Optical transfer function analysis of circular-pupil wavefront coding systems with separable phase masks*

Zhao Ting-Yu(赵廷玉)[a)†], Liu Qin-Xiao(刘钦晓)[b)], and Yu Fei-Hong(余飞鸿)[b)]

[a)] *Department of Physics, Zhejiang Sci-Tech University, Hangzhou* 310018, *China*
[b)] *Optical Engineering Department, Zhejiang University, Hangzhou* 310027, *China*

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This paper proposes a simple method to achieve the optical transfer function of a circular pupil wavefront coding system with a separable phase mask in Cartesian coordinates. Based on the stationary phase method, the optical transfer function of the circular pupil system can be easily obtained from the optical transfer function of the rectangular pupil system by modifying the cut-off frequency and the on-axial modulation transfer function. Finally, a system with a cubic phase mask is used as an example to illustrate the way to achieve the optical transfer function of the circular pupil system from the rectangular pupil system.

Keywords: wavefront coding, circular pupil, separable phase mask, stationary phase method

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1. Introduction

Wavefront coding[1−3] is widely used to extend the depth of field and to correct defocus related aberration. It is a two-step imaging system.[4] Firstly, with a special phase mask[5] added to the pupil, blurred but defocus insensitive images will be obtained in the image plane. Secondly, the defocus-invariant point spread function (PSF) is used as the deconvolution kernel in digital decoding to obtain sharp images. In recent years, several varieties of phase masks have been designed to achieve a defocus insensitive system, which can be divided into two categories, inseparable phase masks[6,7] and separable phase masks in Cartesian coordinates. Separable phase masks seems to be more frequently discussed and analyzed due to its computation convenience, such as cubic type,[1] logarithmic type,[8] sinusoidal type,[9] exponential type,[10] polynomial type,[11] high-order type,[12] etc. The analysis includes optical transfer function (OTF) analysis in the frequency domain[1,8,9,11−13] and PSF analysis in space domain.[14] However, both OTF and PSF analysis are complicated due to the double integral involved. Therefore, a rectangular pupil assumption is usually applied to separable systems because it allows the double integral to be described as the product of two single integrals, which can greatly reduce analysis complexity. Unfortunately, most optical systems have a circular pupil, which have rarely been investigated. A preliminary study of a circular pupil system[15] has focused on the on-axial modulation transfer function (MTF) but ignored the off-axial MTF, let alone the phase transfer function (PTF). In addition, this analysis has been limited to the cubic phase mask and cannot be used in other phase masks.

This paper proposes a method to analyze the two-dimensional (2D) OTF in a circular pupil wavefront coding system with a separable phase mask in Cartesian coordinates. It is based on the stationary phase method,[16] one of the most important analysis tools in wavefront coding systems. The method has been successfully applied to obtain the approximated analytical one-dimensional (1D) OTF[1] and PSF,[14] which are compatible with the exact OTF and PSF calculated by fast Fourier transform (FFT).

The rest of this paper is organized as follows. Sec-

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†Corresponding author. E-mail: zhaotingyu@zstu.edu.cn

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tion 2 gives the derivation process of the 2D OTF of a circular pupil system and a rectangular pupil system. Section 3 proposes the method to obtain the OTF of a circular pupil system from the rectangular pupil system. Section 4 demonstrates the method by an example of a cubic phase wavefront coding system. Conclusions and discussions are given in Section 5.

2. OTF of a rectangular pupil system and a circular pupil system

A separable phase mask in Cartesian coordinates is assumed in a wavefront coding system. The phase function is described as:

$$\Phi(x,y)=\phi_x(x)\,\phi_y(y), \tag{1}$$

where x, y are coordinates in the pupil plane. The pupil function $P(x,y)$ of an arbitrary pupil optical system can be written as:

$$P(x,y)=W(x,y)\exp[\,\mathrm{i}k\,\Phi(x,y)], \tag{2}$$

where $\mathrm{i}=\sqrt{-1}$, $k=2\pi/\lambda$ is the wavenumber, λ is wavelength and W is a window function. The OTF $H(u,v)$ of an optical system is given by the autocorrelation of its pupil function[17]

$$H(u,v)=\frac{\iint_{-\infty}^{+\infty}P\left(x+\frac{u}{2},y+\frac{v}{2}\right)P^*\left(x-\frac{u}{2},y-\frac{v}{2}\right)\mathrm{d}x\mathrm{d}y}{\iint_{-\infty}^{+\infty}P(x,y)\,P^*(x,y)\,\mathrm{d}x\mathrm{d}y}, \tag{3}$$

where u, v are coordinates in the frequency plane; * describes the complex conjugate.

2.1. Rectangular pupil system

The window function W is a Rect function as Eq. (4) shows when using a rectangular pupil

$$\mathrm{Rect}(x,y)=\mathrm{rect}(x)\cdot\mathrm{rect}(y), \tag{4}$$

where the rect function is defined as

$$\mathrm{rect}(x)=\begin{cases}1, & |x|\le D/2,\\ 0, & \text{otherwise.}\end{cases} \tag{5}$$

Here D is the side length of the rectangular pupil. Substituting Eqs. (2) and (4) into Eq. (3), we obtain

$$H(u,v)=H_u(u)\,H_v(v), \tag{6}$$

where

$$\begin{aligned}H_u(u)&=\frac{1}{D}\int_{-(D/2-|u|/2)}^{D/2-|u|/2}\exp(\mathrm{i}\Delta\phi_x)\mathrm{d}x,\\ H_v(v)&=\frac{1}{D}\int_{-(D/2-|v|/2)}^{D/2-|v|/2}\exp(\mathrm{i}\Delta\phi_y)\mathrm{d}y,\\ \Delta\phi_x(x,u)&=\phi_x\left(x+\frac{u}{2}\right)-\phi_x\left(x-\frac{u}{2}\right),\end{aligned} \tag{7}$$

$$\Delta\phi_y(y,v)=\phi_y\left(y+\frac{v}{2}\right)-\phi_y\left(y-\frac{v}{2}\right). \tag{8}$$

Consider the 1D OTF $H_u(u)$ first. The origin OTF is described as Eq. (9) when $u=0$

$$H_u(0)=\frac{1}{D}\int_{-D/2}^{D/2}\exp\{\mathrm{i}[\phi_x(x)-\phi_x(x)]\}\mathrm{d}x=1. \tag{9}$$

According to stationary phase method,[16] the OTF can be approximated as Eq. (10) when $u\neq 0$

$$\begin{aligned}H_u(u)=\frac{1}{D}\sqrt{\frac{2\pi}{|\Delta\phi_x''(x_0,u)|}}\exp\left\{\mathrm{i}\left[\Delta\phi_x(x_0,u)\right.\right.\\ \left.\left.+\frac{\pi}{4}\,\mathrm{sgn}(\Delta\phi_x''(x_0,u))\right]\right\}\end{aligned} \tag{10}$$

with $|x_0|\le D/2-|u|/2$, where $\mathrm{sgn}(x)$ represents the sign function defined as 1 for $x>0$ and -1 for $x<0$. x_0 is the stationary point, which satisfies

$$\begin{aligned}\Delta\phi_x'(x,u)&\equiv\frac{\partial\Delta\phi_x(x,u)}{\partial x}=0,\\ \Delta\phi_x''(x,u)&\equiv\frac{\partial^2\Delta\phi_x(x,u)}{\partial x^2}\neq 0.\end{aligned} \tag{11}$$

The one-dimensional OTF $H_v(v)$ can be obtained in a similar way and y_0 is assumed to be the stationary point. In this way, the 2D OTF of a rectangular pupil system can be written as

$$H(u,v)=\begin{cases}1, & u=0 \text{ and } v=0,\\ \dfrac{1}{D}\sqrt{\dfrac{2\pi}{\left|\Delta\phi_x''(x_0,u)\right|}}\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_x)\right], & v=0 \text{ and } 0<|u|\le D-2|x_0|,\\ \dfrac{1}{D}\sqrt{\dfrac{2\pi}{\left|\Delta\phi_y''(y_0,v)\right|}}\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_y)\right], & u=0 \text{ and } 0<|v|\le D-2|y_0|,\\ \dfrac{2\pi}{D^2}\dfrac{\exp\left[\mathrm{i}\,(\mathrm{PTF}_x+\mathrm{PTF}_y)\right]}{\sqrt{\left|\Delta\phi_x''(x_0,u)\cdot\Delta\phi_y''(y_0,v)\right|}}, & 0<|u|\le D-2|x_0|,\ 0<|v|\le D-2|y_0|,\\ 0, & \text{otherwise,}\end{cases} \tag{12}$$

where

$$\mathrm{PTF}_x=\Delta\phi_x(x_0,u)+\frac{\pi}{4}\operatorname{sgn}\left[\Delta\phi_x''(x_0,u)\right],$$
$$\mathrm{PTF}_y=\Delta\phi_y(y_0,v)+\frac{\pi}{4}\operatorname{sgn}\left[\Delta\phi_y''(y_0,v)\right]. \tag{13}$$

2.2. Circular pupil system

The window function W is a circ function as Eq. (14) shows when using a circular pupil.

$$\operatorname{circ}(x,y)=\begin{cases}1, & x^2+y^2\le r^2,\\ 0, & \text{otherwise,}\end{cases} \tag{14}$$

where r represents the radius of the circular pupil. Notice that the pupil function is separable in Cartesian coordinates, and substituting Eqs. (2) and (14) into Eq. (3)

$$H(u,v)=\frac{1}{\pi r^2}\iint_{-\infty}^{+\infty}\exp\left[\mathrm{i}\,(\Delta\phi_x+\Delta\phi_y)\right]\times\operatorname{circ}\left(x+\frac{u}{2},y+\frac{v}{2}\right)\times\operatorname{circ}^*\left(x-\frac{u}{2},y-\frac{v}{2}\right)\mathrm{d}x\,\mathrm{d}y. \tag{15}$$

The origin OTF is described as Eq. (16) when $u=0$, $v=0$

$$H(0,0)=\frac{1}{\pi r^2}\iint_{-\infty}^{+\infty}\operatorname{circ}(x,y)\times\operatorname{circ}^*(x,y)\,\mathrm{d}x\,\mathrm{d}y=1. \tag{16}$$

Now we consider the on-axis OTF when $u\neq 0$, $v=0$. Notice that $\Delta\phi_y(y,0)=0$, then Eq. (15) is simplified as

$$H(u,0)=\frac{1}{\pi r^2}\iint_{-\infty}^{+\infty}\exp\left[\,\mathrm{i}\Delta\phi_x(x,u)\right]\times\operatorname{circ}\left(x+\frac{u}{2},y\right)\times\operatorname{circ}^*\left(x-\frac{u}{2},y\right)\mathrm{d}x\,\mathrm{d}y. \tag{17}$$

According to the definition of the circ function, Eq. (17) can be written as

$$H(u,0)=\frac{1}{\pi r^2}\int_{-(r-|u|/2)}^{r-|u|/2}\int_{-\sqrt{r^2-(|u|/2+|x|)^2}}^{\sqrt{r^2-(|u|/2+|x|)^2}}\times\exp\left[\,\mathrm{i}\Delta\phi_x(x,u)\right]\mathrm{d}y\,\mathrm{d}x$$
$$=\frac{2}{\pi r^2}\int_{-(r-|u|/2)}^{r-|u|/2}\sqrt{r^2-\left(\frac{|u|}{2}+|x|\right)^2}\times\exp\left[\,\mathrm{i}\Delta\phi_x(x,u)\right]\mathrm{d}x. \tag{18}$$

Equation (18) is approximated as Eq. (19) based on the stationary phase method[16]

$$H(u,0)=\frac{2}{r^2}\sqrt{\frac{2}{\pi}\cdot\frac{r^2-\left(\frac{|u|}{2}+|x_0|\right)^2}{\left|\Delta\phi_x''(x_0,u)\right|}}\times\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_x)\right],\quad |x_0|<r-\frac{|u|}{2}. \tag{19}$$

Similarly, we can obtain $H(0,v)$. Then consider the off-axis OTF when $u\neq 0$, $v\neq 0$. Equation (15) is approximated by using the stationary phase method[16] as

$$H(u,v)=\frac{2\pi\,\mathrm{i}\sigma\exp\left[\,\mathrm{i}\Delta\Phi(x_0,y_0,u,v)\right]}{\pi r^2\sqrt{|ab-c^2|}},\quad \left(x_0\pm\frac{|u|}{2}\right)^2+\left(y_0\pm\frac{|v|}{2}\right)^2\le r^2, \tag{20}$$

where

$$a\equiv\frac{\partial^2\Delta\Phi}{\partial x^2}=\Delta\phi_x''(x,u),$$
$$b\equiv\frac{\partial^2\Delta\Phi}{\partial y^2}=\Delta\phi_y''(y,v),\quad c\equiv\frac{\partial^2\Delta\Phi}{\partial x\partial y}=0, \tag{21}$$

$$\sigma=\begin{cases}+1, & ab>c^2, a>0,\\ -1, & ab>c^2, a<0,\\ -\mathrm{i}, & ab<c^2,\end{cases} \tag{22}$$

the (x_0,y_0) is the stationary point, which satisfies $\partial\Delta\Phi/\partial x=0$, $\partial\Delta\Phi/\partial y=0$ and $ab-c^2\neq 0$. By substituting Eqs. (21) and (22) into Eq. (20), the off-axis OTF of a circular pupil can be written as

$$H(u,v)=\frac{2\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_x+\mathrm{PTF}_y)\right]}{r^2\sqrt{\left|\Delta\phi_x''(x_0,u)\cdot\Delta\phi_y''(y_0,v)\right|}},\quad \left(x_0\pm\frac{|u|}{2}\right)^2+\left(y_0\pm\frac{|v|}{2}\right)^2\le r^2. \tag{23}$$

In summary, the 2D OTF of a circular pupil is written as

$$H(u,v)=\begin{cases}1, & u=0 \text{ and } v=0,\\ \dfrac{2}{r^2}\sqrt{\dfrac{2}{\pi}\cdot\dfrac{r^2-\left(\dfrac{|u|}{2}+|x_0|\right)^2}{|\Delta\phi''_x(x_0,u)|}}\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_x)\right], & v=0 \text{ and } 0<|u|\le 2\,(r-|x_0|),\\ \dfrac{2}{r^2}\sqrt{\dfrac{2}{\pi}\cdot\dfrac{r^2-\left(\dfrac{|v|}{2}+|y_0|\right)^2}{\left|\Delta\phi''_y(y_0,u)\right|}}\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_y)\right], & u=0 \text{ and } 0<|v|\le 2\,(r-|y_0|),\\ \dfrac{2}{r^2}\dfrac{\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_x+\mathrm{PTF}_y)\right]}{\sqrt{|\Delta\phi''_u(x_0,u)\cdot\Delta\phi''_v(y_0,v)|}}, & \left(x_0\pm\dfrac{|u|}{2}\right)^2+\left(y_0\pm\dfrac{|v|}{2}\right)^2\le r^2,\\ 0, & \text{otherwise.}\end{cases}\tag{24}$$

3. Derivation of the OTF of a circular pupil system from a rectangular pupil system

The pupil areas of the circular pupil system and the rectangular pupil system are assumed to be equal for the sake of comparability, i.e.

$$\pi r^2=D^2.\tag{25}$$

By substituting Eq. (25) into Eq. (24), we obtain the 2D OTF of a circular pupil system

$$H(u,v)=\begin{cases}1, & u=0 \text{ and } v=0,\\ \dfrac{1}{D}\dfrac{\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_x)\right]}{\sqrt{|\Delta\phi''_x(x_0,u)|}}\sqrt{8-\dfrac{8\pi}{D^2}\left(\dfrac{|u|}{2}+|x_0|\right)^2}, & v=0 \text{ and } 0<|u|\le 2\left(\dfrac{D}{\sqrt{\pi}}-|x_0|\right),\\ \dfrac{1}{D}\dfrac{\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_y)\right]}{\sqrt{\left|\Delta\phi''_y(y_0,v)\right|}}\sqrt{8-\dfrac{8\pi}{D^2}\left(\dfrac{|v|}{2}+|y_0|\right)^2}, & u=0 \text{ and } 0<|v|\le 2\left(\dfrac{D}{\sqrt{\pi}}-|y_0|\right),\\ \dfrac{2\pi}{D^2}\dfrac{\exp\left[\,\mathrm{i}\,(\mathrm{PTF}_x+\mathrm{PTF}_y)\right]}{\sqrt{|\Delta\phi''_u(x_0,u)\cdot\Delta\phi''_v(y_0,v)|}}, & \left(x_0\pm\dfrac{|u|}{2}\right)^2+\left(y_0\pm\dfrac{|v|}{2}\right)^2\le\dfrac{D^2}{\pi},\\ 0, & \text{otherwise.}\end{cases}\tag{26}$$

Comparing Eq. (12) with Eq. (26), there are similarities and differences between the rectangular pupil system and the circular pupil function. Three aspects will be analyzed in detail: MTF; PTF; and cut-off frequency.

The first one is the MTF. By comparing Eq. (12) with Eq. (26), the original point and off-axial MTF of the circular pupil system is equal that of of the rectangular system. The only difference between the two systems is the on-axial MTF. There is a difference of a factor $2\sqrt{1/\pi-\left(|u|/2+|x_0|\right)^2/\left(D^2\right)}$ on MTF along the u direction, while $2\sqrt{1/\pi-\left(|v|/2+|y_0|\right)^2/\left(D^2\right)}$ on MTF along the v direction, where x_0 and y_0 are the stationary points of $\Delta\phi_x$ and $\Delta\phi_y$, respectively.

Next is the PTF. It is apparent that the two systems have the same PTF.

The third one is the cut-off frequency. The frequency boundaries of a rectangular system form a rectangle as Eq. (12) shows, whose sides are located at

$$\begin{aligned}&u=2\,|x_0|-D, \quad u=D-2\,|x_0|\,,\\ &v=2\,|y_0|-D, \quad v=D-2\,|y_0|\,.\end{aligned}\tag{27}$$

While the frequency boundaries of a circular system are composed by the overlap of two circles as Eq. (28) shows

$$\begin{aligned}&(|u|+2x_0)^2+(|v|+2y_0)^2\le\frac{4D^2}{\pi},\\ &(|u|-2x_0)^2+(|v|-2y_0)^2\le\frac{4D^2}{\pi}.\end{aligned}\tag{28}$$

Therefore, the boundaries of the cut-off frequency can

be approximated as a circle when the stationary points x_0 and y_0 is small. Notice that the cut-off frequency of the on-axial MTF in the rectangular pupil system is $D-2|x_0|$ or $D-2|y_0|$, which is smaller than that of the circular pupil system $2D/\sqrt{\pi}-2|x_0|$ or $2D/\sqrt{\pi}-2|y_0|$.

Based on the above analyses, we propose a simple method to achieve the OTF of the circular pupil system from the rectangular pupil system. Firstly, we establish and analyze the rectangular pupil system using the same phase mask and the same circular pupil system aperture area. Secondly, we modify the cut-off frequency according to Eqs. (27) and (28). Thirdly, we modify the on-axial MTF using the difference factor $2\sqrt{1/\pi-(|u|/2+|x_0|)^2/(D^2)}$ and $2\sqrt{1/\pi-(|v|/2+|y_0|)^2/(D^2)}$. By following these steps, the OTF of a circular pupil system will be achieved.

4. Analysis of the wavefront coding system with a cubic phase mask

A wavefront coding system with a cubic phase mask is used as an example to demonstrate the method of obtaining the OTF of a circular pupil system from the rectangular pupil system. Ignoring the window function, the phase function for the defocused system is described as

$$\Phi(x,y)=\alpha\left(x^3+y^3\right)+W_{20}\left(x^2+y^2\right), \tag{29}$$

where α is the cubic parameter of the phase mask in units of wavelength λ. W_{20} is the defocus aberration in unit of wavelength. Fortunately, the OTF of a rectangular pupil system has already been analyzed[1] and can be rewritten as

$$H(u,v)=\begin{cases}1, & u=0 \text{ and } v=0,\\ \dfrac{1}{D}\sqrt{\dfrac{\pi}{|3\alpha k u|}}\exp\left\{\mathrm{i}\left[\theta(u,0)+\dfrac{\pi}{4}\operatorname{sgn}(u)\right]\right\}, & 0<|u|\le D-\dfrac{|2W_{20}|}{3\alpha} \text{ and } v=0,\\ \dfrac{1}{D}\sqrt{\dfrac{\pi}{|3\alpha k v|}}\exp\left\{\mathrm{i}\left[\theta(0,v)+\dfrac{\pi}{4}\operatorname{sgn}(v)\right]\right\}, & 0<|v|\le D-\dfrac{|2W_{20}|}{3\alpha} \text{ and } u=0,\\ \dfrac{\pi\exp\left[\mathrm{i}\left\{\theta(u,v)+\frac{\pi}{4}\left[\operatorname{sgn}(u)+\operatorname{sgn}(v)\right]\right\}\right]}{3D^2\alpha k\sqrt{|uv|}}, & 0<|u|,|v|\le D-\dfrac{|2W_{20}|}{3\alpha} \text{ and } uv\ne 0,\\ 0, & \text{otherwise,}\end{cases} \tag{30}$$

where

$$\theta(u,v)=k\left[\frac{\alpha}{4}\left(u^3+v^3\right)-\frac{W_{20}^2}{3\alpha}(u+v)\right]. \tag{31}$$

The OTF of the circular system can be achieved by modifying the on-axial MTF and the cut off frequency when $\pi r^2=D^2$. By substituting the stationary point $x_0=-W_{20}/3\alpha$ and $y_0=-W_{20}/3\alpha$ into Eq. (26), the OTF is described as

$$H(u,v)=\begin{cases}1, & u=0 \text{ and } v=0,\\ \dfrac{1}{D}\sqrt{\dfrac{\pi}{|3\alpha v k|}\left[\dfrac{4}{\pi}-\dfrac{4}{D^2}\left(\dfrac{|v|}{2}+\dfrac{|W_{20}|}{3\alpha}\right)^2\right]}\exp\left[\mathrm{i}\left(\theta(0,v)+\dfrac{\pi}{4}\operatorname{sgn}(v)\right)\right], & u=0 \text{ and } 0<|v|\le 2\left(\dfrac{D}{\sqrt{\pi}}-\dfrac{|W_{20}|}{3\alpha}\right),\\ \dfrac{1}{D}\sqrt{\dfrac{\pi}{|3\alpha u k|}\left[\dfrac{4}{\pi}-\dfrac{4}{D^2}\left(\dfrac{|u|}{2}+\dfrac{|W_{20}|}{3\alpha}\right)^2\right]}\exp\left[\mathrm{i}\left(\theta(u,0)+\dfrac{\pi}{4}\operatorname{sgn}(u)\right)\right], & 0<|u|\le 2\left(\dfrac{D}{\sqrt{\pi}}-\dfrac{|W_{20}|}{3\alpha}\right) \text{ and } v=0,\\ \dfrac{\pi}{3D^2\alpha k\sqrt{|uv|}}\exp\left[\mathrm{i}\left(\theta(u,v)+\dfrac{\pi}{4}\left[sgn(u)+\operatorname{sgn}(v)\right]\right)\right], & uv\ne 0 \text{ and } \left(\dfrac{u}{2}\pm\dfrac{|W_{20}|}{3\alpha}\right)^2+\left(\dfrac{v}{2}\pm\dfrac{|W_{20}|}{3\alpha}\right)^2\le\dfrac{D^2}{\pi},\\ 0, & \text{otherwise.}\end{cases} \tag{32}$$

Assume the cubic parameter $\alpha = 90$, the defocus aberration $W_{20} = 5$, the side length D of the rectangular window is 2, the radius r of the circular window is $2/\sqrt{\pi}$. Figure 1(a) demonstrates the rectangular pupil and the circular pupil, while Fig. 1(b) gives the cut-off frequency of the rectangular system and the circular system. Notice that the cut-off frequency is the overlap of Eq. (28), which is demonstrated as solid lines.

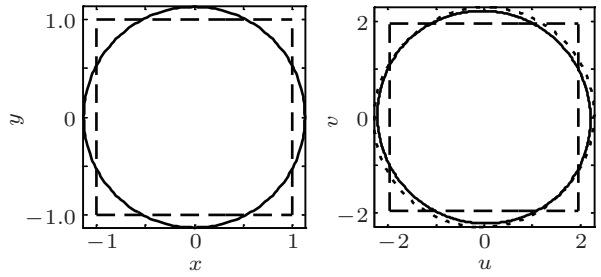


Fig. 1. Pupil and cut-off frequency of the cubic phase wavefront coding system, where $\alpha = 90$ and $W_{20} = 5$. (a) Rectangular pupil (in dashed lines) and circular pupil (in solid lines); (b) boundaries of the cut off frequency of the rectangular pupil system (in dashed lines) and the circular pupil system (in solid lines).

According to Eqs. (30) and (32), the approximated on-axial MTF is given in Fig. 2. The entire MTFs are given in Fig. 3, where Fig. 3(a) demonstrates the approximated MTF of the circular pupil system based on the stationary phase method, while Fig. 3(b) demonstrates the approximated MTF of the rectangular pupil system based on the stationary phase method. Notice that the boundaries of the MTF of the two systems are quite different. The circular pupil system is approximated to be a circle, while the rectangular pupil system is a rectangle. This can be easily explained by Eqs. (27) and (28). The exact MTF calculated by FFT is also given as a contrast. Figure 3(c) illustrates the exact MTF of a circular pupil system, while Fig. 3(d) demonstrates the exact MTF of a rectangular pupil system. Comparing Figs. 3(a) and 3(b) with Figs. 3(c) and 3(d), we find that they are consistent. Although the approximated MTF is smoother than the exact MTF, this is due to the essence of the stationary phase method.[16]

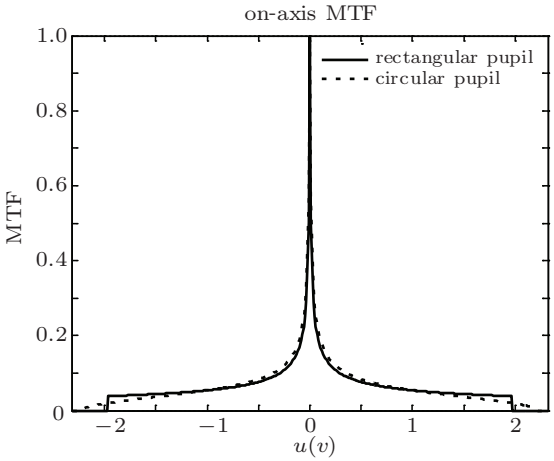


Fig. 2. On-axial MTF of the rectangular pupil system (in solid lines) and the circular pupil system (in dashed lines) with the cubic phase mask, where $\alpha = 90$ and $W_{20} = 5$.

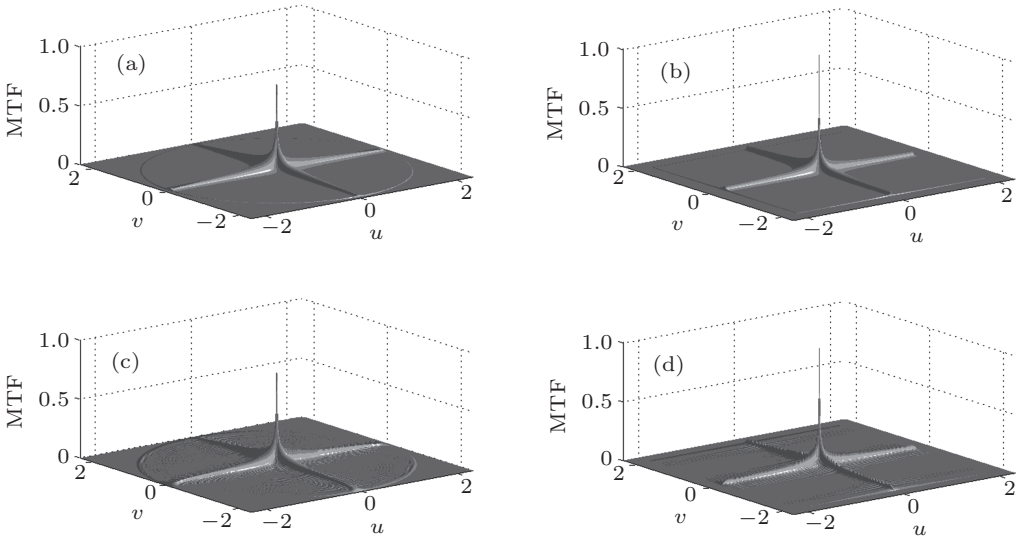


Fig. 3. MTF of the circular pupil system and the rectangular pupil system with the cubic phase mask where $\alpha = 90$ and $W_{20} = 5$. (a) Approximated MTF of the circular pupil system based on stationary phase method; (b) approximated MTF of the rectangular pupil system based on stationary phase method; (c) exact MTF of the circular pupil system calculated by FFT; (d) exact MTF of the rectangular pupil system calculated by FFT.

5. Conclusion and discussion

This paper introduced a simple method to achieve the OTF of a circular pupil wavefront coding system from a rectangular pupil system by modifying the on-axial MTF and cut-off frequency. Because the 1D OTF is always been analyzed in a separable wavefront coding system, this method is simple but effective in analyzing an often encountered circular wavefront coding system. We have to point out that it is not easy to obtain an analytical solution of the stationary points for some complicated phase masks. Fortunately, the numerical solution of a stationary point is also acceptable, which can be always easily solved by using numerical computing software, such as Matlab. In addition, it should be noticed that this method is limited to those phase masks whose $\Delta\phi_x$ and $\Delta\phi_y$ has only one stationary phase point. However, we believe that most of phase masks will satisfy this condition, because more than one stationary points lead to oscillation and zeros in the MTF. It causes information loss and failure of digital decoding, which is not allowed in a wavefront coding system. Consequently, the simple but effective OTF analysis method fits most circular pupil wavefront coding systems with separable phase masks.

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