. [21] and [26].

Example 4 Consider the neural network of the neutral type with the following parameters:[21,22]

$$\boldsymbol{C} = \begin{bmatrix} 2.7644 & 0 & 0 \\ 0 & 1.0185 & 0 \\ 0 & 0 & 10.2716 \end{bmatrix},$$

$$\boldsymbol{A} = \begin{bmatrix} 0.2651 & -3.1608 & -2.4687 \\ 3.1859 & -0.1573 & 0 \\ 2.0368 & -1.3633 & 0.5776 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} -0.7727 & -0.8370 & 3.8019 \\ 0.1004 & 0.6677 & -2.4431 \\ -0.6622 & 1.3109 & -1.8407 \end{bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} 0.2076 & 0.0631 & 0.3915 \\ -0.0780 & 0.3106 & 0.1009 \\ -0.2763 & 0.1416 & 0.3729 \end{bmatrix},$$

$$\boldsymbol{L} = \begin{bmatrix} 0.1019 & 0 & 0 \\ 0 & 0.3419 & 0 \\ 0 & 0 & 0.0633 \end{bmatrix},$$

with $\boldsymbol{f}(\boldsymbol{x}) = \tanh(\boldsymbol{x})$. By using the Matlab LMI toolbox, we solve the LMI (5) with $k = 1$. The feasible solutions for $\tau = 10$ and $d = 0.5$ are

$$\boldsymbol{P}_1 = \begin{bmatrix} 0.1262 & 0.1059 & -0.0285 \\ 0.1059 & 0.2113 & -0.0326 \\ -0.0285 & -0.0326 & 0.1178 \end{bmatrix},$$

$$\boldsymbol{P}_2 = \begin{bmatrix} 0.0068 & 0.0036 & -0.0022 \\ 0.0036 & 0.0064 & -0.0031 \\ -0.0022 & -0.0031 & 0.0107 \end{bmatrix},$$

$$\boldsymbol{P}_3 = \begin{bmatrix} 0.0334 & 0.0182 & -0.0117 \\ 0.0182 & 0.0286 & -0.0127 \\ -0.0117 & -0.0127 & 0.0862 \end{bmatrix},$$

$$\boldsymbol{Q}_1 = \begin{bmatrix} 0.2438 & 0.0941 & -0.0581 \\ 0.0941 & 0.2710 & 0.0784 \\ -0.0581 & 0.0784 & 0.7714 \end{bmatrix},$$

$$\boldsymbol{Q}_2 = \begin{bmatrix} 0.4199 & 0.0190 & -0.0715 \\ 0.0190 & 0.3723 & -0.4034 \\ -0.0715 & -0.4034 & 1.7812 \end{bmatrix},$$

$$\boldsymbol{R}_1 = \begin{bmatrix} 0.0074 & 0.0057 & -0.0028 \\ 0.0057 & 0.0181 & -0.0053 \\ -0.0028 & -0.0053 & 0.0040 \end{bmatrix},$$

$$\boldsymbol{R}_2 = \begin{bmatrix} 0.0005 & 0.0002 & -0.0001 \\ 0.0002 & 0.0010 & -0.0002 \\ -0.0001 & -0.0002 & 0.0002 \end{bmatrix},$$

$$\boldsymbol{S} = \begin{bmatrix} 0.0129 & 0.0059 & -0.0067 \\ 0.0059 & 0.0093 & -0.0074 \\ -0.0067 & -0.0074 & 0.0348 \end{bmatrix}.$$

Therefore, the concerned neural networks with time delays are asymptotically stable.

When the delay is time-invariant, the equilibrium point of the system in this example is asymptotically stable with the maximum delay of $\tau = 1.3044$ according to the criteria in Ref. [21]. While the bound of delay illustrated in Ref. [22] is $\tau = 6.5$. By solving LMI (5) given in Theorem 1 with $d = 0$, the LMI is feasible for any $\tau > 0$. It means that the system is actually delay-independent stable. Therefore, our result presents a less conservative result than those obtained in Refs. [21] and [22].

6. Conclusion

The delay-dependent robust stability problem is addressed for a class of uncertain fuzzy neutral neural networks with time varying delays. By constructing a suitable Lyapunov–Krasovskii functional, some new stability criteria have been presented to guarantee that these delayed neural networks are robustly, globally, and asymptotically stable for all admissible parameter uncertainties. The delay-dependent stability criterion is derived in terms of LMIs, which can be easily solved via the convex optimization algorithm. All these criteria are easy to check. Furthermore, the obtained results improve some existing results, and are less conservative than the previous results. Numerical examples demonstrate the effectiveness of the obtained results.

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