A genetic resampling particle filter for freeway traffic-state estimation*

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On-line estimation of the state of traffic based on data sampled by electronic detectors is important for intelligent traffic management and control. Because a nonlinear feature exists in the traffic state, and because particle filters have good characteristics when it comes to solving the nonlinear problem, a genetic resampling particle filter is proposed to estimate the state of freeway traffic. In this paper, a freeway section of the northern third ring road in the city of Beijing in China is considered as the experimental object. By analysing the traffic-state characteristics of the freeway, the traffic is modeled based on the second-order validated macroscopic traffic flow model. In order to solve the particle degeneration issue in the performance of the particle filter, a genetic mechanism is introduced into the resampling process. The realization of a genetic particle filter for freeway traffic-state estimation is discussed in detail, and the filter estimation performance is validated and evaluated by the achieved experimental data.

Keywords: particle filter, genetic mechanism, traffic-state estimation, traffic flow model

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1. Introduction

Traffic congestion is becoming a serious problem in urban road networks, so it is necessary to reinforce traffic management by advanced technologies. On-line traffic-state estimation is an important part of traffic management systems. Many proposed solutions for estimating aggregated traffic states are based on the application of the Kalman filter or extended Kalman filter.[1−3] Furthermore, most solutions are under the assumption of the Gaussian distribution of the noises and based on the linearization of the state model and the observation model. However, traffic-state data show strong nonlinear features, and the Kalman filter can therefore obtain poor estimation results and even cause serious filter divergence. The particle filter is a promising solution to the general nonlinear and/or non-Gaussian filtering problem.[4−6] The central idea of the particle filter is to represent the posterior probability density functions (PDF) of state by a set of particles with associated weights, and the estimation can be computed as the expected value of the discrete PDF. However, the problem of particle degeneracy exists in particle filters, that is, only a small number of particles have large weights, while most particles will have negligible weights. Then the inference is degraded because it is made by using only a very small number of particles. Resampling is proposed for the degeneracy problem.[4] The idea of resampling is to remove the particle trajectories with small weights and replicate the trajectories with large weights. However, resampling will bring about the loss of particle diversity because the important particles are replicated many times, which reduces the estimation accuracy. So in order to improve the estimation accuracy, the problems of degeneracy and diversity should be considered strongly in the application of the particle filter. In this paper, by applying the genetic mechanism to the resampling,[7,8] the genetic resampling particle filter is investigated and proposed to estimate freeway traffic states. The genetic resampling particle filter can increase particle diversity firstly based on ensuring the effectiveness of the particles, so it has good estimation accuracy for freeway traffic states.

The rest of this paper is organized as follows. In Section 2, the traffic-state characteristics of a freeway section of the northern third ring road in the city of Beijing in China are discussed, and the macroscopic freeway traffic flow model is constructed. In Section 3, the design of a traffic-state estimator is addressed based on the genetic resampling particle filter algo-

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rithm. In Section 4, experiments are conducted to evaluate the filter estimation performance, and finally the main conclusions are summarized in Section 5.

2. The traffic flow model of a freeway

2.1. Description of an example freeway

A freeway section between LianXiangqiao Bridge and JiMenqiao Bridge located in the northern third ring-road freeway in Beijing is considered for the experimental investigations. As shown in Fig. 1, the whole stretch is divided into three segments with three lanes, and each segment is 500 m long. Four detectors, D1, D2, D3, and D4, are used to detect the traffic flow, speed, and density. The location of D1 and D4 are, respectively, at the entrance and exit of the stretch, while D2 and D3 are installed, respectively, at the upper and lower boundaries of the related segment. The traffic data sampling period of each detector is 2 min, and the representative example of the entrance traffic state in 3 h between 7:30 AM and 10:30 AM on 13 May 2010 is shown in Fig. 2.

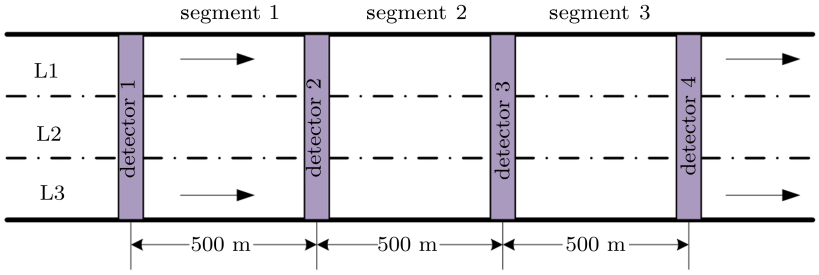


Fig. 1. (colour online) The layout of the experimental freeway.

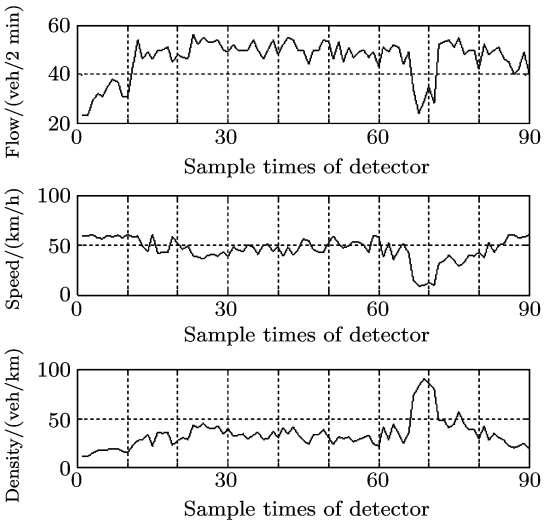


Fig. 2. Entrance traffic-state example.

2.2. The macroscopic traffic flow model of a freeway

The well-known second-order validated macroscopic traffic flow model in the discrete space-time form is employed to describe the dynamic behaviour of traffic flow along a freeway in terms of appropriate aggregated traffic variables, which are traffic density, space mean speed, and traffic flow.[9–11] In general, a stretch of freeway is subdivided into a number of segments (say, N segments) with length L_i, $i = 1, 2, \ldots, N$, while the time discretization is based on time step T and the discrete time indices $k = 0, 1, 2, \ldots$. In Fig. 2, neither the on-ramp nor off-ramp is in the freeway stretch, so for segment i, the stochastic nonlinear difference equations of the second-order macroscopic traffic flow model are simplified as follows:[9]

$$d_i(k+1) = d_i(k) + \frac{T}{\lambda_i L_i}[q_{i-1}(k) - q_i(k)], \quad (1)$$

$$\begin{aligned} v_i(k+1) = {} & v_i(k) + \frac{T}{\tau}[v(d_i(k)) - v_i(k)] \\ & + \frac{T}{L_i} v_i(k)[v_{i-1}(k) - v_i(k)] \\ & - \frac{vT}{\tau L_i}\frac{[d_{i+1}(k) - d_i(k)]}{d_i(k) + \kappa} + \xi_i^v(k), \end{aligned} \quad (2)$$

$$v(d) = v_{\mathrm{f}} \exp\left[-\frac{1}{a}\left(\frac{d}{d_{\mathrm{cr}}}\right)^a\right], \quad (3)$$

$$q_i(k) = d_i(k) v_i(k) \lambda_i + \xi_i^q(k). \quad (4)$$

In the functions (1)–(4), the variables and parameters are denoted as follows.

(i) Traffic density $d_i(k)$ (in (veh/km)/lane) is the number of vehicles in segment i at time kT, divided by the segment length L_i and lane number λ_i.

(ii) Space mean speed $v_i(k)$ (in km/h) is the average speed of all vehicles included in segment i at time kT.

(iii) Traffic flow $q_i(k)$ (in veh/h) is the number of vehicles leaving segment i during the time period $[kT, (k+1)T]$, divided by T.

Variables τ, ν, κ, and a are model parameters which are given the same values for all segments. $v_{\rm f}$ denotes the free speed, $d_{\rm cr}$ is the critical density, and a is the exponent of the stationary speed equation. $\xi_i^v(k)$ and $\xi_i^q(k)$ denote zero-mean Gaussian white noise acting on the empirical speed equation and the approximate flow equation, respectively, to reflect modeling inaccuracies.

The model parameters are normally unknown and may vary with environmental conditions. However, the model results are known to be most sensitive to variations in the free speed, critical density, and exponent.

2.3. The model of traffic measurement

Consider a traffic-state detector installed at the boundary of two adjacent segments, as illustrated in Fig. 2. For the speed of segment i, we have the speed measurement function as

$$Z_i^v(k) = v_i(k) + \xi_i^v(k), \tag{5}$$

where $Z_i^v(k)$ denotes the mean speed measurement during the time period $[(k-1)T, kT]$, and $\xi_i^v(k)$ denotes the corresponding speed measurement noise.

For the density of segment i, we have the density measurement function as

$$Z_i^d(k) = d_i(k) + \xi_i^d(k), \tag{6}$$

where $Z_i^d(k)$ denotes the mean density measurement during the time period $[(k-1)T, kT]$ and $\xi_i^d(k)$ denotes the corresponding density measurement noise.

3. The genetic resampling particle filter for freeway traffic-state estimation

3.1. An outline of the particle filter theory

According to the following discrete-time stochastic model of the dynamic system

$$x_k = f_k(x_{k-1}, w_{k-1}), \tag{7}$$

$$y_k = h_k(x_k, v_k), \tag{8}$$

where x_k represents the state vector at discrete time k, y_k represents the observation vector at discrete time k, f_k is the nonlinear state transition function, and h_k is the nonlinear observation function. The stochastic processes w_k and v_k represent the state noise process and the measurement noise process, respectively. The available information at time step k is the set of measurements $Z_k = \{y_i : i = 1, 2, \ldots, k\}$. It is assumed that the initial probability density function of the state vector $p(x_0|Z_0) = p(x_0)$ is available. According to the Bayesian estimation theory, we have

$$p(x_k|Z_{k-1}) = \int p(x_k|x_{k-1}) p(x_{k-1}|Z_{k-1}) \mathrm{d}x_{k-1}, \tag{9}$$

$$p(x_k|Z_k) = \frac{p(y_k|x_k) p(x_k|Z_{k-1})}{p(y_k|Z_{k-1})}, \tag{10}$$

$$p(y_k|Z_{k-1}) = \int p(y_k|x_k) p(x_k|Z_{k-1}) \mathrm{d}x_k. \tag{11}$$

Because it is very difficult to solve the integration in Eq. (9), the particle filter uses Monte Carlo techniques and the sequence importance sampling (SIS) method to solve the equation. The particle filter estimates the posterior PDF according to the N samples from the important distribution function, that is,

$$p(x_k|Z_k) = \sum_{i=1}^{N} w_k^i \delta(x_k - x_k^i), \tag{12}$$

$$w_k^i = \frac{\bar{w}_k^i}{\sum\limits_{i=1}^{N} \bar{w}_k^i}, \tag{13}$$

$$\bar{w}_k^i = \bar{w}_{k-1}^i \frac{p(y_k|x_k^i) p(x_k^i|x_{k-1}^i)}{q(x_k^i|x_{k-1}^i, y_k)}, \tag{14}$$

where w_k^i is the weight of each particle and w_k^i satisfies $0 \le w_k^i \le 1$, and q is an important distribution function. To obtain a good estimation accuracy, the important distribution function should be close to the real posterior distribution of state.

The particle filter is suitable for both nonlinear and non-Gaussian applications. However, in the freeway state estimation of the particle filter, the degeneration problem should be considered. The genetic resampling particle filter used to solve the problem is investigated to estimate the freeway state.

3.2. The genetic resampling particle filter algorithm

As a theoretical basis of the evolution idea, the genetic mechanism provides a new way to solve the particle degeneration problem. The genetic mechanism not

only selects excellent individuals using the selection operator, but also produces the new individuals using the crossover and mutation operators.[12] Therefore, the genetic mechanism can be used in the resampling procedure, and by appropriately setting the selection, crossover and mutation probabilities, the effectiveness and the diversity of the particles are both embodied. That is, firstly, each particle state is coded to be in binary form. Then each particle state is calculated by the selection, crossover, and mutation operators according to a certain probability, respectively. Finally, after each particle is decoded, the optimized particle set is built. Each optimized particle is duplicated by the resampling process. The realization steps of the genetic resampling particle filter for freeway traffic-state estimation are as follows.

i) **Initialization**

Set the iteration variable $k = 0$. For $i = 0, 1, 2, \ldots, N$, particle set $\{x_0^i, w_0^i\}$ is obtained from the initial distribution $p(x_0)$.

ii) **Particle generation**

For $i = 1, 2, \ldots, N$, sample $x_k^i \sim p(x_k|x_{k-1}^i, Z_k)$ according to Eqs. (1)–(4).

iii) **Weight computation**

On the receipt of a new measurement by the detectors in the segments, and in order to simplify the computation of weight in Eq. (14), the weight is computed

$$\bar{w}_k^i = \bar{w}_{k-1}^i \times p(y_k|x_k^i). \tag{15}$$

The normalized weight w_k^i is computed according to Eq. (13).

iv) **Genetic resampling**

a) Initial population. The size of the population consisting of particles, of which each is coded in the binary form, is N.

b) The prior probability density function from the state function is considered as the fitness function of the particle, and the parent generation particles are selected for the reproduction of the child generation according to the roulette selection method.

c) Crossover operation is executed with the appropriate crossover probability, then the crossover generation is produced.

d) Mutation operation is executed with the appropriate mutation probability, then the mutation generation is produced.

e) Resampling

Drawing a random sample u_i from the uniform distribution over $(0, 1]$. If u_i satisfies $\sum_{j=1}^{m-1} w_k^j < u_i \leq \sum_{j=1}^{m} w_k^j$, the number m particle is resampled as follows:

$$\widetilde{x}_k^i = x_k^m,$$

$$\widetilde{w}_k^i = w_k^m.$$

v) **State estimation**

The approximate posterior PDF is computed according to Eq. (12), then the state estimation is

$$\widehat{x}_k = \sum_{i=1}^{N} \tilde{w}_k^i \widetilde{x}_k^i. \tag{16}$$

vi) If the end condition of the algorithm is not satisfied, k is increased by itself and return to step ii). Otherwise, algorithm running ends.

4. Freeway state estimation experiments

As described in Section 2, the experimental freeway is composed of three segments, each with a length of 500 m, and divided into three lanes. The segment traffic flow, speed, and density are measured by the detectors. The state vector x_k and measurement vector Z_k are set, respectively, as

$$x_k = [v_1(k), d_1(k), v_2(k), d_2(k), v_3(k), d_3(k)]^{\mathrm{T}},$$

$$Z_k = [Z_1^v(k), Z_1^d(k), Z_2^v(k), Z_2^d(k), Z_3^v(k), Z_3^d(k)].$$

The parameters of the traffic model in Section 2 are set to be $T = 120$ s, $v_{\mathrm{f}} = 85$ km/h, $d_{\mathrm{cr}} = 60$ veh/km, $\tau = 20$ s, $v = 40$ km^2/s, and $\kappa = 25$ (veh/km)/lane. The number of particles $N = 500$. The initial state is set to be

$$x_0 = [39, 42, 52, 27, 48, 34]^{\mathrm{T}}.$$

The genetic algorithm performance is influenced by the parameters of population size, crossover probability, and mutation probability. In the experiment, set 500 for population size, 0.45 for crossover probability, and 0.12 for mutation probability. The fitness function is set to be particle weight. The speed and the density of the freeway are estimated by the genetic resampling particle filter algorithm in Section 3.

The relative error (RE), mean square error (MSE), and root-mean-square relative error (RMSRE), expressed, respectively, as formulas (17)–(19), are used to evaluate the estimation performance of the genetic resampling particle filter.

$$\mathrm{RE} = \frac{x_i - \widehat{x}_i}{x_i} \times 100\%, \tag{17}$$

$$\text{MSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \widehat{x}_i)^2}, \tag{18}$$

$$\text{RMSRE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i - \widehat{x}_i}{x_i}\right)^2}, \tag{19}$$

In formulas (17)–(19), x_i represents the true state, and $\widehat{x}_i$ represents the estimated state.

The REs of speed estimation and density estimation for each segment are, respectively, shown in Figs. 3 and 4. In the horizontal axis of the figure, each step time interval is 2 min, which is equal to the detector sample time.

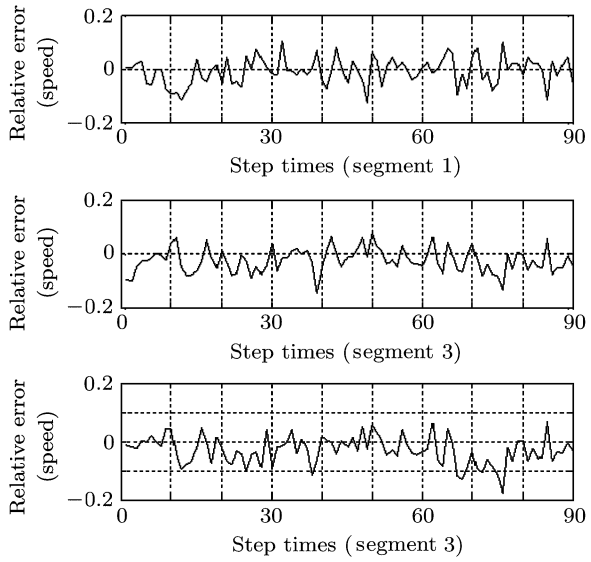


Fig. 3. REs of speed estimation for three segments.

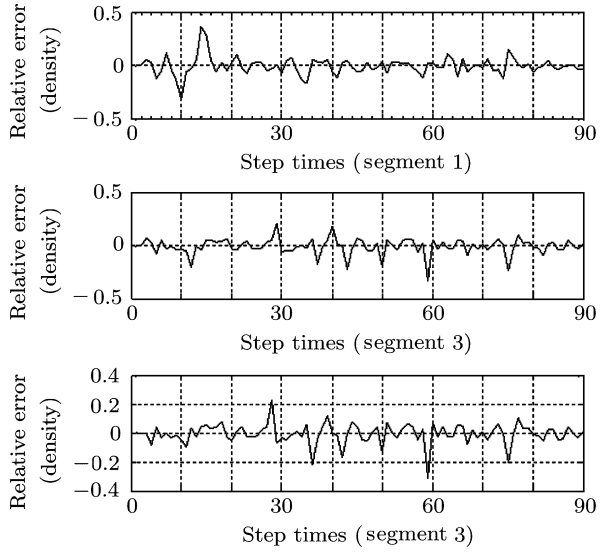


Fig. 4. REs of density estimation for three segments.

The MSE and RMSRE of speed and density estimation for each segment are shown in Table 1.

Table 1. MSE and RMSRE estimation.

Traffic variable	Error indicator			
	Speed		Density	
	MSE	RMSRE	MSE	RMSRE
Segment 1	2.1600	0.0509	3.0034	0.0768
Segment 2	2.9131	0.0511	2.0066	0.0733
Segment 3	3.2961	0.0565	1.8831	0.0701
Average	2.7897	0.0528	2.2977	0.0734

5. Conclusion

In theory, because the genetic mechanism is introduced into the resampling process of the particle filter, the genetic particle filter can effectively restrain particle degeneration and maintain particle diversity. The genetic particle filter can be optimized by three genetic factors, which are selection probability, crossover probability, and mutation probability. Because the nonlinear feature exists in the traffic state and the genetic particle filter has good characteristics for solving the nonlinear issue, the scheme proposed in this paper has good estimation accuracy. The practical experimental results show the validity of the genetic particle filter for freeway traffic-state estimation.

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