The signal synchronization transmission of a spatiotemporal chaos network constituted by a laser phase-conjugate wave*

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(Received 30 November 2011; revised manuscript received 12 December 2011)

The signal synchronization transmission of a spatiotemporal chaos network is investigated. The structure of the coupling function between connected nodes of the complex network and the value range of the linear term coefficient of the separated configuration in state equation of the node are obtained through constructing an appropriate Lyapunov function. Each node of the complex network is a laser spatiotemporal chaos model in which the phase-conjugate wave and the unilateral coupled map lattice are taken as a local function and a spatially extended system, respectively. The simulation results show the effectiveness of the signal synchronization transmission principle of the network.

Keywords: synchronization, complex network, spatiotemporal chaos, phase-conjugate wave

PACS: 42.65.Sf, 05.45.Xt **DOI:** 10.1088/1674-1056/21/6/064217

1. Introduction

The phase-conjugate wave can compensate the wave front distortion of a laser system to achieve high intensity output, and it also shows many attractive prospects in other applications, such as optical communication, image processing, real-time information processing, and so on. Theoretical and experimental investigations have shown that the chaotic behaviour can be observed in the phase-conjugate wave generated by a four-wave mixing process when a lossless Kerr medium is chosen as the phase-conjugation medium to constitute phase-conjugate resonant cavity and under appropriate pump power.[1] With further studying in recent years, it has been found that the synchronization of complex networks constituted by laser chaotic systems, for example, the phase-conjugate wave plays a unique role in substantially increasing its output power, remote communication, and synchronization transition of relay signal. Therefore, the investigation of synchronization of complex networks has become an important focus to date. The fruitful investigations surrounding the chaos synchronization of complex networks and the enhancement of synchronized ability have been achieved to realize global synchronization,[2−9] phase synchronizations,[10,11] and generalized synchronizations[12,13] in random network, small-world network, scale-free networks, and regular network.

Despite the synchronizations of complex networks having been much reported in the literature, however, nodes of complex networks are basically temporal chaotic systems. In reality, spatiotemporal chaotic systems are more common, and they show chaotic behaviour not only with the evolution of time but also with that of space. So the synchronization investigation of networks constructed by spatiotemporal chaotic systems will be more secure and wider, either in theory or in application. Furthermore, it is hoped in practical application not only to achieve complete synchronization among all laser chaotic systems in the complex network connected in some way, but also to realize the synchronization transmission of the laser chaotic system with any external signals. The synchronization theory and technology are less reported in current research. In this paper, the signal synchronization transmission of a spatiotemporal chaos network is investigated. If any given external signal is input to any node in the network, when the network achieves synchronization, all nodes in the network will be synchronized to the external signal, that is, the given external signal realizes the synchroniza-

*Project supported by the Natural Science Foundation of Liaoning Province, China (Grant No. 20082147).
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tion transmission.

2. Model of spatiotemporal chaos in phase-conjugation wave

A phase-conjugate resonant cavity is constructed by a standard reflector, a phase-conjugate mirror, and a lossless Kerr medium as the phase-conjugation medium which can realize multi-level scattering. Considering the threshold effect or switch behaviour, the system evolution can be described as[14]

$$x_{n+1} = M\mathrm{J}_{0/1}^2[(x_n - B)^{1/2}], \tag{1}$$

where M is a control parameter related to the reflection rate of the standard reflector, B is the threshold or a switch parameter, and $\mathrm{J}_{0/1}$ is zero or the first order Bessel function.

When parameter $B = 1$, the bifurcation map and the evolution of the Lyapunov exponent can be obtained from Eq. (1) with control parameter M as shown in Figs. 1 and 2. The system comes into chaos via period-doubling bifurcation.

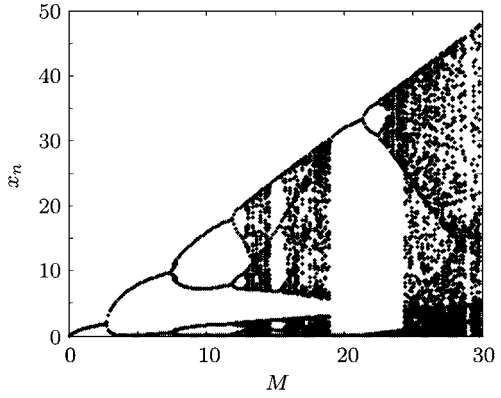


Fig. 1. Bifurcation map of J_0^2 in system (1).

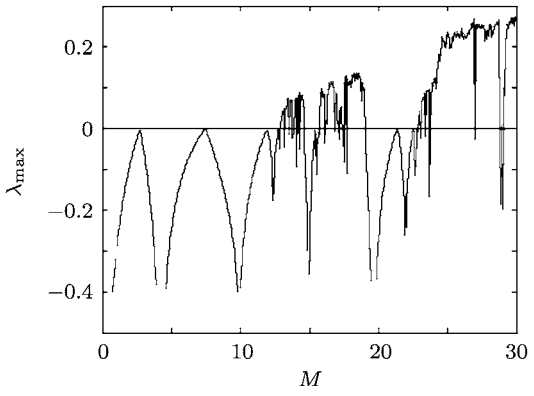


Fig. 2. Maximal Lyapunov exponent of J_0^2 in system (1).

The phase-conjugation wave, equation (1), is taken as a local function $f(x_n(i))$ and the unilateral coupled map lattice[15] as a global function to form a spatially extended system

$$\begin{aligned} x_{n+1}(i) &= (1-\varepsilon)f(x_n(i)) + \varepsilon f(x_n(i-1)) \\ &= (1-\varepsilon)M\mathrm{J}_{0/1}^2[(x_n(i) - B)^{1/2}] \\ &\quad + \varepsilon M\mathrm{J}_{0/1}^2[(x_n(i-1) - B)^{1/2}], \end{aligned} \tag{2}$$

where n is the discrete time, i is the position coordinates of space lattice ($i = 1, 2, \ldots, N$), L is the size of the system, and ε is the coupling strength factor of a discrete lattice point in the system space. When the system size is $L = 100$ and the initial value $x_0(i) = 0.1$, zero Bessel function is taken as the local function. The evolution of the maximal Lyapunov exponent of the phase-conjugation wave spatial extension system with parameter M and the coupling strength ε is shown in Fig. 3. It can be seen in the figure that the maximal Lyapunov exponents are above zero when $\varepsilon = 0.1$ and $M = 28$, indicating that the phase-conjugation wave spatial extension system is in spatiotemporal chaos. The spatiotemporal evolution and the phase map of the system are shown in Figs. 4 and 5.

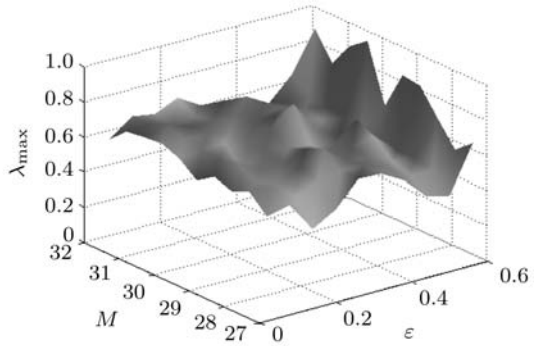


Fig. 3. Evolution of Lyapunov exponent with parameter ε and M.

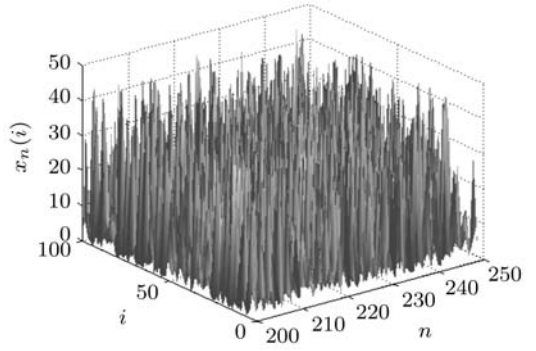


Fig. 4. Spatiotemporal evolution of the phase-conjugation wave spatial extension system (2).

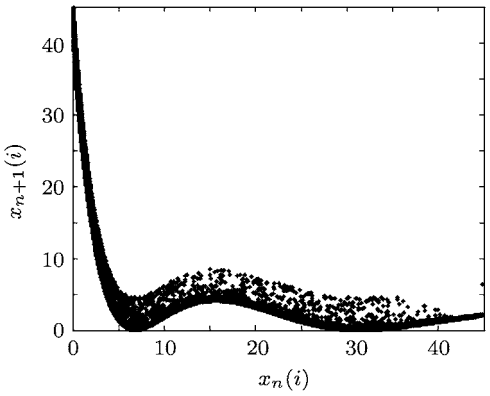


Fig. 5. Phase map of the phase-conjugation wave spatial extension system (2).

3. Signal synchronization transmission of the network

The synchronization of the complex network with m phase-conjugation wave spatial extension systems Eq. (2) acting as nodes is investigated. First, system equation (2) is rewritten into a general form of discrete spatiotemporal chaos systems as shown below

$$x_{n+1}(i) = F(x_n(i)) = Cx_n(i) + Q(x_n(i)), \qquad (3)$$

where C is the linear team coefficient after properly separating the configuration. It is obviously seen that $Q(x_n(i)) = F(x_n(i)) - Cx_n(i)$.

The m discrete spatiotemporal chaos system equation (3) is taken as nodes to constitute a complex network, and the state equation of node j is described as

$$\begin{aligned} x_{n+1}^{(j)}(i) &= Cx_n^{(j)}(i) + Q(x_n^{(j)}(i)) \\ &\quad + \xi_j(x_n^{(1)}(i), x_n^{(2)}(i), \ldots, x_n^{(m)}(i)) \\ &\quad (j = 1, 2, \ldots, m), \end{aligned} \qquad (4)$$

where $\xi_j(x_n^{(1)}(i), x_n^{(2)}(i), \ldots, x_n^{(m)}(i))$ is the coupling function connecting each node, and it is needed to determine.

The errors between the state variables of spatiotemporal chaotic systems at each node are defined as

$$\begin{aligned} e_n^{(j)}(i) &= x_n^{(j)}(i) - x_n^{(j+1)}(i), \\ &\quad (j = 1, 2, \ldots, m-1), \end{aligned} \qquad (5)$$

which is further obtained as

$$\begin{aligned} & e_{n+1}^{(j)}(i) \\ &= x_{n+1}^{(j)}(i) - x_{n+1}^{(j+1)}(i) \\ &= Ce_n^{(j)}(i) + \Delta Q(x_n^{(j)}(i), x_n^{(j+1)}(i)) + \Delta\xi_j, \end{aligned} \qquad (6)$$

where $\Delta Q(x_n^{(j)}(i),$

$$x_n^{(j+1)}(i)) = Q(x_n^{(j)}(i)) - Q(x_n^{(j+1)}(i)),$$

and $\Delta\xi_j = \xi_j - \xi_{j+1}$.

Choose

$$\Delta\xi_j = -\Delta Q(x_n^{(j)}(i), x_n^{(j+1)}(i)), \qquad (7)$$

i.e.

$$\begin{aligned} \xi_k &= \xi_{k-1} + \Delta Q(x_n^{(k-1)}(i), x_n^{(k)}(i)) \\ &= \xi_1 + \sum_{j=1}^{k-1} \Delta Q(x_n^{(j)}(i), x_n^{(j+1)}(i)), \\ &\quad (k = 2, 3, \ldots, m), \end{aligned} \qquad (8)$$

where $\xi_1 = S_{\text{sign}} - Q(x_n^{(1)}(i))$ with S_{sign} being a given external signal.

When the determined coupling function connecting each node is given as Eq. (7), equation (5) can be expressed as

$$e_{n+1}^{(j)}(i) = Ce_n^{(j)}(i). \qquad (9)$$

The Lyapunov function of the network is constructed as

$$V_n = \sum_{j=1}^{m-1} \left| e_n^{(j)}(i) \right|. \qquad (10)$$

Obviously,

$$\begin{aligned} \Delta V &= \sum_{j=1}^{m-1} \left(\left| e_{n+1}^{(j)}(i) \right| - \left| e_n^{(j)}(i) \right| \right) \\ &= (|C| - 1) \sum_{j=1}^{m-1} \left| e_n^{(j)}(i) \right|. \end{aligned} \qquad (11)$$

When $|C| < 1$, it is easy to see from the above equation

$$\Delta V < 0. \qquad (12)$$

According to the Lyapunov theorem,[15] the complete synchronization of the complex network is realized when $n \to \infty$, that is, the synchronization transmission of the given external signals is achieved.

4. Simulation

The synchronization transmission is simulated while the number of the network nodes is given as $m = 4$. The state equation of a single node is described by the phase-conjugation wave spatial extension system Eq. (2). The state variables of the spatial

extension system at the four nodes are $x_n^{(1)}(i)$, $x_n^{(2)}(i)$, $x_n^{(3)}(i)$, and $x_n^{(4)}(i)$, which constitute a complex network according to Eq. (4) and keep all the parameter values of the network unchanged. A given, arbitrary external signal $S_{\rm sign}$ is input to the first node of the network. Suppose that the external signal $S_{\rm sign}$ is a spatiotemporal chaos one produced by the Gibbs electro-optical bistable system[16,17]

$$\begin{aligned} x_{n+1}(i) &= (1-\varepsilon)f(x_n(i)) + \varepsilon f(x_n(i-1)) \\ &= (1-\varepsilon)A\sin^2(x_n(i)-x_b) \\ &\quad + \varepsilon A\sin^2(x_n(i-1)-x_b), \end{aligned} \tag{13}$$

where A and x_b are system parameters, and ε is the coupling strength factor of a discrete lattice point in system space. Taking $A=3$, $x_b=0.85\pi$, and $\varepsilon=0.4$, the phase map of the system is shown in Fig. 6. Comparing Fig. 5 with Fig. 6, it is found that many differences exist between the phase map of the phase-conjugation wave spatial extension system Eq. (2) at the nodes and that of the external signal produced by the Gibbs electro-optical bistable system.

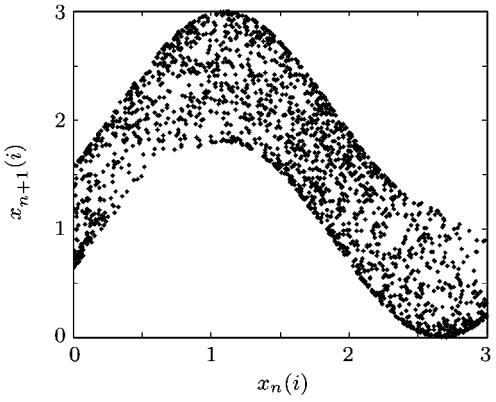


Fig. 6. Phase map of external signal (Eq. (13)).

According to the range of $|C|<1$, the linear team coefficient with separated configuration C is taken to be $C=0.02$. The coupling function ξ_1 is taken to be $\xi_1 = S_{\rm sign} - Q(x_n^{(1)}(i))$, and the other coupling functions ξ_k $(k=2,3,4)$ satisfy Eq. (7). The initial value of the phase-conjugation wave spatial extension systems at four nodes are taken as $x_0^{(1)}(i)=0.1$, $x_0^{(2)}(i)=0.2$, $x_0^{(3)}(i)=0.3$, and $x_0^{(4)}(i)=0.4$. The initial values of the external signal produced by the Gibbs electro–optical bistable system are all taken to be 0.5, i.e., $x_0(i)=0.5$. Coupling connection of the network is processed at the time of 220 s, and the spatiotemporal evolutions of error variables are shown in Figs. 7–9.

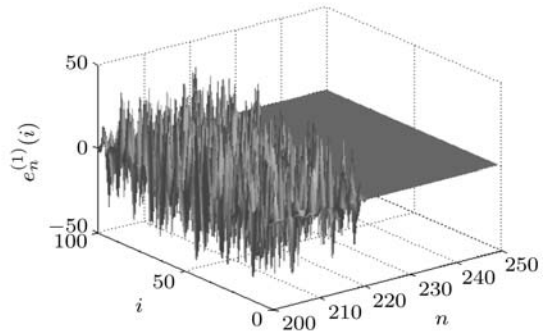


Fig. 7. Spatiotemporal evolution of error variable $e_n^{(1)}(i)$.

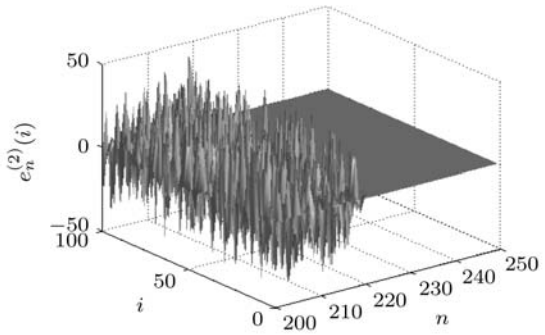


Fig. 8. Spatiotemporal evolution of error variable $e_n^{(2)}(i)$.

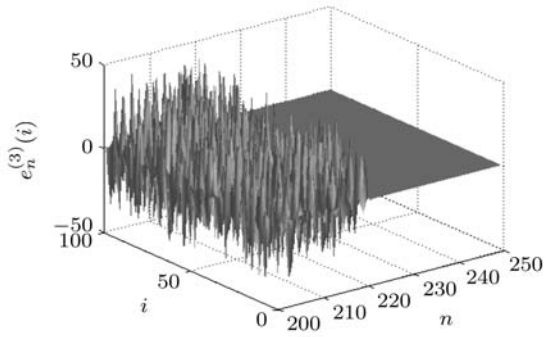


Fig. 9. Spatiotemporal evolution of error variable $e_n^{(3)}(i)$.

It can be seen in Figs. 7–9 that all error signals are concerned with the variables of the phase-conjugation wave spatial extension systems at the four nodes close to zero immediately after the coupling connection, which means that the complete synchronization of the network is achieved. Furthermore, the synchronization transmission effect of the external signal can be seen more clearly from the phase maps in Fig. 10. It is obvious that the phase maps of the four nodes in the network are exactly the same as those of the external signals produced by the Gibbs electro–optical bistable system, indicating that the given external signals are simultaneously transmitted to all the nodes of the complex network.

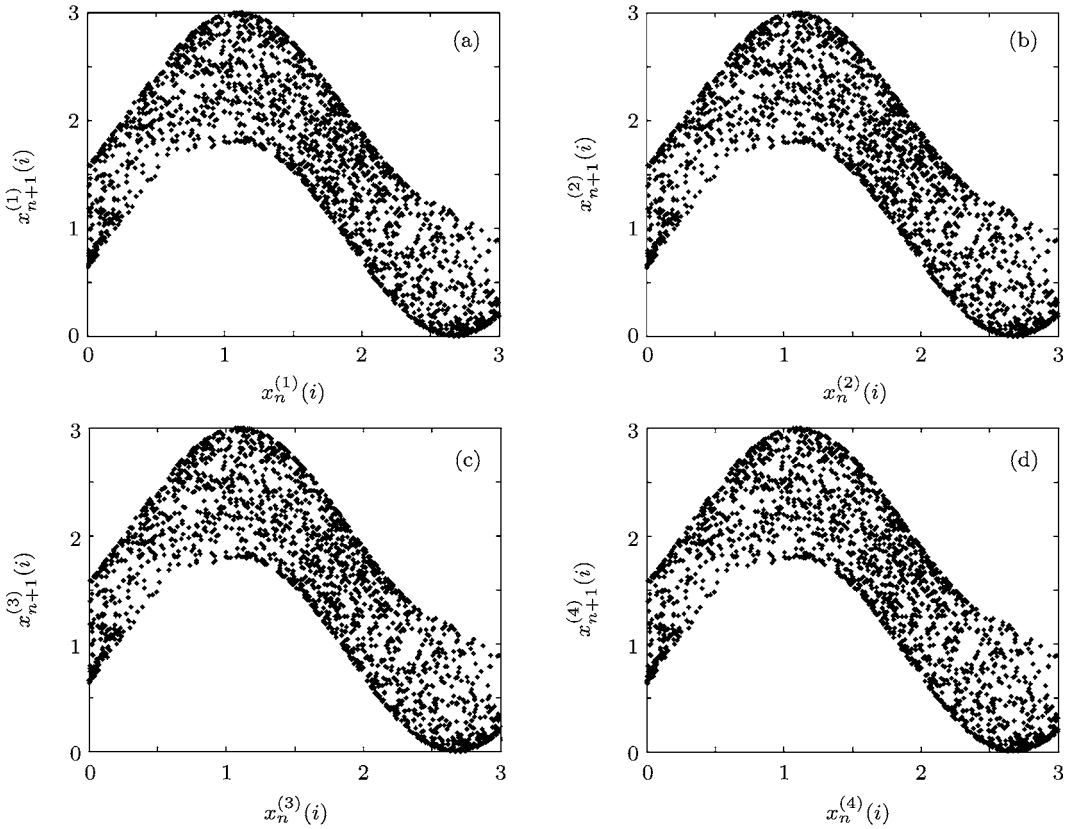


Fig. 10. Phase maps of nodes 1 (a), 2 (b), 3 (c), and 4 (d) after synchronization.

5. Conclusion

The signal synchronization transmission of a spatiotemporal chaos network is investigated. The structure of the coupling function between connected nodes of the complex network and the value range of the linear term coefficient C of separated configuration in state equation of the node are obtained through constructing an appropriate Lyapunov function. Each node of the network is a laser spatiotemporal chaos model in which the phase-conjugate wave and the unilateral coupled map lattice are taken as a local function and a spatial extend system, respectively, and the system is simulated to validate the effectiveness of the signal synchronization transmission principle of the network. The simulation results show that any external signal $S_{\rm sign}$ is input to the first node of the network, such as the spatiotemporal chaos signal produced by the Gibbs electro–optical bistable system, and the synchronization transmission between nodes of the network is realized when the linear term coefficient C of separated configuration in state equation satisfies $|C| < 1$.

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