Three-qubit quantum-gate operation with an SQUID in a cavity*

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We propose a method of realizing a three-qubit quantum gate with a superconducting quantum interference device (SQUID) in a cavity. In this proposal, the gate operation involves the SQUID ground-states and the Fock states of cavity modes $\hat{b}$ and $\hat{c}$. The two field-modes act as the controlling qubits, and the two SQUID states form the target qubit. Since only the metastable lower levels are involved in the gate operation, the gate is not affected by the SQUID decay rates.

Keywords: SQUID, quantum gate, C^2-Phase gate, C^2-Not gate

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1. Introduction

Due to design flexibility, large-scale integration and compatibility with conventional electronics, superconducting devices, including Cooper-pair boxes, Josephson junctions, superconducting quantum interference devices (SQUIDs), and superconducting qubits, have become the most promising candidates for scalable quantum computing.[1−3] For SQUID systems, people have suggested several theoretical methods of realizing a single-qubit gate and a two-qubit controlled-phase (CP) or a controlled-NOT (CNOT) gate.[4−12]

As is well known, any multi-qubit gate or quantum-computing network can be partitioned into a sequence of single-qubit operations and two-qubit gates. However, it will reduce the complexity of the physical realization of scalable quantum circuits if we can directly implement general N-qubit gates. So, recently, attention has been paid to the physical realization of multiqubit gates. There are proposals to implement three-qubit gates with atoms in a cavity.[13−16] The implementation of a three-qubit Fredkin gate based on cavity quantum electrodynamics and optics has also been proposed.[17,18] Recently, the authors have presented different protocols for implementing N-qubit quantum gates in a cavity.[19−22]

In this work, we propose a method to realize a C^2-Phase gate (controlled-, controlled-phase gate) and a C^2-Not gate (controlled- controlled-Not gate) in a cavity-QED system. Our scheme is different from those in Refs. [13]–[15], and the idea may be extended to the direct implementation of N-qubit gates. In this scheme, the gate operation takes place in the lower state of the system and hence the decoherence effect due to radiative damping is minimized.

The rest of the present paper is organized as follows. In Section 2 we give a brief description of the system. In Section 3 we show how to implement a C^2-Phase gate and a C^2-Not gate. The conclusions are given in Section 4.

2. System

We consider a four-level Λ-type SQUID interacting with a three-mode cavity (see Fig. 1). The levels are denoted by $|g\rangle$, $|l\rangle$, $|e_1\rangle$, $|e_2\rangle$, respectively. The transitions from $|g\rangle$ to $|e_1\rangle$, from $|l\rangle$ to $|e_1\rangle$ and from $|l\rangle$ to $|e_2\rangle$ interact with the cavity modes having annihilation operators $\hat{a}$, $\hat{b}$, and $\hat{c}$, respectively. g_1, g_2, and g_3 represent the SQUID-cavity couplings of the a, b, c modes with the corresponding transitions respectively. Δ_i $(i = 1, 2, 3)$ are the respective one-photon

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detunings. The Hamiltonian describing this system in the interaction picture can be written as ($\hbar = 1$)[5]

$$H = g_1 a\, \mathrm{e}^{\mathrm{i}\Delta_1 t}|e_1\rangle\langle g| + g_2 b\, \mathrm{e}^{\mathrm{i}\Delta_2 t}|e_1\rangle\langle l| + g_3 c\, \mathrm{e}^{\mathrm{i}\Delta_3 t}|e_2\rangle\langle l| + \mathrm{H.c.} \quad (1)$$

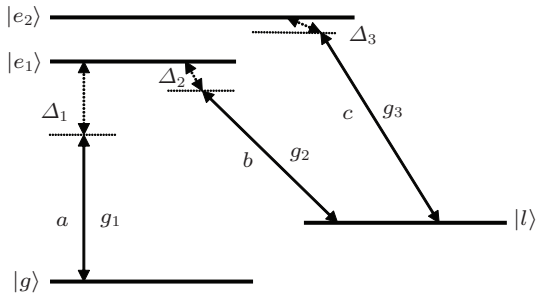


Fig. 1. Sketch of the energy level configuration and the corresponding transitions.

If we assume the initial photon occupation numbers of the three cavity modes to be n_1, n_2, n_3, respectively, then the state vector can be expanded in terms of the possible basis states as follows:

$$|\Psi\rangle = a_1|e_2\rangle|n_1,n_2,n_3\rangle + a_2|l\rangle|n_1,n_2,n_3+1\rangle + a_3|e_1\rangle|n_1,n_2-1,n_3+1\rangle + a_4|g\rangle|n_1+1,n_2-1,n_3+1\rangle, \quad (2)$$

where $a_i(t)$ $(i = 1, 2, 3, 4)$ are the probability amplitudes for the corresponding states. Using the time-dependent Schrödinger equation, we obtain the equations of the motion for the probability amplitude as

$$\begin{aligned}
\mathrm{i}\dot{a}_1 &= g_3 a_2\, \mathrm{e}^{\mathrm{i}\Delta_3 t}\sqrt{n_3+1},\\
\mathrm{i}\dot{a}_2 &= g_3 a_1\, \mathrm{e}^{-\mathrm{i}\Delta_3 t}\sqrt{n_3+1} + g_2 a_3\, \mathrm{e}^{-\mathrm{i}\Delta_2 t}\sqrt{n_2},\\
\mathrm{i}\dot{a}_3 &= g_2 a_2\, \mathrm{e}^{\mathrm{i}\Delta_2 t}\sqrt{n_2} + g_1 a_4\, \mathrm{e}^{\mathrm{i}\Delta_1 t}\sqrt{n_1+1},\\
\mathrm{i}\dot{a}_4 &= g_1 a_3\, \mathrm{e}^{-\mathrm{i}\Delta_1 t}\sqrt{n_1+1}. \quad (3)
\end{aligned}$$

Then we make use of the following transformations to simplify the above equations:

$$a_1\, \mathrm{e}^{-\mathrm{i}(\Delta_1-\Delta_2+\Delta_3)t} \to p_1, \quad a_2\, \mathrm{e}^{-\mathrm{i}(\Delta_1-\Delta_2)t} \to p_2,$$
$$a_3\, \mathrm{e}^{-\mathrm{i}\Delta_1 t} \to p_3, \quad a_4 \to p_4. \quad (4)$$

We will obtain the following set of equations:

$$\begin{aligned}
\dot{p}_1 &= -\mathrm{i}(\Delta_1-\Delta_2+\Delta_3)p_1 - \mathrm{i}g_3 p_2\sqrt{n_3+1},\\
\dot{p}_2 &= -\mathrm{i}(\Delta_1-\Delta_2)p_2 - \mathrm{i}(g_3 p_1\sqrt{n_3+1} + g_2 p_3\sqrt{n_2}),\\
\dot{p}_3 &= -\mathrm{i}\Delta_1 p_3 - \mathrm{i}(g_2 p_2\sqrt{n_2} + g_1 p_4\sqrt{n_1+1}),\\
\dot{p}_4 &= -\mathrm{i}g_1 p_3\sqrt{n_1+1}. \quad (5)
\end{aligned}$$

For simplicity we assume $g_1 = g_2 = g_3 = g$, $\Delta_1 > \Delta_2 > \Delta_3$ and work under the limit of large one-photon detuning, $\Delta_i \gg g$. Then the state $|e_1\rangle$ can be adiabatically eliminated.[23] So we obtain $\dot{p}_3 = 0$ and equation (5) reduces to

$$\begin{aligned}
\dot{p}_1 &= -\mathrm{i}(\Delta_1-\Delta_2+\Delta_3)p_1 - \mathrm{i}gp_2\sqrt{n_3+1},\\
\dot{p}_2 &= -\mathrm{i}(\Delta_1-\Delta_2)p_2 - \mathrm{i}gp_1\sqrt{n_3+1} + \mathrm{i}\frac{g^2}{\Delta_1}(p_2 n_2 + p_4\sqrt{n_2(n_1+1)}),\\
\dot{p}_4 &= \mathrm{i}\frac{g^2}{\Delta_1}[p_2\sqrt{n_2(n_1+1)} + p_4(n_1+1)]. \quad (6)
\end{aligned}$$

We further assume the condition $\Delta_1 - \Delta_2 + \Delta_3 \gg g$. Under such a condition, we set $\dot{p}_1 = 0$, and thus obtain the evolution of the system involving only two lower states such as

$$\begin{aligned}
\dot{p}_2 &= -\Big\{\mathrm{i}(\Delta_1-\Delta_2) - \mathrm{i}\frac{g^2}{\Delta_1}\left[\frac{\Delta_1}{\Delta_1-\Delta_2+\Delta_3}(n_3+1)+n_2\right]\Big\}p_2 + \mathrm{i}\frac{g^2}{\Delta_1}p_4\sqrt{n_2(n_1+1)},\\
\dot{p}_4 &= \mathrm{i}\frac{g^2}{\Delta_1}p_2\sqrt{n_2(n_1+1)} + \mathrm{i}\frac{g^2}{\Delta_1}p_4(n_1+1). \quad (7)
\end{aligned}$$

We use initial condition $n_1 = 0$, $n_2 = 1$, and $n_3 = 0$. So we have

$$\begin{aligned}
p_2 &= \left(\mathrm{i}A\beta\frac{\Delta_1}{g^2} - \mathrm{i}B\alpha\frac{\Delta_1}{g^2} - B\right)\mathrm{e}^{\alpha t}\sin\beta t - \left(\mathrm{i}A\alpha\frac{\Delta_1}{g^2} + \mathrm{i}B\beta\frac{\Delta_1}{g^2} + A\right)\mathrm{e}^{\alpha t}\cos\beta t,\\
p_4 &= A\,\mathrm{e}^{\alpha t}\cos\beta t + B\,\mathrm{e}^{\alpha t}\sin\beta t, \quad (8)
\end{aligned}$$

where

$$\begin{aligned}
A &= p_4(0),\\
B &= \mathrm{i}\frac{g^2}{\Delta_1\beta}p_2(0) - \left(\frac{\alpha}{\beta} - \mathrm{i}\frac{g^2}{\Delta_1\beta}\right)p_4(0), \quad (9)
\end{aligned}$$

and

$$\begin{aligned}
\alpha &= \mathrm{i}\left(\frac{g^2}{\Delta_1} - \frac{\Delta_1-\Delta_2}{2} + \frac{g^2}{2(\Delta_1-\Delta_2+\Delta_3)}\right),\\
\beta &= \frac{1}{2}\left[\left(\Delta_1-\Delta_2-\frac{g^2}{\Delta_1-\Delta_2+\Delta_3}\right)^2 + 4\left(\frac{g^2}{\Delta_1}\right)^2\right]^{1/2}. \quad (10)
\end{aligned}$$

3. Three-qubit quantum-gate operation

In this section, we will perform a C^2-Phase gate and a C^2-Not gate.

3.1. C^2-Phase gate

The solution of Eq. (8) is obtained under the condition of large single-photon detuning such as $\Delta_i \gg g$ and $\Delta_1 - \Delta_2 + \Delta_3 \gg g$. From the solution one can easily see that if we set $p_4(0) = 0$, and $p_2(0) = 1$, then we have

$$p_2 = \left(\frac{\alpha}{\beta} - \frac{\mathrm{i}g^2}{\Delta_1\beta}\right)\mathrm{e}^{\alpha t}\sin\beta t + \mathrm{e}^{\alpha t}\cos\beta t,$$
$$p_4 = \frac{\mathrm{i}g^2}{\Delta_1\beta}\mathrm{e}^{\alpha t}\sin\beta t. \tag{11}$$

We further set the condition

$$\alpha = 0, \quad \beta t = \pi \quad \text{or}$$
$$\frac{2g^2}{\Delta_1} = \Delta_1 - \Delta_2 - \frac{g^2}{\Delta_1 - \Delta_2 + \Delta_3},$$
$$t = \frac{\Delta_1\pi}{\sqrt{2}g^2}. \tag{12}$$

Then, when mode a is in the state $|0\rangle_a$, we can perform the following C^2-Phase gate operation:

$$\begin{aligned}
&|0_b, 0_c, g\rangle \to |0_b, 0_c, g\rangle, \quad |0_b, 0_c, l\rangle \to |0_b, 0_c, l\rangle,\\
&|1_b, 0_c, g\rangle \to |1_b, 0_c, g\rangle, \quad |1_b, 0_c, l\rangle \to |1_b, 0_c, l\rangle,\\
&|0_b, 1_c, g\rangle \to |0_b, 1_c, g\rangle, \quad |0_b, 1_c, l\rangle \to |0_b, 1_c, l\rangle,\\
&|1_b, 1_c, g\rangle \to |1_b, 1_c, g\rangle, \quad |1_b, 1_c, l\rangle \to -|1_b, 1_c, l\rangle,
\end{aligned} \tag{13}$$

which clearly involve the SQUID ground-state basis and the Fock-state basis in $\hat{b}$ and $\hat{c}$ modes. The two field modes $\hat{b}$ and $\hat{c}$ act as the controlling qubits, and the two SQUID states form the target qubit.

3.2. C^2-Not gate

When we apply the Hadamard transformation to the SQUID state before and after the C^2-Phase gate operation (13), we can obtain the following C^2-Not gate

$$\begin{aligned}
&|0_b, 0_c, g\rangle \to |0_b, 0_c, g\rangle, \quad |0_b, 0_c, l\rangle \to |0_b, 0_c, l\rangle,\\
&|1_b, 0_c, g\rangle \to |1_b, 0_c, g\rangle, \quad |1_b, 0_c, l\rangle \to |1_b, 0_c, l\rangle,\\
&|0_b, 1_c, g\rangle \to |0_b, 1_c, g\rangle, \quad |0_b, 1_c, l\rangle \to |0_b, 1_c, l\rangle,\\
&|1_b, 1_c, g\rangle \to |1_b, 1_c, l\rangle, \quad |1_b, 1_c, l\rangle \to |1_b, 1_c, g\rangle.
\end{aligned} \tag{14}$$

Clearly, the field qubits of modes $\hat{b}$ and $\hat{c}$ are controlling qubits and the SQUID qubit is the controlled qubit.

4. Conclusion

In the paper, we present a system in which a four-level SQUID interacts with a three-mode cavity, with the cavity modes being highly detuned from the corresponding single-photon transitions. Using such a system, we obtain the C^2-Phase gate and the C^2-Not gate. Since only the metastable lower levels are involved in the gate operation, the gate is not affected by the SQUID decay rate. Further, the present system may be used to perform the N-qubit quantum gate.

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