Numerical simulation of four-wave mixing efficiency and its induced relative intensity noise*

Chen Wei(陈 伟), Meng Zhou(孟 洲)†,
Zhou Hui-Juan(周会娟), and Luo Hong(罗 洪)

Department of Optic Information Science and Technology, College of Optoelectronic Science and Engineering, National University of Defense Technology, Changsha 410073*, China*

(Received 21 November 2011; revised manuscript received 30 November 2011)

Four-wave mixing, as well as its induced intensity noise, is harmful to wavelength division multiplexing systems. The efficiency and the relative intensity noise of four-wave mixing are numerically simulated for the two-wave and the three-wave fiber transmissions. It is found that the efficiency decreases with the increase of both the frequency spacing and the fiber length, which can be explained using the quasi-phase-matching condition. Furthermore, the relative intensity noise decreases with the increase of frequency spacing, while it increases with the increase of fiber length, which is due to the considerable power loss of the pump light. This investigation presents a good reference for the practical application of wavelength division multiplexing systems.

Keywords: four-wave mixing, efficiency, relative intensity noise, phase matching

PACS: 78.47.nj, 78.20.Bh, 42.81.–i **DOI:** 10.1088/1674-1056/21/6/067802

1. Introduction

Wavelength division multiplexing (WDM) and erbium-doped fiber amplifiers (EDFAs) have been widely used in high-capacity, long-haul fiber sensing systems.[1−3] The use of EDFAs has greatly improved the power of the fiber and extended the transmission length to hundreds of kilometers. As a result, a variety of fiber nonlinearities become important such as four-wave mixing (FWM),[4−12] stimulated Brillouin scattering (SBS),[13−21] stimulated Raman scattering (SRS),[22] modulation instability (MI),[23−27] self-phase modulation (SPM),[28] and cross-phase modulation (XPM),[29] among which FWM is especially significant for WDM systems. When there are three waves of frequencies f_i, f_j, and f_k in the fiber, a new frequency $f_{ijk} = f_i + f_j - f_k$ will be generated through the FWM process. The FWM efficiency is related to the parameters of the fiber and the pump. In addition, intensity noise will also be introduced into the fiber systems due to the newly generated waves. In previous research, Shibata *et al.*[4,5] studied the influence of frequency spacing on generated power and efficiency. However, they did not consider the effect of fiber length. Kaur and Singh[11] and Song and Fan[12] investigated the variations of the signal-to-noise ratio (SNR) in the presence of FWM, while what we focus on is the relative intensity noise (RIN). Besides, none of the research has explained the corresponding phenomena from phase matching.

In this paper, taking two-wave and three-wave fiber transmissions as examples, we numerically analyse the effects of frequency spacing and fiber length on FWM efficiency and its induced RIN. Furthermore, we make an explanation for the simulation results using the quasi-phase-matching condition. Considering the fact that FWM induced RIN is a significant parameter which restricts the performance of the WDM system, the obtained results provide a good reference for the practical design of such systems.

2. Theory analysis

For the two-wave and three-wave fiber transmissions, two and nine new waves are generated through the FWM process, respectively, as shown in Fig. 1. For the latter case, we suppose that the three original waves are equally spaced, thus three of the new waves

*Project supported by the National Natural Science Foundation of China (Grant No. 61177073), the Open Fund of Key Laboratory of Optoelectronic Information and Sensing Technologies of Guangdong Provincial Higher Education Institutes, Jinan University (Grant No. gdol201101), the Fund of Innovation of Graduate School of National University of Defense Technology (Grant No. B110703), and Hunan Provincial Innovation Foundation for Postgraduate, China (Grant No. CX2011B033).

†Corresponding author. E-mail: zhoumeng6806@163.com

© 2012 Chinese Physical Society and IOP Publishing Ltd

overlap the old ones in the spectrum.

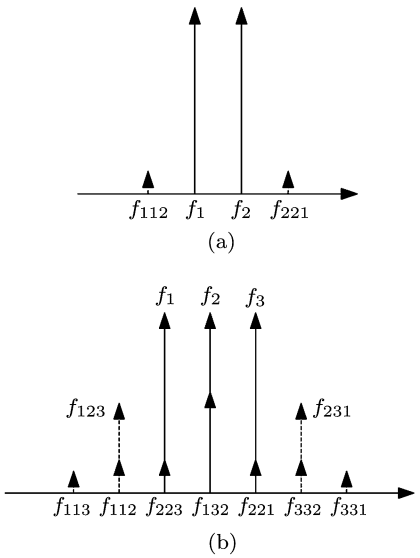


Fig. 1. Sketches of (a) two and (b) three original waves as well as generated waves through the FWM.

The time-averaged power P_{ijk} for frequency f_{ijk} generated by FWM can be written as[4]

$$P_{ijk}(L) = \eta(1024\pi^6/n^4\lambda^2c^2)(DX_{1111})^2(L_{\text{eff}}/A_{\text{eff}})^2 \times P_iP_jP_k\exp(-\alpha L), \quad (1)$$

where

$$\eta = \{\alpha^2/[\alpha^2+(\Delta\beta)^2]\}\{1+4\exp(-\alpha L) \times \sin^2(\Delta\beta L/2)/[1-\exp(-\alpha L)]^2\}, \quad (2)$$

$$\Delta\beta = (2\pi\lambda_k^2/c)\Delta f_{ik}\Delta f_{jk}\{D_c+(\lambda_k^2/2c) \times (\Delta f_{ik}+\Delta f_{jk})[\mathrm{d}D_c(\lambda_k)/\mathrm{d}\lambda]\}, \quad (3)$$

$$\Delta f_{mn} = |f_m - f_n| \quad (m,n=i,j,k), \quad (4)$$

L is the fiber length; η is the FWM efficiency; n is the refractive index of the core; λ is the wavelength; c is the light velocity in free space; D is the degeneracy factor, whose values equal 1, 3, and 6 respectively for the cases of $f_i = f_j = f_k$, $f_i = f_j \neq f_k$, and $f_i \neq f_j \neq f_k$; X_{1111} is the third-order nonlinear susceptibility; L_{eff} is the effective fiber length given as $L_{\text{eff}} = [1-\exp(-\alpha L)]/\alpha$; α is the fiber attenuation coefficient; A_{eff} is the effective area of the fiber core; P_i, P_j, and P_k are the input pump powers; D_c is the fiber-chromatic dispersion value.

According to the law of energy conservation, the power of the old waves decreases with the increasing power of the new ones, and the decrements can be obtained as[12]

$$\Delta P_i(L) = -P_{ijk}(L)\lambda_j\lambda_k/(\lambda_i\lambda_k+\lambda_j\lambda_k-\lambda_i\lambda_j),$$
$$\Delta P_j(L) = -P_{ijk}(L)\lambda_i\lambda_k/(\lambda_i\lambda_k+\lambda_j\lambda_k-\lambda_i\lambda_j),$$
$$\Delta P_k(L) = P_{ijk}(L)\lambda_i\lambda_j/(\lambda_i\lambda_k+\lambda_j\lambda_k-\lambda_i\lambda_j). \quad (5)$$

At the same time, the pump power also experiences a linear loss caused by the fiber attenuation during its transmission. Both the power transfer to the new waves and the linear fiber loss resultantly lead to the final decrease of the pump power. Using the newly generated power and the final pump power, the RIN at an arbitrary fiber length can be achieved for each channel, which is expressed as the ratio between the two powers in the present paper.

For the FWM process where two new photons of angular frequencies ω_3 and ω_4 are generated as two photons of ω_1 and ω_2 are annihilated, the phase-matching condition can be written as[30]

$$\kappa = \Delta k_{\text{M}} + \Delta k_{\text{W}} + \Delta k_{\text{NL}} = 0, \quad (6)$$

$$\Delta k_{\text{M}} = (n_3\omega_3 + n_4\omega_4 - n_1\omega_1 - n_2\omega_2)/c, \quad (7)$$

$$\Delta k_{\text{W}} = (\Delta n_3\omega_3 + \Delta n_4\omega_4 - \Delta n_1\omega_1 - \Delta n_2\omega_2)/c, \quad (8)$$

$$\Delta k_{\text{NL}} = \gamma(P_1+P_2), \quad (9)$$

where Δk_{M}, Δk_{W}, and Δk_{NL} denote the phase mismatch induced by material dispersion, waveguide dispersion, and nonlinear effects, respectively, n_j (j = 1–4) denotes the effective refractive index of the fiber mode, Δn_j (j = 1–4) the variation of the material refractive index caused by the waveguide, and γ the nonlinear coefficient. If the pump wavelength is not near the zero-dispersion wavelength, which is common for the 1.55-μm operating wavelength in the single-mode fiber (SMF), we can obtain

$$\Delta k_{\text{M}} \approx \beta_2(\Delta\omega)^2, \quad (10)$$

where β_2 is the group velocity dispersion (GVD) parameter and $\Delta\omega$ is the frequency spacing. In the SMF, $\Delta k_{\text{W}} \approx 0$, which is due to the fact that all of the waves have nearly the same values of Δn. In addition, the influence of the phase mismatch induced by the nonlinear effects can be ignored when the input pump power is not very large.

It needs to be noticed that FWM can also occur even if the phase is not matched rigorously. The coherent length is defined as[30]

$$L_{\text{coh}} = 2\pi/|\Delta\kappa|, \quad (11)$$

where $\Delta\kappa$ is the maximal phase mismatch that is permitted. FWM can occur effectively only if $L < L_{\text{coh}}$, which is the so-called quasi-phase-matching condition.

Supposing that material dispersion is dominant in the FWM process, we can obtain

$$L_{\rm coh} \approx 2\pi/|\Delta k_{\rm M}| \approx 2\pi/[\beta_2(\Delta\omega)^2]. \quad (12)$$

3. Numerical simulations and discussion

The two-wave and three-wave fiber transmissions are considered. For the three-wave transmission, as mentioned above, three new frequencies overlap the old ones and we can investigate the RINs of the three old channels. However, for the two-wave transmission, as shown in Fig. 1(a), the new frequencies do not overlap the old ones. In this situation, we do not distinguish these different channels when we investigate the RIN, in other words, we treat the old and the new channels as a whole. We suppose that the input pump powers of different channels are the same, and the parameters used in the simulations are summarized in Table 1.

Table 1. Parameters used in the simulations.

Parameter	Value
n	1.465
λ_1/μm	1.55
$c/\mathrm{m\cdot s^{-1}}$	3×10^8
$X_{1111}/\mathrm{m^3\cdot J^{-1}}$	6×10^{-14}
$\alpha/\mathrm{dB\cdot km^{-1}}$	0.2
$A_{\rm eff}/\mu\mathrm{m}^2$	80
$P/$mW	1
$D_c/\mathrm{ps\cdot km^{-1}\cdot nm^{-1}}$	16

3.1. Influence of frequency spacing

The variations of the FWM efficiency with frequency spacing for two-wave and three-wave fiber transmissions are shown in Figs. 2 and 3, respectively. The fiber length is assumed to be 50 km. It can be found that the efficiency decreases with the increase of frequency spacing. For the three-wave transmission, the variations can be divided into three sorts and their critical frequency spacings, when the efficiency is close to zero, are different. The decrease of the efficiency can be explained by Eq. (12). As the frequency spacing increases, the coherent length decreases. At the same time, FWM occurs only when the quasi-phase-matching condition $L < L_{\rm coh}$ is satisfied, thus for a given fiber length, the FWM efficiency also decreases. The efficiency decreasing with the increase of frequency spacing has been experimentally validated in previous researches.[4,5] The variations of

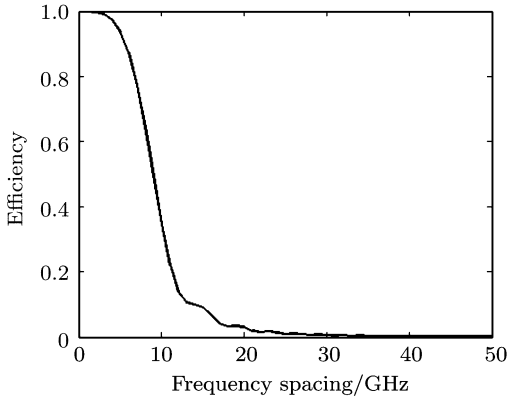


Fig. 2. FWM efficiency versus frequency spacing for two-wave transmission.

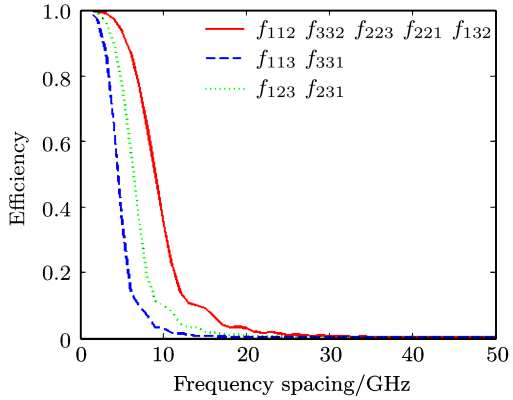


Fig. 3. (colour online) FWM efficiencies versus frequency spacing for three-wave transmission.

the RIN with the increase of frequency spacing for the two-wave and three-wave transmissions are shown in Figs. 4 and 5, respectively. It is apparent that the RIN also decreases with the increase of frequency spacing. For the three-wave transmission, the RIN of channel 1 is the same as that of channel 3, while it is much lower than that of channel 2. It was found in Ref. [11] that the SNR increases with the increase of frequency spacing. According to the above RIN definition, its variation is just the opposite to SNR, thus the RIN decreasing with the increase of frequency spacing is confirmed in Ref. [11] from the other point of view. In the practical application of the WDM system, the frequency spacing is always an integral multiple of 100 GHz, even if in the dense wavelength division multiplexing (DWDM) systems, the minimal frequency spacing at the present time is merely 50 GHz. From Figs. 2–5, we find that the efficiency and RIN are almost zero when the frequency spacing is more

than 25 GHz, thus the influence of FWM on the RIN of present WDM/DWDM systems can be neglected, which has been discussed in our other paper.[6] On the contrary, for applications where FWM is used as a frequency conversion, we should reduce the frequency spacing to acquire a larger efficiency.

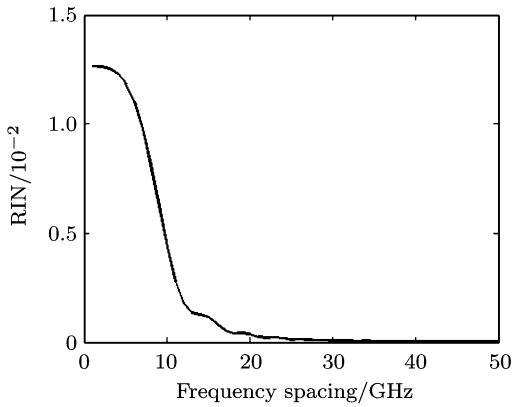


Fig. 4. FWM induced RIN versus frequency spacing for two-wave transmission.

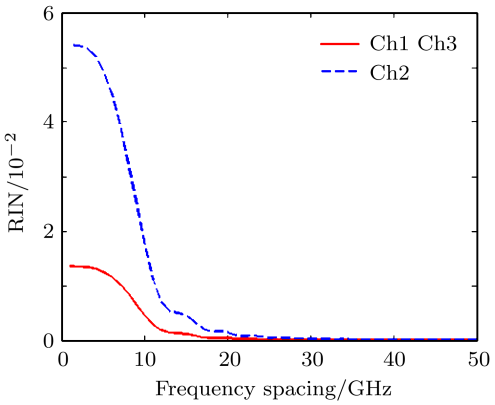


Fig. 5. (colour online) FWM induced RINs versus frequency spacing for three-wave transmission.

3.2. Influence of fiber length

The variations of the FWM efficiency with the fiber length for two-wave and three-wave fiber transmission are shown in Figs. 6 and 7, respectively. Considering the context of the practical applications, the frequency spacing is assumed to be 100 GHz. It can be found that the efficiency first decreases with the fiber length increasing and then becomes close to zero. For the three-wave transmission, the variations are similar to those in Fig. 3, which can also be divided into three sorts in the same way. As mentioned above, FWM occurs only when $L < L_{\rm coh}$, thus for a given $L_{\rm coh}$, the efficiency is lower when L is larger, and it approaches zero when L is large enough. The FWM efficiencies at different fiber lengths were also calculated in Ref. [4], and similar results were obtained, which confirms our above conclusion. The variation of the RIN with fiber length for the two-wave and three-wave transmission are shown in Figs. 8 and 9, respectively. Obviously, the RIN increases as the fiber length increases, which is opposite to those in Figs. 4 and 5. This phenomenon can be explained as follows: when the fiber is long enough, the linear fiber loss becomes dominant, leading the pump power to be lower at a longer fiber length. Therefore, the RIN is significant in spite of having a low FWM efficiency when the fiber is long. It can also be found that the RIN of channel 2 is larger than those of channels 1 and 3, which is the same as that in Fig. 5, thus we can infer that channel 2 is the worst channel for equal-spaced three-wave transmission. It was also shown in Ref. [31] that increasing the transmission distance can lead to the noise introduced by the FWM to grow rapidly, which is similar to our predicted case. In practical applications, the fiber length of the system is always fixed, thus it is better to achieve a low RIN combined with the method of enlarging the frequency spacing.

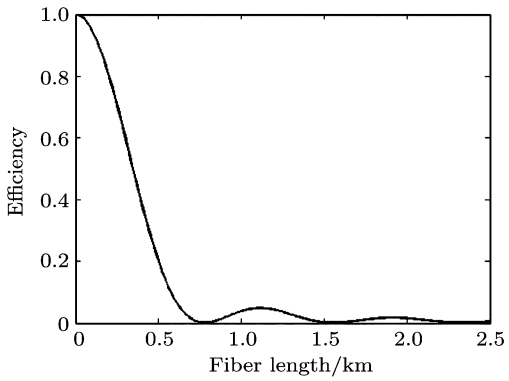


Fig. 6. FWM efficiency versus fiber length for two-wave transmission.

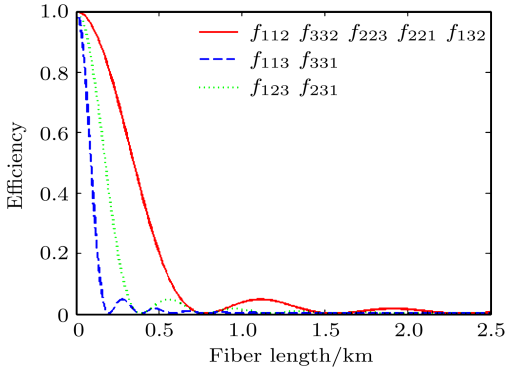


Fig. 7. (colour online) FWM efficiency versus fiber length for three-wave transmission.

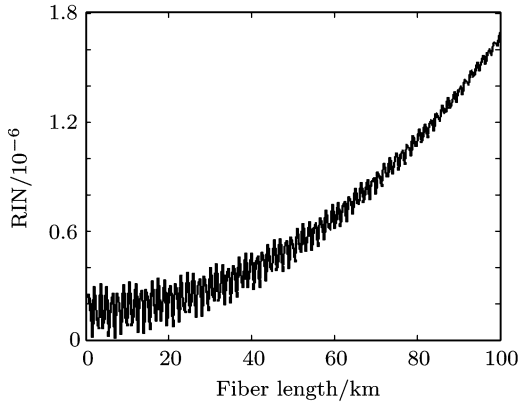


Fig. 8. FWM induced RIN versus fiber length for two-wave transmission.

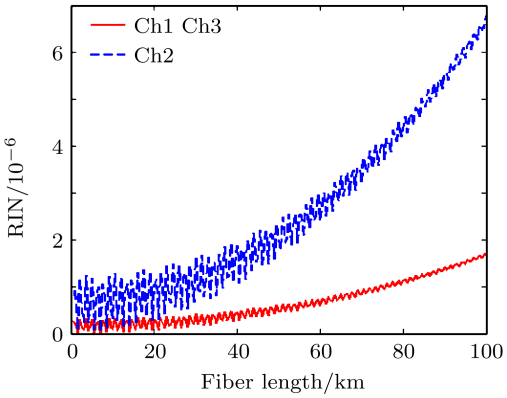


Fig. 9. (colour online) FWM induced RINs versus fiber length for three-wave transmission.

4. Conclusion

The FWM efficiencies and their induced RINs are numerically simulated for two-wave and three-wave fiber transmissions. It is shown that the efficiency decreases with the increase in frequency spacing and fiber length, both of which can be explained by the quasi-phase-matching condition. It is also shown that the RIN decreases with the increase in frequency spacing. For 100-GHz frequency spacing, which is common in the present WDM systems, the influence of FWM on the RIN can be neglected. Furthermore, considering the fact that the linear loss of the pump power in a long fiber is significant, the RIN increases with the increase in fiber length. The obtained results can serve as a reference for the practical applications of the WDM and DWDM systems.

References

[1] Nash P, Strudley A, Crickmore R and DeFreitas J 2009 *Proc. SPIE* **7503** 75037T
[2] Kringlebotn J T, Nakstad H and Eriksrud M 2009 *Proc. SPIE* **7503** 75037U
[3] Rao Y, Feng S, Jiang Q and Ran Z 2009 *Proc. SPIE* **7503** 75031Q
[4] Shibata N, Braun R P and Waarts 1987 *IEEE J. Quantum Electronics* **QE-23** 1205
[5] Shibata N, Azuma Y, Tateda M and Nakano Y 1988 *Electron. Lett.* **24** 1528
[6] Chen W and Meng Z 2011 *Opt. Laser Technol.* **43** 1270
[7] Yang D and Kumar S 2009 *J. Lightwave Technol.* **27** 2916
[8] Ito Y, Tamo T and Numai T 2009 *Opt. Commun.* **282** 3989
[9] Yuan J H, Sang X Z, Yu C X, Xin X J, Li S G, Zhou G Y and Hou L T 2010 *Chin. Phys.* B **19** 074218
[10] Xin X J, Ma J X, Zhang Q, Deng C G, Wang K R, Yu C X and Liu B 2009 *Chin. Phys.* B **18** 3449
[11] Kaur G and Singh M L 2009 *Optik* **120** 268
[12] Song J and Fan C C 1996 *J. China Institute Commun.* **17** 120 (in Chinese)
[13] Shimizu T, Nakajima K, Shiraki K, Ieda K and Sankawa I 2008 *Opt. Fiber Technol.* **14** 14
[14] Chen W, Wang K and Meng Z 2010 *Chin. Opt. Lett.* **8** 365
[15] Chen W and Meng Z 2010 *Chin. Opt. Lett.* **8** 1124
[16] Chen W and Meng Z 2011 *Chin. J. Lasers* **38** 0305002 (in Chinese)
[17] Chen W and Meng Z 2011 *J. Phys.* B: *At. Mol. Opt.* **44** 165402
[18] Chen W and Meng Z 2011 *Proc. SPIE* **7753** 77532G
[19] Wang S H, Ren L Y and Liu Y 2009 *Acta Phys. Sin.* **58** 3943 (in Chinese)
[20] Zhao L J 2010 *Acta Phys. Sin.* **59** 6219 (in Chinese)
[21] Chen X D, Shi J W, Liu J, Liu B, Xu Y X, Shi J L and Liu D H 2010 *Acta Phys. Sin.* **59** 1047 (in Chinese)
[22] Singh M L 2010 *Optik* **121** 778
[23] Xiang Y, Wen S, Dai X, Tang Z, Su W and Fan D 2007 *J. Opt. Soc. Am.* B **24** 3058
[24] Dai X, Xiang Y, Wen S and Fan D 2009 *J. Opt. Soc. Am.* B **26** 564
[25] Klinger J, Martin H and Chen Z 2001 *Opt. Lett.* **26** 271
[26] Zhong X Q and Xiang A P 2010 *Chin. Phys.* B **19** 064212
[27] Xu W C, Zhang S M, Chen W C, Luo A P, Guo Q and Liu S H 2002 *Chin. Phys.* **11** 39
[28] Sheetala A, Sharmab A K and Kaler R S 2010 *Optik* **121** 246
[29] Costa N M S and Cartaxo A V T 2008 *J. Lightwave Technol.* **26** 3640
[30] Agrawal G P 2001 *Nonlinear Fiber Optics* (San Diego: Academic Press)
[31] Yuan H, Xu A S, Wang C and Wu D M 1996 *Commun. Nonlinear Sci. & Numer. Simul.* **1** 1