Modulation of atomic exit and injection rates on the phase-dependent gain without inversion in a Doppler broadened open four-level system*

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It is shown that in a Doppler broadened open N-type four-level atomic system with spontaneously generated coherence (SGC), the gain without inversion (GWI) is very sensitive to the variation of the relative phase between the probe field and the driving field; the atomic exit rate (R_0) and the ratio (S) of the atomic injection rates have a considerable modulation effect on the phase-dependent GWI. GWI first increases and then decreases with R_0 increasing; in a certain value range of S, GWI increases monotonically with S increasing; by adjusting the values of R_0 and S, in an open system a much larger GWI can be obtained than in the corresponding closed system [2011 *Phys. Rev.* A **83** 043805]. The modulation effects of R_0 and S on the phase-dependent GWI in the case with the counter-propagating probe and driving fields are stronger than those in the co-propagating case, GWI in the co-propagating case is much larger than that in the counter-propagating case.

Keywords: open N-type four-level system, spontaneously generated coherence, relative phase, Doppler broadening, gain without inversion

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1. Introduction

Since the seminal works of Kocharovskaya and Khanin,[1] Harris,[2] and Scully *et al.*,[1−3] the study of lasing without inversion (LWI) has attracted tremendous attention,[4−12] as this study has an important scientific significance and wide potential applications. LWI originates from quantum coherence and interference in an atomic system. There are many ways to generate the quantum coherence and interference. Generally, they are produced by a coherent driving field or by initial coherence, and the interference between different spontaneous emission pathways can also lead to the generation of coherence, which is usually called spontaneously generated coherence (SGC). It has been shown that optical properties and phenomena in an atomic system with SGC are sensitive to the phase of the applied field. At present, effects of phase of the applied field on many important physical phenomena, such as LWI, subluminal and superluminal propagation of light pulse, optical bistability (OB), electromagnetically induced transparency (EIT), absorption (EIA), etc. have been extensively investigated.[13−33] However, these researches concentrated mainly on the three-level atomic systems, only a few papers dealt with more complicated system — the four-level atomic system due to mathematical complexity. Revealed in Ref. [19] was the phase-dependent properties for absorption and dispersion in a V-type four-level system. In Ref. [21] were studied the effects of phase on single- and two-photon absorption in a Y-type four-level atomic system. Reference [30] dealt with the effects of phase on EIT, refractive property and group velocity in an inverted-Y type four-level atomic system. In Refs. [31] and [32] the authors analysed the phase control of EIA in an N-type four-level atomic system. Moreover, in these studies about four-level systems only the static atomic systems were considered. In fact, in many experimental researches on optical properties and phenomena (e.g. LWI) a gaseous atomic medium is used. In these cases atoms are motive and Doppler broadening is prominent. The results show that Doppler broadening has a great influence on gain in an LWI system without SGC.[34−39] One important subject of

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the study about LWI is how to enhance LWI gain (also referred to as gain without inversion (GWI)). So investigating the effect of phase of the applied field on GWI in a Doppler broadening four-level atomic system with SGC is necessary. Very recently, we[40] studied the effect of the relative phase between the probe field and the driving field on GWI in a closed Doppler broadened N-type four-level atomic system with SGC and found that GWI is very sensitive to the variation of the relative phase: generally speaking, GWI decreases with Doppler width increasing; but at some special values of Doppler width, by choosing an appropriate value of the relative phase, GWI larger than that in the case without Doppler broadening can be obtained. According to this study, in this paper we investigate the influence of the relative phase on GWI and specially the modulation effects of atomic exit and injection rates on the phase-dependent GWI in a corresponding open Doppler broadened N-type four-level atomic system with SGC. To the best of our knowledge, this is the first time that the effect of the relative phase on GWI in a Doppler broadened open four-level system has been investigated.

2. Model and equation of motion

An open N-type four-level atomic system considered here is shown in Fig. 1. The transition $|1\rangle \leftrightarrow |3\rangle$ with frequency ω_{31} is coupled by a weak probe field ($\boldsymbol{\varepsilon}_{\rm p} = \boldsymbol{E}_{\rm p}\exp(-\mathrm{i}\,\omega_{\rm p}t) + \text{c.c}$) with Rabi frequency $\Omega_{\rm P} = \boldsymbol{\mu}_{31}\cdot\boldsymbol{E}_{\rm p}/(2\hbar)$, while the transition $|2\rangle \leftrightarrow |3\rangle$ with frequency ω_{32} is coupled by a strong driving field ($\boldsymbol{\varepsilon}_{\rm c} = \boldsymbol{E}_{\rm C}\exp(-\mathrm{i}\,\omega_{\rm c}t) + \text{c.c.}$) with Rabi frequency $\Omega_{\rm C} = \boldsymbol{\mu}_{32}\cdot\boldsymbol{E}_{\rm C}/(2\hbar)$. A coherent pump field ($\boldsymbol{\varepsilon}_{\rm s} = \boldsymbol{E}_{\rm S}\exp(-\mathrm{i}\,\omega_{\rm s}t) + \text{c.c.}$) with Rabi frequency $\Omega_{\rm S} = \boldsymbol{\mu}_{41}\cdot\boldsymbol{E}_{\rm S}/(2\hbar)$ is applied between levels $|1\rangle$ and $|4\rangle$. In the above representatives of Rabi frequencies, $\boldsymbol{\mu}_{ij} = \langle i| - e\boldsymbol{r}|j\rangle$ is the matrix element of the electrodipole moment between the levels $|i\rangle$ and $|j\rangle$. $\Delta_{\rm p}$ ($=\omega_{31}-\omega_{\rm p}$), $\Delta_{\rm c}$ ($=\omega_{32}-\omega_{\rm p}$), and $\Delta_{\rm S}$ ($=\omega_{41}-\omega_{\rm S}$) denote the detunings of the probe, driving, and pump fields from their relevant atomic transitions, respectively. The atomic injection rates for levels $|1\rangle$ and $|2\rangle$ are R_1 and R_2, respectively. The atomic exit rate from the cavity is R_0. In the following discussion, we always make $R_0 = R_1 + R_2$ for keeping the total number of the atoms as a constant, and introduce the ratio of the atomic injection rates, $S = R_2/R_1$.

Following the procedure in Ref. [40], we can obtain the density matrix equations of motion of the open N-type four-level atomic system with effects of SGC and phases of the applied fields as follows:

$$\begin{aligned}
\dot{\tilde{\rho}}_{11} &= \gamma_{31}\tilde{\rho}_{33} + \gamma_{41}\tilde{\rho}_{44} - \mathrm{i}\,\Omega'_{\rm P}\tilde{\rho}_{13} + \mathrm{i}\,\Omega'_{\rm P}\tilde{\rho}_{31} \\
&\quad - \mathrm{i}\,\Omega'_{\rm S}\tilde{\rho}_{14} + \mathrm{i}\,\Omega'_{\rm S}\tilde{\rho}_{41} + R_1 - R_0\tilde{\rho}_{11}, \qquad (1a)\\
\dot{\tilde{\rho}}_{22} &= \gamma_{32}\tilde{\rho}_{33} + \gamma_{42}\tilde{\rho}_{44} \\
&\quad - \mathrm{i}\,\Omega'_{\rm C}\tilde{\rho}_{23} + \mathrm{i}\,\Omega'_{\rm C}\tilde{\rho}_{32} + R_2 - R_0\tilde{\rho}_{22}, \qquad (1b)\\
\dot{\tilde{\rho}}_{33} &= -(\gamma_{31}+\gamma_{32})\tilde{\rho}_{33} + \mathrm{i}\,\Omega'_{\rm P}\tilde{\rho}_{13} - \mathrm{i}\,\Omega'_{\rm P}\tilde{\rho}_{31} \\
&\quad + \mathrm{i}\,\Omega'_{\rm C}\tilde{\rho}_{23} - \mathrm{i}\,\Omega'_{\rm C}\tilde{\rho}_{32} - R_0\tilde{\rho}_{33}, \qquad (1c)\\
\dot{\tilde{\rho}}_{44} &= -(\gamma_{41}+\gamma_{42})\tilde{\rho}_{44} + \mathrm{i}\,\Omega'_{\rm S}\tilde{\rho}_{14} \\
&\quad - \mathrm{i}\,\Omega'_{\rm S}\tilde{\rho}_{41} - R_0\tilde{\rho}_{44}, \qquad (1d)\\
\dot{\tilde{\rho}}_{12} &= -\mathrm{i}(\Delta_{\rm P} - \Delta_{\rm C})\tilde{\rho}_{12} + \mathrm{i}\,\Omega'_{\rm P}\tilde{\rho}_{32} \\
&\quad + \mathrm{i}\,\Omega'_{\rm S}\tilde{\rho} - \mathrm{i}\,\Omega'_{\rm C}\tilde{\rho}_{13} + \sqrt{\gamma_{31}\gamma_{32}}\varsigma_1\eta_1\tilde{\rho}_{33} \\
&\quad + \sqrt{\gamma_{41}\gamma_{42}}\varsigma_2\eta_2\tilde{\rho}_{44}, \qquad (1e)\\
\dot{\tilde{\rho}}_{13} &= -[(\gamma_{31}+\gamma_{32})/2 - \mathrm{i}\,\Delta_{\rm P}]\tilde{\rho}_{13} - \mathrm{i}\,\Omega'_{\rm S}\tilde{\rho}_{43} \\
&\quad - \mathrm{i}\,\Omega'_{\rm S}\tilde{\rho}_{12} + \mathrm{i}\,\Omega'_{\rm p}(\tilde{\rho}_{33} - \tilde{\rho}_{11}), \qquad (1f)\\
\dot{\tilde{\rho}}_{14} &= -[(\gamma_{41}+\gamma_{42})/2 - \mathrm{i}\,\Delta_{\rm S}]\tilde{\rho}_{14} + \mathrm{i}\,\Omega'_{\rm P}\tilde{\rho}_{34} \\
&\quad + \mathrm{i}\,\Omega'_{\rm S}(\tilde{\rho}_{44} - \tilde{\rho}_{11}), \qquad (1g)\\
\dot{\tilde{\rho}}_{23} &= -[(\gamma_{31}+\gamma_{32})/2 - \mathrm{i}\,\Delta_{\rm C}]\tilde{\rho}_{23} \\
&\quad + \mathrm{i}\,\Omega'_{\rm P}\tilde{\rho}_{21} + \mathrm{i}\,\Omega'_{\rm C}(\tilde{\rho}_{33} - \tilde{\rho}_{22}), \qquad (1h)\\
\dot{\tilde{\rho}}_{24} &= -[(\gamma_{41}+\gamma_{42})/2 - \mathrm{i}(\Delta_{\rm P} - \Delta_{\rm C} - \Delta_{\rm S})]\tilde{\rho}_{24} \\
&\quad + \mathrm{i}\,\Omega'_{\rm C}\tilde{\rho}_{34} - \mathrm{i}\,\Omega'_{\rm S}\tilde{\rho}_{21}, \qquad (1i)\\
\dot{\tilde{\rho}}_{34} &= -[(\gamma_{31}+\gamma_{32}+\gamma_{41}+\gamma_{42})/2 \\
&\quad + \mathrm{i}(\Delta_{\rm P} - \Delta_{\rm S})]\tilde{\rho}_{34} \\
&\quad + \mathrm{i}\,\Omega'_{\rm p}\tilde{\rho}_{14} + \mathrm{i}\,\Omega'_{\rm C}\tilde{\rho}_{24} - \mathrm{i}\,\Omega'_{\rm S}\tilde{\rho}_{31}. \qquad (1j)
\end{aligned}$$

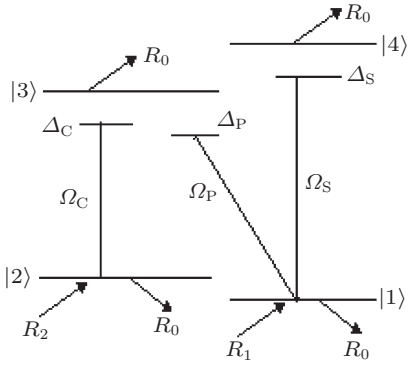


Fig. 1. Open N-type four-level atomic system.

The above equations are constrained by $\tilde{\rho}_{mn} = \tilde{\rho}^*_{nm}$ and $\tilde{\rho}_{11} + \tilde{\rho}_{22} + \tilde{\rho}_{33} + \tilde{\rho}_{44} = 1$. Here $\tilde{\rho}_{mm}$ is the atomic population of state $|m\rangle$; $\tilde{\rho}_{mn}$ ($m \neq n$) is the atomic polarization between states $|m\rangle$ and $|n\rangle$; γ_{mn} represents the spontaneous decay rate from level $|m\rangle$ to $|n\rangle$; $\zeta_1 = p_1\exp(\mathrm{i}\,\phi)$ and $\zeta_2 = p_2\exp(\mathrm{i}\,\phi)$; $\Omega'_{\rm C} = \Omega_{\rm C}\exp(-\mathrm{i}\,\phi_{\rm c})$, $\Omega'_{\rm p} = \Omega_{\rm p}\exp(-\mathrm{i}\,\phi_{\rm p})$, and $\Omega'_{\rm S} = \Omega_{\rm S}\exp(-\mathrm{i}\,\phi_{\rm s})$; $\phi_{\rm c}$, $\phi_{\rm p}$, and $\phi_{\rm s}$ denote the phases of

the driving, the probe field, and pump field, respectively; $\phi = \phi_{\rm p} - \phi_{\rm c}$ denotes the relative phase between the probe field and the driving field; $\Omega'_{\rm C}$, $\Omega'_{\rm p}$, and $\Omega'_{\rm s}$ are real parameters. If levels $|1\rangle$ and $|2\rangle$ lie so closely that the SGC effect must be taken into account, then $\eta_1(\eta_2) = 1$, otherwise $\eta_1(\eta_2) = 0$. The parameter p_1 is defined as $p_1 \equiv \boldsymbol{\mu}_{31} \cdot \boldsymbol{\mu}_{32}/|\boldsymbol{\mu}_{31}||\boldsymbol{\mu}_{32}| = \cos\theta_1$, with θ_1 representing the angle between the two dipole moments $\boldsymbol{\mu}_{31}$ and $\boldsymbol{\mu}_{32}$; $p_2 \equiv \boldsymbol{\mu}_{41} \cdot \boldsymbol{\mu}_{42}/|\boldsymbol{\mu}_{41}||\boldsymbol{\mu}_{42}| = \cos\theta_2$, with θ_2 denoting the angle between the two dipole moments $\boldsymbol{\mu}_{41}$ and $\boldsymbol{\mu}_{42}$. If $p_1 = p_2$=0, the SGC effect is absent. When $R_1 = R_2 = R_0 = 0$, equations (1a)–(1j) become the equations for the closed N-type four-level atomic system discussed in Ref. [40].

In the steady-state case, $\mathrm{d}\tilde{\rho}_{mn}/\mathrm{d}t = 0$, the new equations reduce to a set of coupled algebraic equations. We can obtain the exact nonlinear analytical expressions for $\mathrm{Im}\,\tilde{\rho}_{13}$ and $\tilde{\rho}_{mm}$ $(m = 1, 2, 3, 4)$ by using the symbolic package Mathematica or Maple. For simplicity, we do not consider the effect of the pump field phase on the system properties and make $\phi_{\rm s} = 0$. When Doppler broadening is present, the gain (absorption) of the probe field and population can be represented by

$$G = \frac{N|\mu_{31}|^2}{2\varepsilon_0\hbar\Omega'_{\rm p}}\,\mathrm{Im}\int_{-\infty}^{\infty}\tilde{\rho}_{13}(\delta_{\rm p})\rho(\delta_{\rm p})\,\mathrm{d}\delta_{\rm p}, \tag{2}$$

$$\bar{\rho}_{ii} = \int_{-\infty}^{\infty}\tilde{\rho}_{ii}(\delta_{\rm p})\rho(\delta_{\rm p})\,\mathrm{d}\delta_{\rm p}, \tag{3}$$

where

$$\rho(\delta_{\rm p}) = \exp[-(\delta_{\rm p} - \Delta_{\rm p})^2/2D^2]/\sqrt{2\pi D^2}, \tag{4a}$$

$$\delta_{\rm p}(\upsilon) = \Delta_{\rm p} \mp \omega_{\rm p}\upsilon/c, \tag{4b}$$

$$\delta_{\rm c}(\upsilon) = \Delta_{\rm c} \pm \omega_{\rm c}[\delta_{\rm p}(\upsilon) - \Delta_{\rm p}]/\omega_{\rm p}, \tag{4c}$$

$$\delta_{\rm s}(\upsilon) = \Delta_{\rm s} \pm \omega_{\rm s}[\delta_{\rm p}(\upsilon) - \Delta_{\rm p}]/\omega_{\rm p}, \tag{4d}$$

and N is the medium density; $\hbar$ is the Planck constant; $\rho(\delta_{\rm p})$ is the Maxwell–Boltzmann distribution for the atomic velocities, D denotes the Doppler width; μ_{31} is the electric dipole matrix element of the optical transition $|3\rangle \to |1\rangle$; υ and c are the speeds of the atom and light, respectively; $\delta_{\rm c}(\upsilon)$, $\delta_{\rm p}(\upsilon)$ and $\delta_{\rm s}(\upsilon)$ are the detunings of driving, probe, and pump fields from the respective transitions; $\delta_{\rm c}(\upsilon)$ and $\delta_{\rm s}(\upsilon)$ have been represented by functions of $\delta_{\rm p}(\upsilon)$, respectively; the negative (positive) sign corresponds to the propagation direction of the field the same as (opposite to) the motion direction of the atom. For convenience, in the following calculations we assume that the driving and pump fields all propagate in the direction that the atoms move along, so co-propagating (counter-propagating) probe and driving fields correspond to the propagation direction of the probe field the same (opposite) as the motion direction of the atom. Under this condition, we only need take the negative sign in Eqs. (4c) and (4d).

3. Numerical calculation results and discussion

If $G > 0$, the system exhibits gain for the probe field; if $G < 0$, the probe field is attenuated. When $\bar{\rho}_{33} - \bar{\rho}_{11} < 0$ and $G > 0$ are simultaneously satisfied, GWI can be obtained, and then LWI can be realized. Under the condition of $\Delta_{\rm c} = 0$ and $\Delta_{\rm s} = 0$, we explore effects of the relative phase (ϕ) between the probe and driving fields, the atomic exit rate (R_0) and the ratio (S) of the atomic injection rates on GWI by using the numerical results from Eqs. (2) and (3). It should be pointed out that in the following discussion, the inversionless condition $\bar{\rho}_{33} - \bar{\rho}_{11} < 0$ is always satisfied; if $G > 0$, we always obtain GWI. So in the following discussion the gain always means GWI. In addition, in the following discussion (including that in all figures), the gain G ($G_{\rm max}$) is scaled by $N|\mu_{31}|^2/(2\varepsilon_0\hbar\Omega'_{\rm p})$, all other parameters (except p_1, p_2, and ϕ) are scaled by γ_{31}. For the sake of clarity, we analyse the probe field and the driving field first in the case where both of them are in the same propagation directions (co-propagating), and then in the case where they are in the opposite propagation directions (counter-propagating).

3.1. Co-propagating probe and driving fields

Figure 2 shows the values of gain G versus the probe detuning $\Delta_{\rm p}$ for different values of the relative phase ϕ in a closed system with $R_0 = 0$ and $S = 0$ (which is equivalent to $R_1 = R_2 = R_0 = 0$) and an open system with $R_0 = 0.4$ and $S = 9$ when Doppler width D equals 0.8.

It can be seen from Fig. 2 that (i) the size of gain and the probe detuning region in which gain exists are very sensitive to variation of the value of ϕ; correspondingly for a fixed value of ϕ there exists a gain maximum ($G_{\rm max}$), which always occurs when the value of $\Delta_{\rm p}$ is close to zero, i.e. the probe field is close to the resonant value; the gain maximum ($G_{\rm max}$) corresponding to $\phi = \pi/2$ also is the largest gain value; (ii) the variations of R_0 and S also have a dramatic effect on the phase-dependent gain and the probe detuning region in which gain exists. The largest gain

value in the open system with $R_0 = 0.4$ and $S = 9$ is considerably larger than that in the closed system with $R_0 = 0$ and $S = 0$.

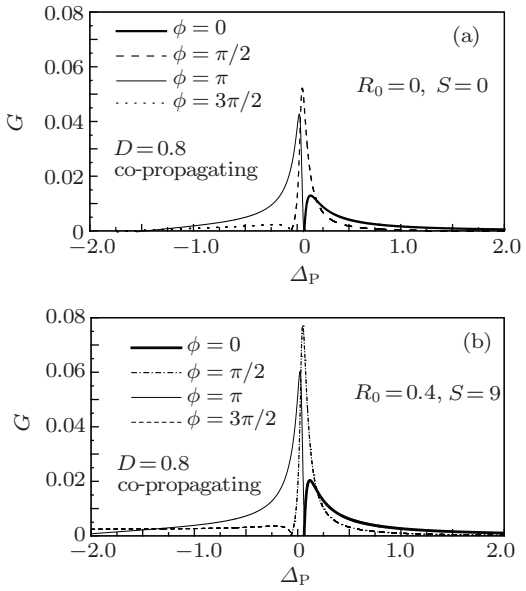


Fig. 2. The G as a function of Δ_p for different values of ϕ: (a) the closed system ($R_0 = 0$, $S = 0$), (b) the open system ($R_0 = 0.4$, $S = 9$) when the probe and driving fields are co-propagating. The parameters' values are $D = 0.8$, $\Omega_S = 0.3$, $\Omega_c = 0.5$, $\Omega_p = 0.01$, $\gamma_{31} = \gamma_{32} = 1$, $\Delta_s = 0$, $\eta_1 = \eta_2 = 1$, $p_1 = p_2 = 1$, $\gamma_{41} = \gamma_{42} = 1.2$, $\Delta_c = 0$, $\Delta_s = 0$. In all the figures from Fig. 2 to Fig. 8, G (G_{max}) is scaled by $N|\mu_{31}|^2/(2\varepsilon_0\hbar\Omega'_p)$, all other parameters (except of p_1, p_2, and ϕ) are scaled by γ_{31}.

Figure 3 shows that i) G_{max} varies periodically with ϕ. The period is 2π. Variation trend of G_{max} versus ϕ in the open system is the same as that in the closed system; but the value of G_{max} in open system is obviously larger than that in the closed system; this means that varying values of R_0 and S can change the size of gain but do not change the variation trend of G_{max} versus ϕ; ii) the value of G_{max} decreases with the value of D increasing.

Now we discuss in detail the effects of atomic exit rate and ratio of the atomic injection rates on phase-dependent gain.

Figure 4 illustrates the values of gain maximum G_{max} versus the atomic exit rate R_0 for different values of ϕ with $S = 9$ and $D = 0.8$ in the open system. Figure 4 shows that varying value of ϕ will obviously changing the size of G_{max} and varying value of R_0 also has a considerable effect on the phase-dependent gain maximum value. For a fixed value of ϕ, there is a specific value R'_0 of the atomic exit rate; when $R_0 < R'_0$, G_{max} increases with R_0 increasing; when $R_0 > R'_0$, G_{max} decreases with R_0 increasing; when $R_0 = R'_0$, we obtain the largest value of G_{max}, i.e. the largest gain value. The curves corresponding to different values of ϕ have different values of R'_0.

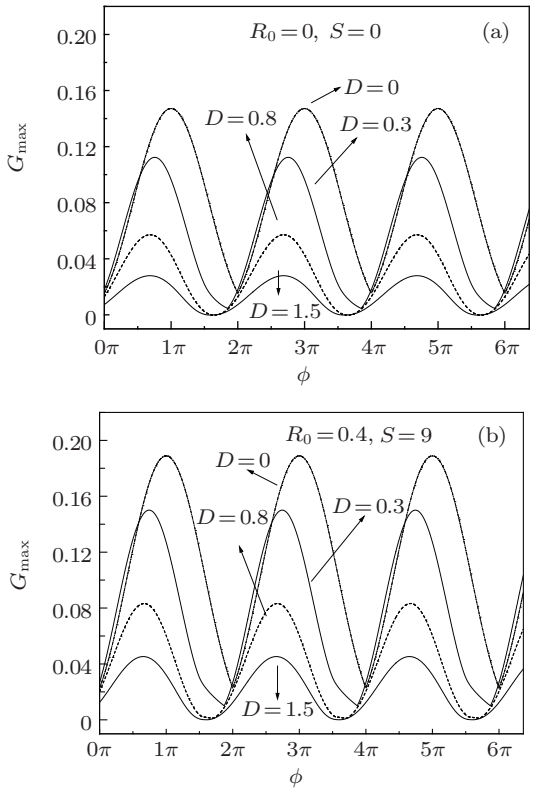


Fig. 3. The G_{max} as a function of ϕ at different values of D for (a) the closed system ($R_0 = 0$, $S = 0$), and (b) the open system ($R_0 = 0.4$, $S = 9$). Other parameters have the same values as those in Fig. 2. The probe and driving fields are co-propagating.

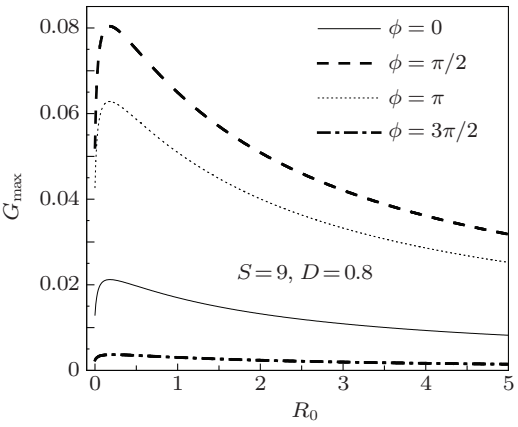


Fig. 4. The G_{max} as a function of R_0 at different values of ϕ with $D = 0.8$ and $S = 9$ and other parameters have the same values as those in Fig. 2. The probe field and the driving field are co-propagating.

Figure 5 shows the values of gain maximum G_{max} versus the ratio S of the atomic injection rates for different values of ϕ with $R_0 = 0.4$ and $D = 0.8$ in the open system. From Fig. 5 we can see that the value

of $G_{\max}$ is determined mainly by the value of ϕ while the variation of value of S also has an apparent modulation effect on the phase-dependent gain maximum value. In a certain value range of S, $G_{\max}$ increases monotonically with S increasing.

When the values of other parameters are fixed, by selecting appropriate values of R_0 and S, in the open system we can obtain a much larger gain than in the corresponding closed system. For example, from Fig. 2 we know that in the closed system ($R_0 = 0$ and $S = 0$), when $\phi = \pi/2$, we obtain the largest gain value (about 0.052). From Figs. 4 and 5 we can see clearly that in the open system ($R_0 \neq 0$ and $S \neq 0$), when $\phi = \pi/2$, we can have a much larger gain value than 0.052 by selecting appropriate values of R_0 and S.

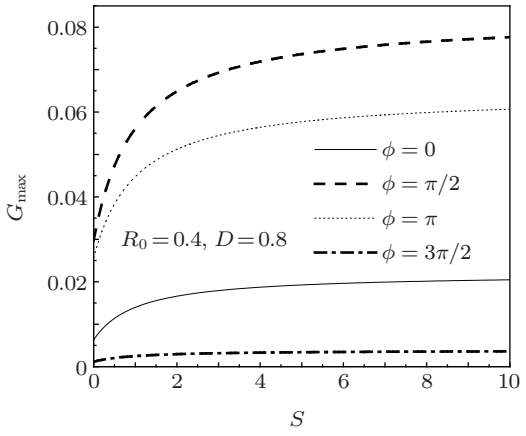


Fig. 5. The $G_{\max}$ as a function of S at different values of ϕ with $D = 0.8$ and $R_0 = 0.4$ and other parameters have the same values as those in Fig. 2. The probe field and the driving field are co-propagating.

3.2. Counter-propagating probe and driving fields

In the following we analyse the case with the counter-propagating probe and driving fields.

Figure 6 shows the curves of gain G versus probe detuning $\Delta_{\rm p}$ for different values of ϕ in the closed and open systems when $D = 0.8$ in the counter-propagating case. Comparing Fig. 6 with Fig. 2 (note that the ordinate scales in Figs. 6 and Figs. 2 are different), we can see the following points. (I) Similar to that in the co-propagating case, in the counter-propagating case the variation of ϕ also has a remarkable effect on gain and probe detuning region. (II) In the co-propagating case, for each value of ϕ, gain curve corresponding to $R_0 = 0.4$ and $S = 9$ is always similar to that corresponding to $R_0 = 0$ and $S = 0$; in both cases the values of R_0 and S are zero (Fig. 6(a)) and not (Fig. 6(b)), the largest gain value always appears when $\phi = \pi/2$. However, in the counter-propagating case, when $\phi = \pi$ or $\phi = 3\pi/2$, the two gain curves corresponding to different values of R_0 and S are much different: for $R_0 = 0$ and $S = 0$, the largest gain value appears when $\phi = \pi/2$; but for $R_0 = 0.4$ and $S = 9$, the largest gain value appears when $\phi = \pi$. This means that varying values of R_0 and S in the counter-propagating case has a stronger effect on phase-dependent gain than that in the co-propagating case. (III) No matter whether the values of R_0 and S are zero, for a same value of ϕ, gain curve in the counter-propagating case is much different from that in the co-propagating case; the largest gain value in the counter-propagating case is much smaller than that in the co-propagating case.

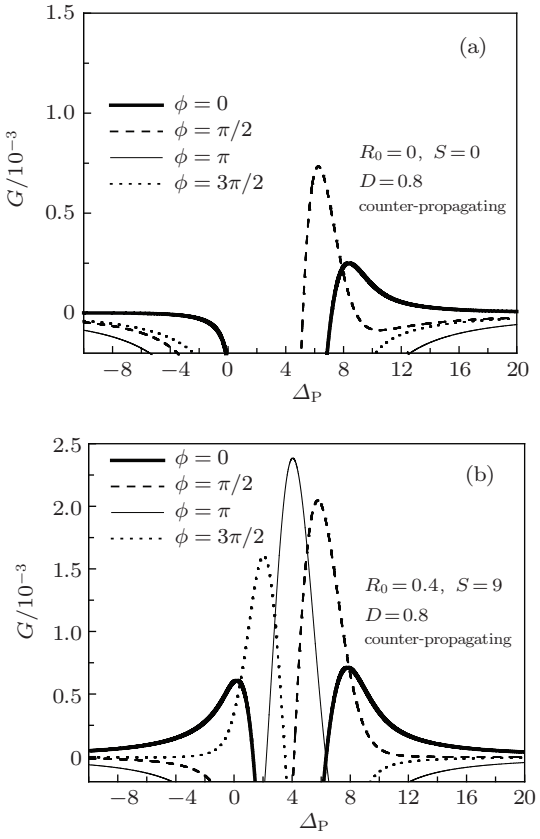


Fig. 6. The G as a function of $\Delta_{\rm p}$ at different values of ϕ for (a) the closed system ($R_0 = 0$, $S = 0$) and (b) the open system ($R_0 = 0.4$, $S = 9$). Other parameters have the same values as those in Fig. 2. The probe and driving fields are counter-propagating.

Our numerical calculation result shows the following points. I) Similar to in the co-propagating case, in the counter-propagating case $G_{\max}$ varies periodically with ϕ varying, the period is 2π; the value of $G_{\max}$ decreases with the value of D increasing; the value of $G_{\max}$ in the open system ($R_0 \neq 0$ and $S \neq 0$) is obviously larger than that in the closed system

($R_0 = 0$ and $S = 0$). II) Different from that in the co-propagating case, in the counter-propagating case varying values of R_0 and S can obviously change not only the size of $G_{\max}$ but also the variation trend of $G_{\max}$ with ϕ varying; when the values of other parameters keep unchanged, the value of $G_{\max}$ in the counter-propagating case is considerably smaller than that in the co-propagating case.

Figure 7 shows the values of gain maximum $G_{\max}$ versus the atomic exit rate R_0 for different values of ϕ in the open system with $D = 0.8$ and $S = 9$ when the probe field and driving field are counter-propagating. Comparing Fig. 7 with Fig. 4, we can obtain the following conclusions. 1) Similar to that in the co-propagating case, in the counter-propagating case varying value of ϕ will obviously change the size of $G_{\max}$ and varying value of R_0 also has a considerable effect on the phase-dependent gain; for a same value of ϕ, by choosing suitable value of R_0, we can obtain visibly larger gain. 2) In the co-propagating case, curves of $G_{\max}$ corresponding to different values of ϕ do not cross each other; but in the counter-propagating case, when R_0 takes a special value, two curves of $G_{\max}$ corresponding to $\phi = 0$ and $3\pi/2$ ($\phi = \pi/2$ and π) cross. 3) The largest value of $G_{\max}$ in the counter-propagating case is much smaller than that in the co-propagating case.

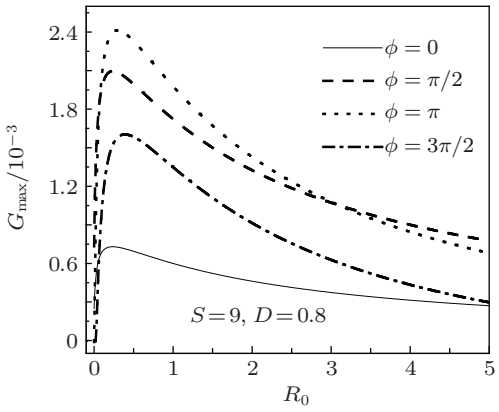


Fig. 7. The $G_{\max}$ as a function of R_0 for different values of ϕ with $D = 0.8$ and $S = 9$ and other parameters have the same values as those in Fig. 2. The probe field and driving field are ounter-propagating.

Figure 8 shows the curves for gain maximum $G_{\max}$ versus the ratio S of the atomic injection rates for different values of ϕ in the open system with $D = 0.8$ and $R_0 = 0.4$ when the probe and driving fields are counter-propagating. Comparing Fig. 8 with Fig. 5, we can obtain the similar conclusion as that obtained by comparing Fig. 7 with Fig. 4, only it is needed that in the above conclusion R_0 is replaced by S.

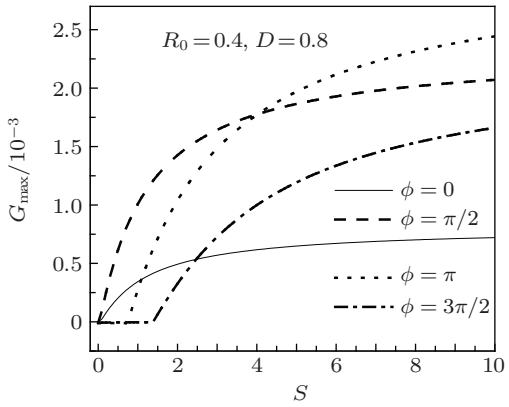


Fig. 8. The $G_{\max}$ as a function of S for different values of ϕ with $D = 0.8$ and $R_0 = 0.4$ and other parameters have the same values as those in Fig. 2. The probe field and the driving field are counter-propagating.

Similar to that in the co-propagating case, in the counter-propagating case, when the values of other parameters are fixed, by selecting appropriate values of R_0 and S, we can also obtain a much larger gain than that in the closed system ($R_0 = 0$ and $S = 0$). For example, from Fig. 6 we know that in the closed system, when $\phi = \pi/2$ we have the largest gain value (about 0.00075). It is obvious from Figs. 7 and 8 that in the open system ($R_0 \neq 0$ and $S \neq 0$), not only when $\phi = \pi/2$ but also when $\phi = \pi$ and $\phi = 3\pi/2$ can we obtain a much larger gain value than 0.00075 by selecting appropriate values of R_0 and S.

Why in both co- and counter-propagating probe and driving fields cases, when the values of other parameters are fixed, by selecting appropriate values of R_0 and S, a much larger gain in the open system than in the corresponding closed system ($R_0 = 0$ and $S = 0$) can be obtained? We try to give a brief physical explanation for this in the following.

In an atomic system, quantum constructive interference leads to gain and quantum destructive interference leads to absorption. In a closed system, quantum destructive or constructive interference is determined mainly by the phases, propagating directions, and detunings of the driving field, the probe field, and the Doppler width. Compared with that in the corresponding closed system, in the open system the atomic injection and exit can remarkably affect the quantum destructive or constructive interference. In some situations, quantum destructive interference arises and leads to absorption; and in other situations, quantum constructive interference appears and leads to gain. By selecting appropriate values of R_0 and S, the quan-

tum constructive interference can significantly be enhanced, so a much larger gain than in the corresponding closed system can be obtained.

4. Conclusion

The effect of the relative phase (ϕ) between the probe field and the driving field on the GWI in a Doppler broadened open N-type four-level atomic system with SGC for the cases of both co- and counter-propagating probe and driving fields is studied and compared with that in the corresponding closed system. The results are as follows. (A) GWI and the probe detuning region in which GWI exists are very sensitive to the variation of the relative phase; when the values of the other parameters are kept unchanged, by adjusting the value of ϕ, the largest GWI can be obtained. GWI varies periodically with ϕ varying, the period is 2π. (B) Varying ratio (S) of the atomic injection rate to atomic exit rate (R_0) also has obvious effect on phase-dependent GWI and the probe detuning region in which GWI exists. In general, GWI first increases and then decreases with R_0 increasing; in a certain value range of S, GWI increases monotonically with S increasing; by adjusting the values of R_0 and S, in the open system a much larger GWI than in the closed system can be obtained. (C) Different from the case without Doppler broadening, when Doppler broadening exists, varying propagation directions of the probe field and the driving field have a remarkable effect on GWI. Modulation effects of R_0 and S on the phase-dependent GWI in the case of the counter-propagating probe and driving fields are stronger than those in the co-propagating case, GWI in the co-propagating case is much larger than that in the counter-propagating case.

References

[1] Kocharovskaya O and Khanin Ya I 1988 *JEPT Lett.* **48** 630

[2] Harris S E 1989 *Phys. Rev. Lett.* **62** 1033

[3] Scully M O, Zhu S Y and Gavridiles A 1989 *Phys. Rev. Lett.* **62** 2813

[4] Mompart J and Corbalan R 2000 *J. Opt.* B: *Quantum Semiclass. Opt.* **2** R7

[5] Scully M O 2002 *Phys. Rev. Lett.* **88** 050602

[6] Rostovtsev Y, Trendafilov S, Artemiev A, Kapale K, Kurizki G and Scully M O 2003 *Phys. Rev. Lett.* **90** 214802

[7] Li C R, Li Y C, Men F K, Pao C H, Tsai Y C and Wang J F 2005 *Appl. Phys. Lett.* **86** 201112

[8] Wieczorek S and Chow W W 2006 *Phys. Rev. Lett.* **97** 113903

[9] Waldmueller I, Wanke M C and Chow W W 2007 *Phys. Rev. Lett.* **99** 117401

[10] Kilin S Y, Kapale K T and Scully M O 2008 *Phys. Rev. Lett.* **100** 173601

[11] Liberato S D and Ciuti C 2009 *Phys. Rev. Lett.* **102** 136403

[12] Scully M O 2010 *Phys. Rev. Lett.* **104** 207701

[13] Paspalakis E and Knight P L 1998 *Phys. Rev. Lett.* **81** 293

[14] Zhou P and Swain S 1998 *Phys. Rev. Lett.* **82** 2500

[15] Ansari N A, Gea-Banacloche J and Zubairy M S 1990 *Phys. Rev.* A **41** 5179

[16] Ghafoor F, Zhu S and Zuairy M S 2000 *Phys. Rev.* A **62** 013811

[17] Bortman-Arbiv D, Wilson-Gordon A D and Friedmann H 2001 *Phys. Rev.* A **63** 043818

[18] Wu J H and Gao J Y 2002 *Phys. Rev.* A **65** 063807

[19] Xu W H, Zhang H F, Gao J Y and Zhang B 2003 *J. Opt. Soc. Am.* B **20** 2377

[20] Goren C, Wilson-Gordon A D, Rosenbluh M and Friedmann H 2003 *Phys. Rev.* A **68** 043818

[21] Hou B P, Wang S J, Yu W L and Sun W L 2004 *Phys. Rev.* A **69** 053805

[22] Sahrai M, Tajalli H, Kapale K T and Zubairy M S 2004 *Phys. Rev.* A **70** 023813

[23] Sahrai M, Tajalli H, Kapale K T and Zubairy M S 2005 *Phys. Rev.* A **72** 013820

[24] Bai A F, Guo H and Han D A 2005 *J. Opt.* B: *Quantum Semiclass. Opt.* **7** 35

[25] Fan X J, Cui N, Tian S F, Ma H, Gong S Q and Xu Z Z 2005 *J. Mod. Opt.* **52** 2759

[26] Kapale K T and Zubairy M S 2006 *Phys. Rev.* A **73** 023813

[27] Yang J H and Zhu S Y 2007 *Phys. Rev.* A **76** 062322

[28] Fan X J, Li A Y, Tian S F, Tong D M, Gong S Q and Xu Z Z 2007 *Eur. Phys. J.* D **42** 483

[29] Yang Y L, Liu Z B, Wang L, Tong D M and Fan X J 2009 *Chin. Phys.* B **18** 1054

[30] Osman K I, Hassan S S and Joshi A 2009 *Eur. Phys. J.* D **54** 119

[31] Li A J, Song X L, Wei G X, Wang L and Gao J Y 2008 *Phys. Rev.* A **77** 053806

[32] Wang L, Li G Q, Xiao S W and Zheng C B 2010 *Acta Phys. Sin.* **59** 8512 (in Chinese)

[33] Yang L J, Ma L J, Lu D Q and Zhang L S 2011 *Acta Phys. Sin.* **60** 104205 (in Chinese)

[34] Karawajczyk A and Zajrzewski J 1995 *Phys. Rev.* A **51** 830

[35] Vemuri G and Agarwal G S 1996 *Phys. Rev.* A **53** 1060

[36] Mompart J, Peters C and Corbalan R 1998 *Phys. Rev.* A **57** 2163

[37] Boon J R, Zekou E, McGloin D and Dunn M H 1998 *Phys. Rev.* A **58** 2560

[38] Ahufinger V, Mompart J and Corbalan R 1999 *Phys. Rev.* A **60** 614

[39] Fan X J, Ma H, Liu Z B and Tong D M 2009 *Chin. Phys.* B **18** 5342

[40] Fan X J, Liu Z B, Liang Y, Jia K N and Tong D M 2011 *Phys. Rev.* A **83** 043805