Kinetic evolutionary behavior of catalysis-select migration*

Wu Yuan-Gang(吴远刚), Lin Zhen-Quan(林振权)†, and Ke Jian-Hong(柯见洪)

Department of Physics, Wenzhou University, Wenzhou 325035, *China*

(Received 15 September 2011; revised manuscript received 5 January 2012)

We propose a catalysis-select migration driven evolution model of two-species (A- and B-species) aggregates, where one unit of species A migrates to species B under the catalysts of species C, while under the catalysts of species D the reaction will become one unit of species B migrating to species A. Meanwhile the catalyst aggregates of species C perform self-coagulation, as do the species D aggregates. We study this catalysis-select migration driven kinetic aggregation phenomena using the generalized Smoluchowski rate equation approach with C species catalysis-select migration rate kernel $K(k;i,j) = Kkij$ and D species catalysis-select migration rate kernel $J(k;i,j) = Jkij$. The kinetic evolution behaviour is found to be dominated by the competition between the catalysis-select immigration and emigration, in which the competition is between JD_0 and KC_0 (D_0 and C_0 are the initial numbers of the monomers of species D and C, respectively). When $JD_0 - KC_0 > 0$, the aggregate size distribution of species A satisfies the conventional scaling form and that of species B satisfies a modified scaling form. And in the case of $JD_0 - KC_0 < 0$, species A and B exchange their aggregate size distributions as in the above $JD_0 - KC_0 > 0$ case.

Keywords: catalysis-select migration, kinetic evolution behavior, rate equation, scaling law

PACS: 82.20.–w, 05.40.–a, 68.43.Jk, 89.75.Da **DOI:** 10.1088/1674-1056/21/6/068201

1. Introduction

The phenomenon of aggregation growth is very ubiquitous in a wide range of fields, such as physics, chemistry, biology, and demography. A multitude of growth phenomena in physical processes are driven by the exchange of particles (or units) between clusters: the droplet growth via evaporation and then condensation,[1] island growth in deposition processes,[2] phase ordering,[3,4] exchange processes have been used to model social and economical systems including the segregation of heterogeneous population,[5−8] the distribution of wealth in a society,[9] the growth of urban populations,[10] and so on. Aggregation is central to natural science not only for its abundant kinetic evolutional behavior arising from its complex mechanisms but it can also provide predictions concerning the evolution of aggregate system. The most original and popular studied mechanism is the pure cluster and cluster aggregation process,[11−15] and research has been extended steadily to aggregation phenomena with much more complex mechanisms.[16−18]

The kinetic evolutionary behavior of aggregates are of theoretical and practical significance for the understanding of the exchange migration phenomena by some complex mechanisms. In exchange aggregation processes, clusters are composed of atoms (monomers). Monomers detach from one cluster and reattach to another cluster (from one community to anther community) within a timescale. In the last few years, the exchange migration between clusters of two different species has been studied. The system consists of an ensemble of clusters, and clusters evolve via a transfer of a single monomer from one cluster to another.[19−21] In these models, irreversible growth of aggregates takes place through biased migration or unbiased exchange mechanisms. The biased migration mechanism can be described by an irreversible reaction scheme, $A_k + A_l \xrightarrow{K(k,l)} A_{k-1} + A_{l+1}$, where A_k denotes an aggregate characterized only by its size k, and the migration rate kernel $K(k,l)$ represents the rate of a monomer migrating from an aggregate of size k to another aggregate of size l. In an unbiased exchange model, an aggregate is equally likely to gain or lose a monomer. Recently, much attention has been devoted to generalized aggregation phenomena in sociology and economy. Ke and Lin investigated the kinetics of the general unbiased migration-driven aggregation system,[22] a variety of catalysis-driven aggre-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10875086 and 10775104).

†Corresponding author. E-mail: linzhenquan@yahoo.com.cn

© 2012 Chinese Physical Society and IOP Publishing Ltd

gation mechanisms,[23,24] and Lin and Ke further generalized the research to the exchange migration-driven aggregations with births and deaths to mimic the evolution of city populations and individual wealth.[25] Ben-Naim and Krapivsky made a general research on exchange-driven growth with the product and the generalized homogeneous rate kernel.[26] Lin and Wang investigated the competition between the catalyzed birth and death in the three species exchange-driven growth.[27] The results show that evolutionary behavior of the aggregate size distribution is drastically different from that for the corresponding system in normal space.

Consider a real system in nature and society, apart from the catalyzed birth process as in population-asset aggregation[28] or competition-driven individual's fitness advancement,[29] there may be catalyzed death processes which will play important roles in the aggregation growth of systems containing more than one species. Even more, in some special cases, mutually catalyzed death of two aggregates of different species will play important roles. Such as, in the commercial circle, there exists unavoidable competition between two corporations with different manufacture levels in the same industry catalogue. Based as much as possible on the profit they may obtain, both sides of the competition will take some measures to achieve their aims. Thus, the competition will become more and more drastic, and as a result, one of the two corporations will collapse. We can describe this phenomenon simply, referring to the two corporations as A and B. Let A_k and B_k respectively represent the aggregates of concentration in the two corporations of A and B. Aiming at a broad and general applicability of the model and results, it can be described by the exchange migration species reaction scheme: $A_k + B_l \xrightarrow{K_1(k,l)} A_{k+1} + B_{l-1}$, and $B_k + A_l \xrightarrow{K_2(k,l)} B_{k+1} + A_{l-1}$, with reaction rates $K_1(k,l), K_2(k,l)$. But, now there are some new phenomena in our life. For example, we present an interacting many-agent asset migration model with different companies that can be quantitatively analyzed using statistical physics tools. In this model, the interaction between the two agents results in a redistribution of their assets. We regard an asset as any economic attribute: cash, goods, or other materials, that contributes to overall individual wealth. The asset exchange migration with different firms or the same company in different departments is driven by the same benefits. In view of this, we consider the model that two different companies or two departments in the same firm cannot migrate directly. If they can migrate, they must need some extra elements to affect (drive), such as wealth migration in the different firms with different benefits driven. The different elements turn into different results for the firm to select. Here, for the convenience of solving the society phenomenon, we use A species and B species to denote the two companies or two departments. They can be migration under the two different catalysts, which we use C and D species to denote the two different catalysts, respectively.

More and more rate equations[30−32] have been studied. In this paper, we approach Smoluchowski's rate equations to study the behavior of the catalysis-select migration driven evolution model. The present investigation is based on the mean-field theory, which can be applied to the case where spatial dimension of the system is greater than a critical dimension or equal to a critical dimension d_c. Thus, the fluctuations in densities of the reactants are ignored and the aggregates are considered to be homogeneously distributed in the space throughout the process. Here we assume that the spatial dimension of this two-interacting species system is greater than the corresponding critical dimension and therefore, the mean-filed assumption is valid.

The rest of this paper is organized as follows. In Section 2, we introduce a model of the kinetic competing behavior of the two species of catalysis-select migration driven evolution studied by the generalized Smoluchowski's rate equation approach. In Section 3, we analyze the kinetic behavior of species A and B by the competition between the catalysis-select immigration and emigration, which inlaid the competition between JD_0 and KC_0. At last, a brief summary is given in Section 4.

2. A model of kinetic evolution behavior of catalysis-select migrations

In this more complex migration-driven evolution system, there are aggregates of two different species, which are denoted as species A and B. The migrations happen between two aggregates of different species, and the migration direction is dominated by a catalyst-selection. Under the catalysis of an aggregate of species C, one unit of an aggregate of species A migrates to an aggregate of species B with the catalyst aggregate remaining unchanged, while under the

catalysis of an aggregate of species D the catalysis–migration reaction will become one unit of an aggregate of species B migrating to an aggregate of species A and the catalyst aggregate also remains unchanged. Meanwhile the catalyst aggregates of species C perform self-coagulations, and so do the species D aggregates. Denoting A_k, B_k, C_k, and D_k as the aggregates of size k of species A, B, C, and D, respectively. The above catalysis-select migrations between aggregates of species A and B can be described as $A_i + B_j + C_k \xrightarrow{K(i,j;k)} A_{i-1} + B_{j+1} + C_k$ with reaction rate kernel $K(i,j;k)$ and $A_i + B_j + D_k \xrightarrow{J(i,j;k)} A_{i+1} + B_{j-1} + D_k$ with rate kernel $J(i,j;k)$, respectively. In particular, if one unit migrates out from an aggregate of size $k = 1$, the aggregate will become an empty aggregate and disappears, which gives rise to a reduction in the total number of aggregates of this species. The reaction schemes of the self-coagulations of the catalyst aggregates of species C and D are $C_i + C_j \xrightarrow{I_1(i,j)} C_{i+j}$ and $D_i + D_j \xrightarrow{I_2(i,j)} D_{i+j}$ respectively, where $I_1(i,j)$ and $I_2(i,j)$ are the self-coagulation rate kernels of species C and D.

The main way to study the kinetic aggregation evolution behaviour of such systems is to investigate the kinetic aggregation evolution behaviour of the concentration of the aggregate. Based on the mean-field assumption, we can investigate the evolution behaviour of the aggregate concentrations by means of the generalized Smoluchowski rate equation approach. The concentration (here it is the total number) of aggregates A_k is denoted as $a_k(t)$, and so $b_k(t)$, $c_k(t)$, and $d_k(t)$ for the aggregates of B_k, C_k, and D_k, respectively.

In general, the rate kernels of catalysis-select migrations are dependent on both the reactant aggregate's size and the catalyst aggregate's size, and so do for the self-coagulation rate kernels. Here for the convenience of solving the rate equations and to perform a basic investigation on the evolution, we focus on typical catalysis-select migration kernels such as $K(i,j;k) = Kijk$ and $J(i,j;k) = Jijk$, which are proportional to the sizes of all the reactant aggregates and K and J are two constants. The self-coagulation rate kernels are simply considered as $I_1(i,j) = I_1$ and $I_2(i,j) = I_2$, which are both constants.

We generalize the rate equations for the catalysis-select migration processes in the mean-field frame on the assumption that for any reaction it proceeds at a rate proportional to the reactant concentrations and write the corresponding rate equations for our system as follows:

$$\begin{aligned}
\frac{\mathrm{d}a_k}{\mathrm{d}t} &= K(k+1)a_{k+1}\sum_{l=1}^{\infty} jb_j \sum_{l=1}^{\infty} lc_l \\
&\quad - Kka_k \sum_{l=1}^{\infty} jb_j \sum_{l=1}^{\infty} lc_l \\
&\quad + J(k-1)a_{k-1}\sum_{l=1}^{\infty} jb_j \sum_{l=1}^{\infty} ld_l \\
&\quad - Jka_k \sum_{l=1}^{\infty} jb_j \sum_{l=1}^{\infty} ld_l, \qquad (1)\\
\frac{\mathrm{d}b_k}{\mathrm{d}t} &= K(k-1)b_{k-1}\sum_{l=1}^{\infty} ja_j \sum_{l=1}^{\infty} lc_l \\
&\quad - Kkb_k \sum_{l=1}^{\infty} ja_j \sum_{l=1}^{\infty} lc_l \\
&\quad + J(k+1)b_{k+1}\sum_{l=1}^{\infty} ja_j \sum_{l=1}^{\infty} ld_l \\
&\quad - Jkb_k \sum_{l=1}^{\infty} ja_j \sum_{l=1}^{\infty} ld_l, \qquad (2)\\
\frac{\mathrm{d}c_k}{\mathrm{d}t} &= \frac{1}{2}I_1 \sum_{i+j=k}^{\infty} c_i c_j - I_1 \sum_{j=1}^{\infty} c_k c_j, \qquad (3)\\
\frac{\mathrm{d}d_k}{\mathrm{d}t} &= \frac{1}{2}I_2 \sum_{i+j=k}^{\infty} d_i d_j - I_2 \sum_{j=1}^{\infty} d_k d_j. \qquad (4)
\end{aligned}$$

These equations describe a different catalysis-select migration system, without any spatial correlations, as infinite dimensional lattice systems or complete graphs. In the following, equations (1)–(4) are discussed, and from their properties, we gain intuition on the effects of stochasticity.

Several methods have been developed to solve the rate equations in different competing exchange or migration processes. In this paper, we employ the special ansatz

$$\begin{aligned}
&a_k(t) = A(t)[a(t)]^{k-1}, \quad b_k(t) = B(t)[b(t)]^{k-1}, \\
&c_k(t) = C(t)[c(t)]^{k-1}, \quad d_k(t) = D(t)[d(t)]^{k-1} \qquad (5)
\end{aligned}$$

to solve the rate equations to study the evolution behavior under the monodisperse initial conditions:

$$\begin{aligned}
&a_k(0) = A_0\delta_{k1}, \quad b_k(0) = B_0\delta_{k1}, \\
&c_k(0) = C_0\delta_{k1}, \quad d_k(0) = D_0\delta_{k1}, \qquad (6)
\end{aligned}$$

where A_0, B_0, C_0, and D_0 are initial concentrations of species A, B, C, and D, respectively. The special ansatz was first used by Krapivsky in the aggregation–annihilation process[33,34] and the applicability of the ansatz was analyzed completely in Ref. [27]. Here we find that our current rate Eqs. (1)–(4) can be solved with the help of the ansatz.

The catalyst aggregates of species C and D perform only the same irreversible aggregation processes, so their evolutionary kinetic behaviours are the same and are relatively simple. The kinetics of the simple irreversible aggregation process has been studied in the former work under the monodisperse initial condition $c_k(0) = C_0\delta_{k1}$, $d_k(0) = D_0\delta_{k1}$. The accurate solution to the aggregate size distribution of species C is[35]

$$c_k(t) = 4C_0(2 + I_1C_0t)^{-2}\left(1 - \frac{2}{2 + I_1C_0t}\right)^{k-1}. \quad (7)$$

It satisfies the conventional scaling law in the longtime limit[31]

$$c_k(t) \simeq 4C_0^{-1}I_1^{-2}t^{-2}\Phi(x), \quad x = \frac{k}{S_C(t)}, \quad (8)$$

where $\Phi(x) = \exp(-x)$ is the scaling function and $S_C(t) = I_1C_0t/2$ is the characteristic size of aggregates of species C, which plays a role similar to the correlation length in critical phenomenon. Moreover, the total number and the total mass of C aggregates are derived respectively as

$$M_0^C(t) = \sum_{k=1}^{\infty} c_k(t) = 2C_0(2 + I_1C_0t)^{-1} \simeq 2I_1^{-1}t^{-1},$$
$$M_1^C(t) = \sum_{k=1}^{\infty} kc_k(t) = C_0. \quad (9)$$

The total number decays linearly with time as t^{-1}, and the total mass is conserved.

Species D has a similar kinetic evolution behavior, and the total number and the total mass of D aggregates are

$$M_0^D(t) = \sum_{k=1}^{\infty} d_k(t) = 2D_0(2 + I_2D_0t)^{-1} \simeq 2I_2^{-1}t^{-1},$$
$$M_1^D(t) = \sum_{k=1}^{\infty} kd_k(t) = D_0. \quad (10)$$

Under the ansatz (5), the infinite set of rate equations for $a_k(t)$ $(k = 1, 2, \ldots)$ are reduced to two ordinary differential equations for $a(t)$ and $A(t)$ as

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -Ka(1-a)M_1^BM_1^C + J(1-a)M_1^BM_1^D, \quad (11)$$
$$\frac{\mathrm{d}A}{\mathrm{d}t} = 2KAaM_1^BM_1^C - KAM_1^BM_1^C - JAM_1^BM_1^D, \quad (12)$$

where the moments M_1^A and M_1^B are the total masses of aggregates of species A and B, respectively. The infinite set of rate equations for $b_k(t)$ $(k = 1, 2, \ldots)$ are also reduced to two ordinary differential equations for $b(t)$ and $B(t)$ as

$$\frac{\mathrm{d}b}{\mathrm{d}t} = -Jb(1-b)M_1^AM_1^D + K(1-b)M_1^AM_1^C, \quad (13)$$
$$\frac{\mathrm{d}B}{\mathrm{d}t} = 2JBbM_1^AM_1^D - JBM_1^AM_1^D - KBM_1^AM_1^C. \quad (14)$$

On the ansatz (5), the monodisperse initial conditions (6) for a_k and b_k are transformed correspondingly into

$$a = b = 0, \quad A = A_0, \quad B = B_0, \quad \text{at } t = 0. \quad (15)$$

By the ansatz (5), the first two moments of the aggregate size distribution $a_k(t)$, i.e., the total number and the total mass of aggregates of species A, can be expressed as

$$M_0^A(t) = \sum_{k=1}^{\infty} a_k(t) = A(t)\sum_{k=1}^{\infty}[a(t)]^{k-1} = \frac{A(t)}{1-a(t)}, \quad (16)$$
$$M_1^A(t) = \sum_{k=1}^{\infty} ka_k(t) = A(t)\sum_{k=1}^{\infty} k[a(t)]^{k-1} = \frac{A(t)}{[1-a(t)]^2}. \quad (17)$$

And the first two moments of the aggregate size distributions $b_k(t)$ can also be expressed in the similar forms,

$$M_0^B(t) = \sum_{k=1}^{\infty} b_k(t) = B(t)\sum_{k=1}^{\infty}[b(t)]^{k-1} = \frac{B(t)}{1-b(t)}, \quad (18)$$
$$M_1^B(t) = \sum_{k=1}^{\infty} kb_k(t) = B(t)\sum_{k=1}^{\infty} k[b(t)]^{k-1} = \frac{B(t)}{[1-b(t)]^2}. \quad (19)$$

From Eqs. (16), (17) and Eqs. (18), (19), we derive $M_0^A/M_1^A = 1 - a$ and $M_0^B/M_1^B = 1 - b$, respectively.

On the above expression for the total aggregate numbers and the total aggregate masses of species A and B, we can derive the evolutionary differential equations for these moments from the rate equations (1) and (2),

$$\frac{\mathrm{d}\ln M_0^A}{\mathrm{d}t} = \frac{1}{1-a}\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}t} = -K\frac{M_0^A}{M_1^A}M_1^BM_1^C, \quad (20)$$

$$\frac{\mathrm{d}\ln M_1^A}{\mathrm{d}t} = \frac{2}{1-a}\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}t} = (JM_1^D - KM_1^C)M_1^B, \quad (21)$$

$$\frac{\mathrm{d}\ln M_0^B}{\mathrm{d}t} = \frac{1}{1-b}\frac{\mathrm{d}b}{\mathrm{d}t} + \frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = -J\frac{M_0^B}{M_1^B}M_1^A M_1^D, \quad (22)$$

$$\frac{\mathrm{d}\ln M_1^B}{\mathrm{d}t} = \frac{2}{1-b}\frac{\mathrm{d}b}{\mathrm{d}t} + \frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = -(JM_1^D - KM_1^C)M_1^A. \quad (23)$$

Correspondingly, the initial conditions for these moments are $M_0^A(0) = M_1^A(0) = A_0$, $M_0^B(0) = M_1^B(0) = B_0$.

Using the conservation results $M_1^C(t) = C_0$ and $M_1^D(t) = D_0$, equations (20)–(23) can further be written into

$$\frac{\mathrm{d}\ln M_0^A}{\mathrm{d}t} = -KC_0\frac{M_1^B}{M_1^A}M_0^A, \quad (24)$$

$$\frac{\mathrm{d}\ln M_1^A}{\mathrm{d}t} = (JD_0 - KC_0)M_1^B, \quad (25)$$

$$\frac{\mathrm{d}\ln M_0^B}{\mathrm{d}t} = -JD_0\frac{M_1^A}{M_1^B}M_0^B, \quad (26)$$

$$\frac{\mathrm{d}\ln M_1^B}{\mathrm{d}t} = -(JD_0 - KC_0)M_1^A. \quad (27)$$

From Eqs. (25) and (27), we find that the total mass of the system is conserved, $M_1^A + M_1^B = M_1^A(0) + M_1^B(0) = A_0 + B_0$. This is a natural result because the catalysis-select migrations give rise to the exchange of mass between species A and B, which does not change the total mass of the system. Using this solution, equation (25) can be further rewritten into

$$\frac{\mathrm{d}\ln M_1^A}{\mathrm{d}t} = (JD_0 - KC_0)(A_0 + B_0 - M_1^A). \quad (28)$$

The total mass of species A can be obtained from the exact solution of the above equation

$$M_1^A(t) = \frac{A_0 + B_0}{1 + (B_0/A_0)\,\mathrm{e}^{-(JD_0 - KC_0)(A_0+B_0)t}}. \quad (29)$$

And the total mass of species B can be obtained

$$M_1^B(t) = \frac{A_0 + B_0}{1 + (A_0/B_0)\,\mathrm{e}^{(JD_0 - KC_0)(A_0+B_0)t}}. \quad (30)$$

From these solutions we can find that the competition between the catalysis-select immigration and emigration laid in the competition between JD_0 and KC_0. If $JD_0 - KC_0 > 0$, the net result of the catalysis-select migrations is some units of species B becoming units of species A, and the total mass of A grows while the total mass of B declines. On contrast, If $JD_0 - KC_0 < 0$, the net result of the catalysis-select migrations is some units of species A becoming units of species B. Meanwhile, in the marginal case of $JD_0 - KC_0 = 0$, there are no net units transforming between species A and B.

The self-coagulations of the catalysis aggregates of species C and D play no role in kinetic evolution of species A and B. In fact, this comes from the catalysis-select migration rate kernels $K(i,j;k) = Kijk$ and $J(i,j;k) = Jijk$, which are proportional to the size of the catalyst aggregates. From the rate equations (1) and (2), it is apparent that if the rate kernels $K(i,j;k)$ and $J(i,j;k)$ are not proportional to the size of the catalyst aggregates, the self-coagulation of the catalysis aggregate of species C and D will play a role on the kinetic evolution of species A and B.

Now we investigate the evolution behavior of the total aggregate number M_0^A and M_0^B. Inserting the solutions of M_1^A and M_1^B into Eqs. (24) and (26), we derive the solutions for M_0^A and M_0^B as follows:

$$M_0^A(t) = \frac{A_0(JD_0 - KC_0)(A_0 + B_0)}{(JD_0 - KC_0)(A_0 + B_0) + KC_0B_0[1 - \mathrm{e}^{-(JD_0 - KC_0)(A_0+B_0)t}]}, \quad (31)$$

$$M_0^B(t) = \frac{B_0(JD_0 - KC_0)(A_0 + B_0)}{(JD_0 - KC_0)(A_0 + B_0) + JD_0A_0[\mathrm{e}^{(JD_0 - KC_0)(A_0+B_0)t} - 1]}. \quad (32)$$

We introduce a constant m, $m = (JD_0 - KC_0)(A_0 + B_0)$, which reflects the competition between the catalysis-select immigration and the emigration of species A. Equations (29)–(32) can then be written simply as

$$M_1^A(t) = \frac{A_0 + B_0}{1 + (B_0/A_0)\,\mathrm{e}^{-mt}}, \quad (33)$$

$$M_1^B(t) = \frac{A_0 + B_0}{1 + (A_0/B_0)\,\mathrm{e}^{mt}}, \quad (34)$$

$$M_0^A(t) = \frac{A_0 m}{m + KC_0B_0(1 - \mathrm{e}^{-mt})}, \quad (35)$$

$$M_0^B(t) = \frac{B_0 m}{m + JD_0A_0(\mathrm{e}^{mt} - 1)}. \quad (36)$$

Using solutions of Eqs. (33) and (35) with the

total numbers and the total masses of species A aggregates, we derive the solution for $a(t)$:

$$a(t) = 1 - \frac{M_0^A}{M_1^A} = 1 - \frac{(JD_0 - KC_0)(A_0 + B_0\,\mathrm{e}^{-mt})}{KB_0C_0(1 - \mathrm{e}^{-mt}) + m}. \tag{37}$$

With the relationship $A = M_0^A(1-a)$, we obtain the accurate solution to the aggregate size distribution of A species

$$a_k(t) = \frac{A_0 m(JD_0 - KC_0)(A_0 + B_0\,\mathrm{e}^{-mt})}{[m + KB_0C_0(1 - \mathrm{e}^{-mt})]^2} \times \left[1 - \frac{(JD_0 - KC_0)(A_0 + B_0\,\mathrm{e}^{-mt})}{m + KB_0C_0(1 - \mathrm{e}^{-mt})}\right]^{k-1}. \tag{38}$$

In the same way, we obtain the kinetic behavior of the aggregate size distribution of B species as follows:

$$b_k(t) = \frac{B_0 m(JD_0 - KC_0)(B_0 + A_0\,\mathrm{e}^{mt})}{[m + JD_0A_0(\mathrm{e}^{mt} - 1)]^2} \times \left[1 - \frac{(JD_0 - KC_0)(B_0 + A_0\,\mathrm{e}^{mt})}{m + JD_0A_0(\mathrm{e}^{mt} - 1)}\right]^{k-1}. \tag{39}$$

In the next section, in order to thoroughly investigate the kinetic behavior of the system on the aggregate size distribution of species A and B, we solve Eqs. (38) and (39) in different cases.

3. Kinetic evolutionary behavior of catalysis-select migrations

3.1. Case of $JD_0 > KC_0$

In this case, $m = (JD_0 - KC_0)(A_0 + B_0) > 0$, for species A the catalysis-select immigration dominates the evolution and the catalysis-select emigration dominates the evolution of species B.

In the scaling region of $k \gg 1$ and $t \gg 1$, the aggregate size distribution of species A becomes

$$a_k(t) \simeq A_0B_0^{-2}C_0^{-2}K^{-2}t^{-2}\exp\left(-\frac{k}{KB_0C_0t}\right). \tag{40}$$

It approaches the conventional scaling form:

$$a_k(t) \simeq Ct^{-w}\Phi[k/S_A(t)], \quad S_A(t) \propto t^z \tag{41}$$

with the scaling function $\Phi(x) = \exp(-x)$ and the scaling parameters are $w = 2$ and $z = 1$. Here C is a constant, $C = A_0B_0^{-2}C_0^{-2}K^{-2}$. The characteristic aggregate size of A species is

$$S_A(t) = KB_0C_0t, \tag{42}$$

which grows with time as $S_A(t) \propto t$.

And in the scaling region of $k \gg 1$ and $t \gg 1$, the aggregate size distribution of species B becomes

$$b_k(t) \simeq B_0A_0^{-2}D_0^{-2}J^{-2}\,\mathrm{e}^{-mt}\exp\left(-\frac{k}{JA_0D_0\,\mathrm{e}^{mt}}\right). \tag{43}$$

It obeys a generalized scaling form

$$b_k(t) \simeq C[S_B(t)]^{-1}\Phi[k/S_B(t)], \tag{44}$$

with the scaling function $\Phi(x) = \exp(-x)$, the constant $C = B_0A_0^{-1}D_0^{-1}J^{-1}$, and the characteristic aggregate size is

$$S_B(t) = JA_0D_0\,\mathrm{e}^{mt}, \tag{45}$$

which grows exponentially.

3.2. Case of $JD_0 < KC_0$

In this case, $m = (JD_0 - KC_0)(A_0 + B_0) < 0$, for species A the catalysis-select emigration dominates the evolution and the catalysis-select immigration dominates the evolution of species B.

In the scaling region of $k \gg 1$ and $t \gg 1$, the aggregate size distribution of species A becomes

$$a_k(t) \simeq A_0B_0^{-2}C_0^{-2}K^{-2}\,\mathrm{e}^{mt}\exp\left(-\frac{k}{KB_0C_0\,\mathrm{e}^{-mt}}\right). \tag{46}$$

It approaches the generalized scaling form:

$$a_k(t) \simeq C[S_A(t)]^{-1}\Phi[k/S_A(t)], \tag{47}$$

with the scaling function $\Phi(x) = \exp(-x)$, the constant $C = A_0B_0^{-1}C_0^{-1}K^{-1}$, and the characteristic aggregate size is

$$S_A(t) = KB_0C_0\,\mathrm{e}^{-mt}. \tag{48}$$

In the same way, we obtain the accurate solution to the aggregate size distribution of B species, with the scaling region of $k \gg 1$ and $t \gg 1$, its $b_k(t)$ becomes:

$$b_k(t) \simeq B_0A_0^{-2}D_0^{-2}J^{-2}t^{-2}\exp\left(-\frac{k}{JA_0D_0t}\right). \tag{49}$$

It approaches the conventional scaling form:

$$b_k(t) \simeq Ct^{-w}\Phi[k/S_B(t)], \quad S_B(t) \propto t^z \tag{50}$$

with the scaling function $\Phi(x) = \exp(-x)$ and the scaling parameters are $w = 2$ and $z = 1$. Here C is a constant, $C = B_0A_0^{-2}D_0^{-2}J^{-2}$. And $S_B(t)$ denotes the characteristic aggregate size of B species, which

plays a role analogous to the correlation length in ordinary critical phenomena. Here it is

$$S_B(t) = JA_0D_0t, \tag{51}$$

which changes with time as $S_B(t) \propto t$ and changes much slower than $S_A(t)$.

3.3. Case of $JD_0 = KC_0$

In this case, the characteristic aggregate size of A and B species is a constant. There are no net units transforming between species A and B.

4. Summary

In summary, we have introduced the kinetic behavior of the catalysis-select migration evolution model of two-species (A and B species) aggregates under two-catalysts (C and D clusters) in social and economic systems. In this model, A and B species aggregates perform to compete with each other and with the catalyst, while the catalyst aggregates of the species C and D perform self-coagulation processes with the self-coagulation rate kernel. By means of the generalized Smoluchowski rate equation approach based on the mean-field assumption, we studied the kinetic behavior of the system with the catalysis-select migrations rate kernel and investigated the evolution behavior of the two species aggregates to study the competition scheme under the different catalysts and the reaction rate coefficients. We found that the competition between the catalysis-select immigration and emigration inlaid the competition between JD_0 and KC_0. When $JD_0 - KC_0 > 0$, the characteristic aggregate size of A species changes slower than the characteristic aggregate size of B species. It means that the net result of the catalysis-select migration is some units of species B becoming units of species A, and the total mass of A grows while the total mass of B declines. The $a_k(t)$ satisfies the conventional scaling form and the $b_k(t)$ satisfies the generalized scaling form. In contrast, when $JD_0 - KC_0 < 0$, the characteristic aggregate size of A species changes faster than the characteristic aggregate size of B species. That means the net result of the catalysis-select migration is some units of species A becoming units of species B. The $a_k(t)$ satisfies the generalized scaling form and the $b_k(t)$ satisfies the conventional scaling form. Meanwhile, in the marginal case of $JD_0 - KC_0 = 0$, there are no net units transforming between species A and B.

References

[1] Meakin P F 1998 *Scaling and Growth Far from Equilibrium* (New York: Cambridge University Press)

[2] Zangwill A 1998 *Physics at Surface* (New York: Cambridge University Press)

[3] Lifshitz I M and Slyozov V V 1961 *J. Phys. Chem. Solids* **19** 35

[4] Bdv A J 1994 *Phys. Rep.* **43** 357

[5] Anderson P W, Arrow k J and Pines D 1988 *The Economy as an Evolving Complex System* (Redwood: Addison-Wesley)

[6] Brenner Y S, Reijinders J P G and Spithoven A H G M 1988 *The Theory of Income and Wealth Distribution* (New York: St. Martin Press)

[7] Amaral L A N, Gopikrishnan P, Plerou V and Stanly H E 2001 *Physica* A **299** 127

[8] Schelling T 1971 *Math. Soc.* **1** 61

[9] Ispolatov S, Krapivsky P L and Redner S 1998 *Eur. Phys. J.* B **2** 267

[10] Leyvraz F and Redner S 2002 *Phys. Rev. Lett.* **88** 068301

[11] Ernst M H, Hendriks E M and Ziff R M 1982 *J. Phys.* A: *Math. Gen.* **15** l743

[12] Kang K and Redner S 1984 *Phys. Rev. Lett.* **52** 955

[13] Vicsek T and Family F 1984 *Phys. Rev. Lett.* **52** 1669

[14] Meakin P, Vicsek T and Family F 1985 *Phys. Rev.* B **31** 564

[15] Ernst M H and Van Dongen P G J 1985 *Phys. Rev. Lett.* **54** 1396

[16] Chen Z and Redner S 1990 *J. Phys.* A: *Math. Gen.* **23** 1233

[17] Leyvraz F 2003 *Phys. Rep.* **383** 95

[18] Ke J H and Lin Z Q 2002 *Phys. Rev.* E **65** 051107

[19] Smoluchowski M V 1917 *Z. Phys. Chem. Stoechiom. Verwandtschaftsl.* **92** 215

[20] Chandrasekhar S 1943 *Rev. Mod. Phys.* **15** 1

[21] Leyvraz F and Tschudi H R 1981 *J. Phys.* A: *Math. Gen.* **14** 3389

[22] Ke J H and Lin Z Q 2002 *Phys. Rev.* E **66** 050102(R)

[23] Ke J H and Lin Z Q 2004 *J. Phys.* A: *Math. Gen.* **37** 3967

[24] Chen Y, Han A J, Ke J H and Lin Z Q 2006 *Chin. Phys.* **15** 1896

[25] Lin Z Q and Ke J H 2003 *Phys. Rev.* E **66** 031103

[26] Ben-Naim E and Krapivsky P L 2003 *Phys. Rev.* E **68** 031104

[27] Lin Z Q, Ke J H and Ye G X 2006 *Phys. Rev.* E **75** 046113

[28] Wang H F, Lin Z Q and Ke J H 2007 *Phys. Rev.* E **75** 046108

[29] Ben-Naim E and Redner S 2005 *J. Stat. Mech.* **1** 11002

[30] Wang H, Liu G Q, Yue J C, Luan J H and Qin X G 2009 *Acta Phys. Sin.* **58** 137 (in Chinese)

[31] Lin Y F, Zhang G, Zhu H Y, Huang C H, Li A H and Wei Y 2009 *Acta Phys. Sin.* **58** 3909 (in Chinese)

[32] Tang J W, Huang D Z and Yi Y G 2010 *Acta Phys. Sin.* **59** 7769 (in Chinese)

[33] Ben-Naim E and Krapivsky P L 1995 *Phys. Rev.* E **52** 6066

[34] Krapivsky P L 1993 *Physica* A **198** 135

[35] Ke J H and Lin Z Q 2002 *Phys. Rev.* E **66** 062101