Point defect states of a hollow cylinder in two-dimensional phononic crystal*

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Point defect states in two-dimensional phononic crystal of a hollow mercury cylinder in a water host are studied. An improved plane expansion method combined with the supercell technique is used to calculate the band gaps and the pressure distribution at the defect position. The sonic pressure of defect modes shows that the waves are localized at or near the defect. As the filing fraction increases, more defect modes appear in the band gaps.

Keywords: phononic crystal, point defect, cavity defect, hollow cylinder

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1. Introduction

In the past decade, a great deal of effort has been devoted to the study of phononic crystals constituted by a periodical repetition of inclusions in a matrix background because of their novel physical properties and many potential applications.[1−17] Phononic crystals may exhibit absolute band gaps in which the sound and vibrations are all forbidden in all directions. The absolute band gaps have been found to be present in the binary composite materials with large contrast between their mass density and wave velocities (or elastic constants) or in the three-component materials.[18−20] With those attracting properties, phononic band gap materials have possible applications such as ultrasonic filter noise control and improvement on the design of transducers, as well as the understanding of the fundamental physics concerned with the Anderson localization of sound and vibrations. The absolute band gap of phononic crystal has been confirmed both theoretically and experimentally.[21,22] When the periodicity of a phononic crystal is broken by introducing defects, it is probable to engender highly localized defects within the absolute band gaps, which are analogous to localized modes in photonic crystals and localized impurity states in semiconductors.[23−30]

Defect modes can be used to design waveguides. Much work on searching for the optimal conditions of the appearance of the elastic wave (acoustic wave) has been done, but only a little work has been devoted to the defects. Sigalas[23] reported on the point and the linear defect states in two-dimensional (2D) phonic crystals composed of solid cylinders in an air or a solid matrix by the plane-wave expansion (PWE) method.[23] In order to study the linear waveguides in 2D elastic wave band gap materials constituted by either the fluid or solid constituents, Khelif *et al.*[22] calculated the band structures and the transmission coefficients, using the PWE and the finite difference time domain (FDTD) method, respectively. Wu *et al.* have studied the point defect modes of an acoustic wave in 2D square arrays of square water rods in a mercury host.[27]

However, in all the above reports, only the two kinds of (circular and square) cross sections of the cylinders were considered. The point defects are produced by changing the radius of a cylinder, or simply removing several cylinders (one cylinder) to form lin-

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ear defects (point defect). In this paper, we deal with longitudinal waves propagating in a 2D periodic system consisting of infinitely long hollow mercury cylinders of circular cross section, which are arranged in a square lattice and filled with some water. The effects of the filling ratio on the band gap, the point defect state and the pressure distribution are investigated.

2. Theory

Figure 1 shows the cross sections of the supercell of the phononic crystal with one defect at (0.0, 0.0), which is created by removing the middle hollow cylinder in a supercell. The lattice constant is a, and the inner radius of the hollow cylinder is r, the outer radius is R, thus the filling fraction is $f = \pi(R^2 - r^2)/a^2$. Setting R to be the maximum value $(a/2)$, we change the filling fraction by varying the inner radius, in searching for point defect states. The calculations in the present paper are performed with the plane expansion method combined with the supercell technique. And the supercell has a size of 9×9, 1089 plane waves are used in calculation, which convergence rate of the numerical results is less than 5%.

$$p = -\lambda(\boldsymbol{r}_{\rm t})\nabla \cdot \boldsymbol{u}. \tag{1}$$

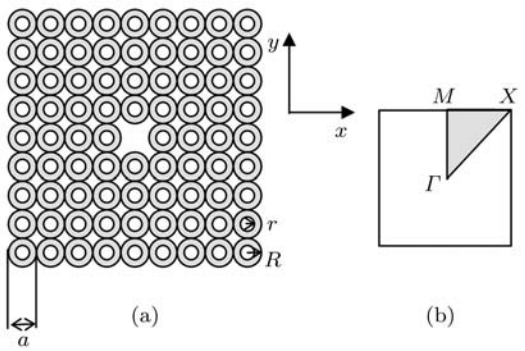


Fig. 1. (a) Cross section of xy plane in a 9×9 supercell, where the hollow cylinder defect is located at (0.0, 0.0) and the lattice parameter is denoted as a. (b) The corresponding first Brillouin zone.

The basic equations which describe the wave motion then reduce to a simple scalar equation as

$$-\frac{\omega^2}{\lambda(\boldsymbol{r}_{\rm t})}p = \nabla \cdot \left(\frac{1}{\rho(\boldsymbol{r}_{\rm t})}\nabla p\right), \tag{2}$$

where $\lambda(\boldsymbol{r}_{\rm t})$, $\rho(\boldsymbol{r}_{\rm t})$, and p are the bulk modulus, the mass density, and pressure of the fluid, respectively. The process of the solution to the above equation can be found in Ref. [31].

In the present paper, we study the point defect states in the 2D phononic crystal composed of hollow mercury (with longitudinal velocities $c_{\rm l} = 14.5$ km/s and density $\rho = 13.5 \times 10^3$ kg/m^3) cylinders each with a square array embedded in water (with longitudinal velocities $c_{\rm l} = 1.45$ km/s and density $\rho = 1.48 \times 10^3$ kg/m^3) host.

The reciprocal lattice vectors are $\boldsymbol{G} = (2\pi/a)(n_x\hat{x} + n_y\hat{y})$ for the square lattice, n_x and n_y are integers. The n_x and n_y, in the reciprocal lattice vector $\boldsymbol{G}$, are permitted to take the values from –16 to +16 (i.e., 1089 plane waves). In this case of periodic fluid–fluid system, shearing can be neglected ($\mu = 0$), and we can remove the coupling difficulty by using another variable, the pressure p, to determine a pure scalar dilatation motion.

3. Result and discussion

It is necessary to compare the band gaps of perfect phononic crystal and those of point defect. First we calculate the band gaps of the perfect phononic crystal. The band structures for the perfect phononic crystal with hollow mercury cylinder in water with different filling ratios are shown in Fig. 2.

The filling fraction changes from 0.20 to 0.78 (we consider only the lowest 350 band gaps). It is found that the absolute band gaps begin to appear when the filling fraction is more than 0.2. There are many pseudogaps in the band structures, which are distributed below normalized frequency 0.2 ($\omega a/2\pi c_{\rm l}$). As the filling ratio increases, the variations of the position and the extent of the band gaps can be clearly seen in Fig. 2. There are high band gaps when the filling ratio increases from 0.2 to 0.5. When the filling ratio equals 0.55, the high band gaps disappear, as the filling ratio increases, the high band gaps open again, finally becomeing pseudo gaps. When filling ratio equals 0.65, four band gaps are apparent.

In order to calculate the point defect states, we break the periodicity of the structure by introducing a defect. This is produced by removing the central cylinder to form a cavity as depicted in Fig. 1(a). The band gaps of a point defect located at the central phononic crystal with different values of the filling ratio are exhibited in Fig. 3.

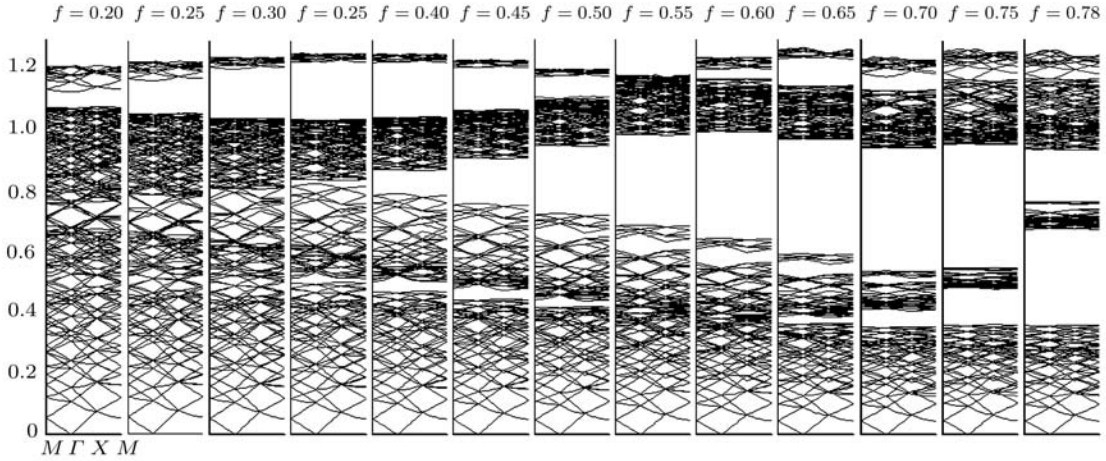


Fig. 2. Band structures of the perfect phononic crystals consisting of a hollow mercury cylinder with circular cross section in water with filling fraction changing from 0.20 to 0.78.

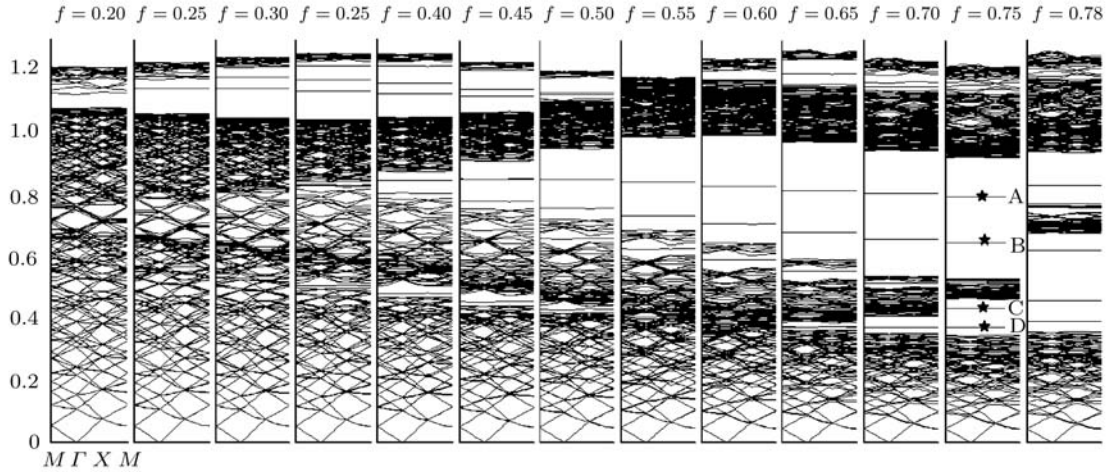


Fig. 3. Band structure of the phononic crystal with one defect varying with filling fraction.

The pass band gaps (defect states) occur when the filling ratio changes from 0.2 to 0.78. When the filling ratio reaches 0.75, four flat branches (labeled as D, C, B, A) appear in the band gap regions. Two of them appear in the first band gap and the remaining two flat branches occur in the second one. They are localized around the effect region with a longitudinal group velocity ($\mathrm{d}\omega/\mathrm{d}k$) equal to zero. This is a clear indication that guided modes exist for pass band-gap frequencies inside the band gaps. In order to illustrate the localization of defect modes, the eigenvectors for all flat branches are calculated. The sonic pressure distributions associated with the four defect modes with filling ratio equal to 0.75 at Γ point in 9×9 supercell are plotted in Fig. 4.

The degree of localization for the A mode is much higher than that for the other three modes, and the degree of localization for C mode is the lowest in the four modes. We can see that the frequencies of the defect modes decrease as the filling fraction increases. The number of defect bands increases from 1 to 4 as the filling fraction increases from 0.2 to 0.78. The defect bands disappear at very low filling fractions (lower than 0.2). These band gaps are compared with those of perfect phonic crystal which is displayed in Fig. 2; we can see that one defect hardly changes the band structure and causes the defect modes to occur. Except for the pass bands, the band gaps obtained with the supercell are almost identical to those obtained with elementary cell. This demonstrates that the band structural properties of the phononic crystal are not significantly perturbed by the inclusion of the periodic point defect. The pass branches that appear inside the band structures are those of defect modes and are representative of their dispersion as a function of the filling ratio. It can be seen that the phononic crystal with a hollow mercury cylinder possesses several guided modes that exit within the different absolute band gaps.

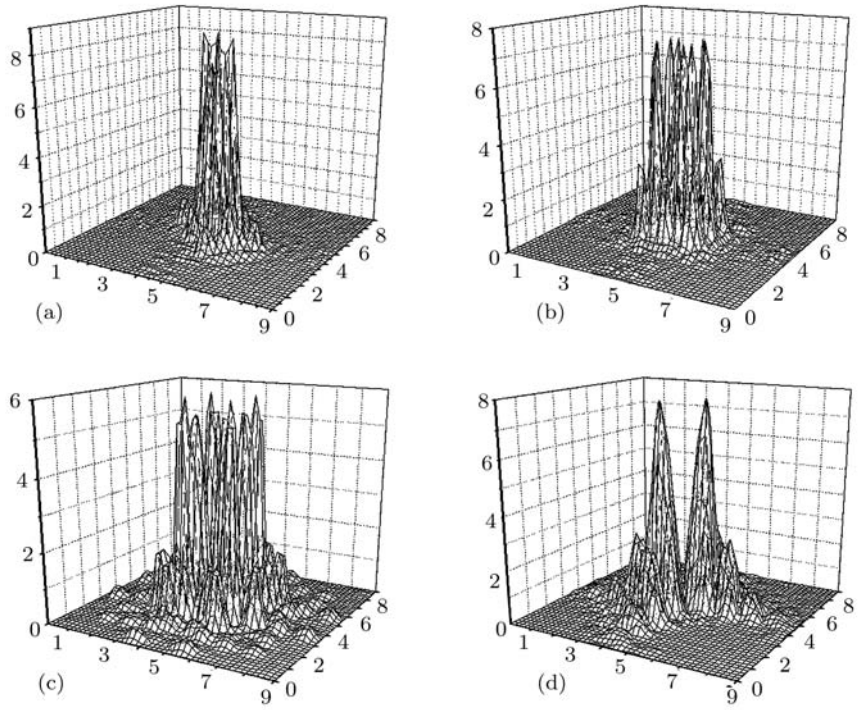


Fig. 4. Sonic pressure distributions in the 9×9 supercell at the Γ point of the modes marked as A, B, C, and D, respectively in panels (a), (b), (c), and (d) at $\omega a/2\pi c_{\rm l} = 0.7955$ (a), 0.6443 (b), 0.4349 (c), and 0.3746 (d).

4. Conclusions

Based on the improved PWE method combined with the supercell technique, the band gaps of the 2D phononic crystals of hollow mercury cylinder in a water host with a point defect are calculated. The defect modes are shown to be strongly dependent on the filling fraction of the scattering objects. For a given filling fraction, the number of the defect modes changes from 0 to 4 in a certain range of the filling fraction, and the frequencies of the defect bands increase with filling fraction increasing. The sonic pressure distributions exhibit that the sonic waves in point defect are highly localized at or near the defect, and the degrees of localization are different for different modes. Those results provide a theoretical foundation to design acoustic filters in engineering applications.

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