Arbitrary full-state hybrid projective synchronization for chaotic discrete-time systems via a scalar signal

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In this paper we present a new projective synchronization scheme, where two chaotic (hyperchaotic) discrete-time systems synchronize for any arbitrary scaling matrix. Specifically, each drive system state synchronizes with a linear combination of response system states. The proposed observer-based approach presents some useful features: i) it enables exact synchronization to be achieved in finite time (i.e., dead-beat synchronization); ii) it exploits a scalar synchronizing signal; iii) it can be applied to a wide class of discrete-time chaotic (hyperchaotic) systems; iv) it includes, as a particular case, most of the synchronization types defined so far. An example is reported, which shows in detail that exact synchronization is effectively achieved in finite time, using a scalar synchronizing signal only, for any arbitrary scaling matrix.

Keywords: chaos synchronization, full-state hybrid projective synchronization, observer-based synchronization, chaotic discrete-time systems, dead beat control, attractor scaling

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1. Introduction

Chaos is a ubiquitous and extremely complex nonlinear phenomenon.[1−3] Since Carroll and Pecora[4] introduced a method of synchronizing two identical chaotic systems, chaos control and synchronization have played an important role in nonlinear science (see Refs. [5]–[15] and the references therein). In particular, in Ref. [4] it has been shown that a drive system and a driven (response) system achieve synchronized dynamics when the Lyapunov exponents of the driven system are negative. Different synchronization methods have been successively proposed in Refs. [7]–[15]. Among them, some techniques have focused on the synchronization of chaotic systems via a scalar synchronizing signal.[8−13]

The projective synchronization was introduced in 1999.[16] In projective synchronization the response system states are scaled replicas of the drive system states.[17] Since in Ref. [16] the scaling factor was not predictable, some control methods have been successively developed for choosing any desired scaling factor.[17−22]

A variation of the projective synchronization has been recently developed in Refs. [23]–[27], where the concept of full state hybrid projective synchronization (FSHPS) has been introduced. In this type of synchronization the scaling factor can be different for each state variable. Namely, the single scaling parameter originally introduced in Ref. [16] has been replaced by a diagonal scaling matrix in Refs. [23]–[28]. In particular, in Refs. [25] and [26] an active control scheme was utilized to achieve FSHPS in continuous-time systems, whereas in Ref. [27] an active control is used to synchronize a class of chaotic maps, which includes the so-called Grassi–Miller map. Moreover, in Ref. [28] the Lyapunov stability theory is utilized to achieve FSHPS (called therein modified generalized projective synchronization) for a new chaotic system.

Even though the FSHPS methods developed in Refs. [23]–[28] are interesting, they present the drawback that synchronization is achieved using several control signals, i.e., one control input for each system equation. On the other hand, chaos-based communications benefit from the use of a scalar synchronizing signal.[10] Moreover, a desirable feature in discrete-time system is to achieve exact synchronization in finite time (i.e., dead-beat synchronization).[29] Previous discrete-time FSHPS methods, such as those described in Refs. [24] and [27], did not present this feature.

According to these considerations, in this paper we present a further contribution to the topic of synchronization by introducing a novel scheme, where two chaotic (hyperchaotic) discrete-time systems synchronize for any arbitrary scaling matrix. Specifically, each

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drive system state synchronizes with a linear combination of response system states. Since the adoption of an arbitrary scaling matrix represents a generalization of FSHPS (where the scaling matrix is a diagonal one), the proposed method is called herein arbitrary full-state hybrid projective synchronization (AFSHPS). The approach presents some useful features: i) it enables exact synchronization to be achieved in finite time (dead-beat synchronization); ii) it exploits a scalar synchronizing signal; iii) it can be applied to a wide class of discrete-time chaotic (hyperchaotic) systems; iv) it includes, as a particular case, most of the synchronization types defined so far. The rest of the present paper is organized as follows. In Section 2 we define the proposed observer-based AFSHPS and present a Theorem, which assures that AFSHPS is achieved when a structural condition in the considered class of discrete-time systems is satisfied. Finally, in order to show the effectiveness of the approach, in Section 3 we illustrate an example of AFSHPS involving the recently introduced hyperchaotic double scroll map.[30]

2. Arbitrary full state hybrid projective synchronization

Definition 1 Given the discrete-time drive and response systems described by the n-dimensional state vectors $\boldsymbol{x}(k)$ and $\hat{\boldsymbol{x}}(k)$, respectively, those two systems are in arbitrary full-state hybrid projective synchronization (AFSHPS) when, for an initial condition, there is a matrix

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \dots & \dots & \dots & \dots \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{bmatrix} \in \Re^{n\times n}, \tag{1}$$

such that

$$\|e_i(k)\| = \left\| \left(x_i(k) - \sum_{j=1}^{n} \alpha_{ij}\hat{x}_j(k) \right) \right\| \to 0$$
$$\text{as} \quad k \to \infty, \quad i = 1, 2, \dots, n. \tag{2}$$

Condition (2) means that each drive system variable $x_i(k)$ synchronizes with a linear combination of response system variables $(\alpha_{i1}\hat{x}_1(k) + \alpha_{i2}\hat{x}_2(k) + \cdots + \alpha_{in}\hat{x}_n(k))$, $i = 1, 2, \dots, n$.

Referring to the systems to be synchronized, in this paper we focus our attention on the class of dynamical systems defined by the following state and output equations, respectively:

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}f(\boldsymbol{x}(k)) + c, \tag{3}$$
$$y(k) = f(\boldsymbol{x}(k)) + \boldsymbol{k}\boldsymbol{x}(k), \tag{4}$$

where $\boldsymbol{x}(k) \in \Re^{n\times 1}$, $\boldsymbol{A} \in \Re^{n\times n}$, $\boldsymbol{b} \in \Re^{n\times 1}$, $\boldsymbol{c} \in \Re^{n\times 1}$, $f : \Re^n \to \Re$, $y(k)$ is a scalar synchronizing output, and $\boldsymbol{k} \in \Re^{1\times n}$ is a gain vector to be determined.

Remark 1 As pointed out in Ref. [13], the class described by Eq. (3) includes several chaotic and hyperchaotic discrete-time systems, that is, the logistic map, the cubic map, the Duffing map, the Gauss map, the Lozi map, and the generalized Henon map. Class (3) also includes the so-called Grassi–Miller map,[27] the gingerbreadman map,[29] and the recently introduced double scroll map.[30]

Now, the following assumption is made.

Assumption 1 Given the matrices $\boldsymbol{A}$ and $\boldsymbol{b}$ in Eq. (3), the matrix

$$\left[\boldsymbol{b} \ \boldsymbol{A}\boldsymbol{b} \ \boldsymbol{A}^2\boldsymbol{b} \ \dots \ \boldsymbol{A}^{n-1}\boldsymbol{b} \right] \tag{5}$$

is full rank.

Given drive system (3), the observed-based approach enables the state $x(k)$ to be reconstructed by a response system using a scalar function of the states in the form of Eq. (4). By utilizing the fact that the response system need not be a copy of the drive system in order to obtain synchronized dynamics (see Refs. [8]–[12]), a theorem that AFSHPS is achievable using a scalar synchronizing signal can be developed.

Theorem 1 (dead-beat AFSHPS)

If Assumption 1 is satisfied, the nonlinear dynamical system

$$\hat{\boldsymbol{x}}(k+1) = \boldsymbol{\alpha}^{-1}A\boldsymbol{\alpha}\hat{\boldsymbol{x}}(k) + \boldsymbol{\alpha}^{-1}\boldsymbol{b}f(\hat{x}(k)) + \boldsymbol{\alpha}^{-1}c + \boldsymbol{\alpha}^{-1}\boldsymbol{b}\left(y(k) - \hat{y}(k)\right), \tag{6}$$
$$\hat{y}(k) = f(\hat{\boldsymbol{x}}(k)) + \boldsymbol{k}\boldsymbol{\alpha}\hat{\boldsymbol{x}}(k), \tag{7}$$

and drive systems (3) and (4) achieve dead-beat AFSHPS for any arbitrary scaling matrix $\boldsymbol{\alpha}$, provided that the eigenvalues of $[\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}]$ are placed at zero by suitable $\boldsymbol{k}$.

Proof First of all, it is worth while noting that error (2) can be written in matrix form as

$$\|\boldsymbol{e}(k)\| = \|(\boldsymbol{x}(k) - \boldsymbol{\alpha}\hat{\boldsymbol{x}}(k))\| \to 0. \tag{8}$$

By considering error (8) and by taking into account equations (3), (4), and (6), (7), it follows that the error dynamics is described by:

$$\begin{aligned} \boldsymbol{e}(k+1) &= (\boldsymbol{x}(k+1) - \boldsymbol{\alpha}\hat{\boldsymbol{x}}(k+1)) \\ &= \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}f(\boldsymbol{x}(k)) + \boldsymbol{c} - \boldsymbol{A}\boldsymbol{\alpha}\hat{\boldsymbol{x}}(k) \end{aligned}$$

$$
\begin{aligned}
&-\boldsymbol{b}f(\hat{\boldsymbol{x}}(k))-\boldsymbol{c}-\boldsymbol{b}\left(y(k)-\hat{y}(k)\right)\\
=\;&\boldsymbol{A}\left(\boldsymbol{x}(k)-\boldsymbol{\alpha}\hat{\boldsymbol{x}}(k)\right)+\boldsymbol{b}f(x(k))-\boldsymbol{b}f(\hat{\boldsymbol{x}}(k))\\
&-\boldsymbol{b}\left(f(\boldsymbol{x}(k))+\boldsymbol{k}\boldsymbol{x}(k)-f(\hat{\boldsymbol{x}}(k))-\boldsymbol{k}\boldsymbol{\alpha}\hat{\boldsymbol{x}}(k)\right)\\
=\;&\boldsymbol{A}\left(\boldsymbol{x}(k)-\boldsymbol{\alpha}\hat{\boldsymbol{x}}(k)\right)-\boldsymbol{b}\boldsymbol{k}\left(x(k)-\boldsymbol{\alpha}\hat{\boldsymbol{x}}(k)\right)\\
=\;&\boldsymbol{A}\boldsymbol{e}(k)-\boldsymbol{b}\boldsymbol{k}\boldsymbol{e}(k). \qquad (9)
\end{aligned}
$$

Error dynamics (9) can be written as

$$\boldsymbol{e}(k+1)=\boldsymbol{A}\boldsymbol{e}(k)+\boldsymbol{b}u(k),\ \text{with}\ u(k)=-\boldsymbol{k}\boldsymbol{e}(k). \qquad (10)$$

Since matrix (5) is the controllability matrix of the linear time-invariant error system (10), by virtue of Assumption 1 all the eigenvalues of Eq. (10) can be placed at the origin by suitable $\boldsymbol{k}$, indicating that dead beat control is achieved.[31] Thus, the error dynamics will reach exactly zero in n steps. This means that deadbeat AFSHPS is achieved, i.e., the drive system and the response system will exactly synchronize with an arbitrary scaling matrix $\boldsymbol{\alpha}$ in n steps.

Remark 2 It can be readily verified that matrix (5) is full rank for several well-known chaotic (hyperchaotic) discrete-time systems. Specifically, all the maps mentioned in Remark 1 satisfy Assumption 1. As a consequence, according to Theorem 1, all these maps achieve dead beat AFSHPS.

Remark 3 The proposed scheme represents a general framework to achieve most of the synchronization types defined so far. Namely, when $\alpha_{11}=\alpha_{22}=\cdots=\alpha_{nn}=1$ and $\alpha_{ij}=0$, $i\neq j$, identical (complete) synchronization is achieved, whereas for $\alpha_{11}=\alpha_{22}=\cdots=\alpha_{nn}=-1$ and $\alpha_{ij}=0$, $i\neq j$, anti-phase synchronization is obtained. Moreover, when $\alpha_{11}=\alpha_{22}=\cdots=\alpha_{nn}=\alpha$ and $\alpha_{ij}=0$, $i\neq j$, the projective synchronization originally proposed in Ref. [16] is obtained. On the other hand, when $\alpha_{11}\neq\alpha_{22}\neq\cdots\neq\alpha_{nn}$ and $\alpha_{ij}=0$, $i\neq j$, the scaling matrix $\boldsymbol{\alpha}$ is diagonal, and therefore the full state hybrid projective synchronization is achieved (see Refs. [26]–[28]).

Remark 4 The proposed observer-based method shares the same advantages illustrated in Ref. [8], i.e., it is rigorous and systematic, it can be applied to a wide class of discrete-time systems, and it does not require computation of Lyapunov exponents in order to check synchronization. In addition, note that the author has successfully implemented in hardware a similar discrete-time observer (though without a scaling ability),[13] indicating that the observer-based method is able to be electronically realized.

3. Example

A new piecewise linear discrete-time system capable of generating hyperchaotic attractors has been recently presented in Ref. [30]. The attractor generated by the map has been called "the discrete hyperchaotic double-scroll" due to its shape and the adoption of the same nonlinearity as that used in the well-known chaotic Chua's circuit.[32] The map is described by the following equations:[30]

$$\begin{cases} x_1(k+1)=x_1(k)-ah\left(x_2(k)\right), \\ x_2(k+1)=bx_1(k), \end{cases} \qquad (11)$$

where a and b are the bifurcation parameters whereas

$$
\begin{aligned}
h\left(x_2(k)\right)=\;&(2m_1x_2(k)+(m_0-m_1)\\
&\times(|x_2(k)+1|-|x_2(k)-1|))/2 \qquad (12)
\end{aligned}
$$

is the piecewise linear function originally introduced in Chua's circuit (see Ref. [32]). The map (11) can be written in the form (3) as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ b & 0 \end{bmatrix}\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}+\begin{bmatrix} -a \\ 0 \end{bmatrix}h\left(x_2(k)\right). \qquad (13)$$

When $a=3.36$, $b=1$, $m_0=-0.43$, and $m_1=0.41$, the system exhibits the discrete hyperchaotic double scroll shown in Fig. 1.

From Eq. (13) it can be readily shown that matrix (5) is full rank. Since the eigenvalues of $[\boldsymbol{A}-\boldsymbol{b}\boldsymbol{k}]$ can be placed at zero by $k=\begin{bmatrix} -0.2976 & 0 \end{bmatrix}$, the scalar synchronizing signal (4) can be written as

$$y(k)=h(x_2(k))-0.2976x_1(k). \qquad (14)$$

Thus, the nonlinear observer (6) and (7) is described by

$$
\begin{aligned}
\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \end{bmatrix}=\;&\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}^{-1}\begin{bmatrix} 1 & 0 \\ b & 0 \end{bmatrix}\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}\begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix}+\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}^{-1}\begin{bmatrix} -a \\ 0 \end{bmatrix}h(\hat{x}_2(k))\\
&+\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}^{-1}\begin{bmatrix} -a \\ 0 \end{bmatrix}\left(y(k)-\hat{y}(k)\right), \qquad (15)
\end{aligned}
$$

$$\hat{y}(k) = h(\hat{x}_2(k)) + \begin{bmatrix} -0.2976 & 0 \end{bmatrix} \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \times \begin{bmatrix} \hat{x}_1(k) & \hat{x}_2(k) \end{bmatrix}^{\mathrm{T}}. \quad (16)$$

According to Theorem 1, dead-beat AFSHPS is achieved between systems (13) and (15), that is, the error dynamics will reach exactly zero after two steps, regardless of the scaling matrix $\boldsymbol{\alpha}$. In order to show the effectiveness of the approach, some simulations have been carried out for different scaling matrices. For example, by taking the following (full) matrix:

$$\boldsymbol{\alpha} = \begin{bmatrix} 0.5 & -1 \\ 1 & 2 \end{bmatrix}, \quad (17)$$

the results are shown in Fig. 1, which depicts the attractor of drive system (13) in the (x_1, x_2)-plane and the attractor of the response system (15) in the ($\hat{x}_1$, $\hat{x}_2$)-plane.

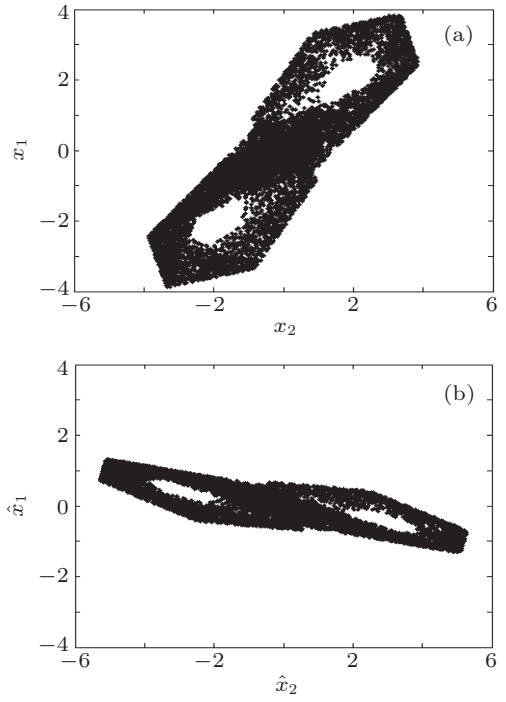


Fig. 1. Attractor of the drive system (13) in the (x_1, x_2)-plane (a) and attractor of the observer (15) in the ($\hat{x}_1$, $\hat{x}_2$)-plane (b) for the scaling matrix (17). Note that the two figures have the same scale, so that sizes and positions of the two attractors can be compared.

Note that the response system attractor has been rotated with respect to the drive system attractor. Moreover, the response system states have different ranges from the corresponding drive system states have. This is because, according to Eq. (2), each drive system state synchronizes with a linear combination of response system states. Namely, the error vector is given by

$$e_1(k) = x_1(k) - 0.5\hat{x}_1(k) + \hat{x}_2(k),$$
$$e_2(k) = x_2(k) - \hat{x}_1(k) - 2\hat{x}_2(k). \quad (18)$$

The corresponding error dynamics are described in Fig. 2. This figure clearly shows that dead-beat AFSHSP is achieved between systems (13) and (15), since the error is exactly zero after two steps.

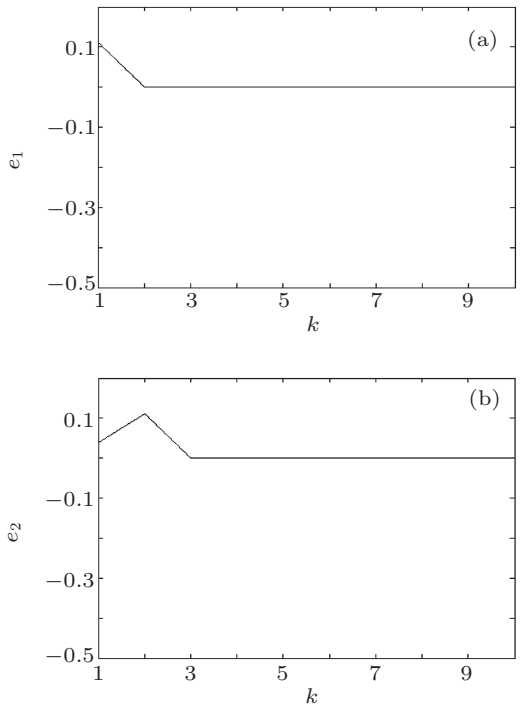


Fig. 2. Errors $e_1(k)$ (a) and $e_2(k)$ (b) each as a function of k, for the scaling matrix (17). Note that the errors are zero after two steps, i.e., the errors are zero from step three on.

Now the scaling matrix is chosen as a triangular matrix:

$$\boldsymbol{\alpha} = \begin{bmatrix} 2 & 1 \\ 0 & 0.5 \end{bmatrix}. \quad (19)$$

Figure 3 shows the attractor of drive system (13) in the (x_1, x_2)-plane and the attractor of the response system (15) in the ($\hat{x}_1$, $\hat{x}_2$)-plane.

The error vector is given by

$$e_1(k) = x_1(k) - 2\hat{x}_1(k) - \hat{x}_2(k),$$
$$e_2(k) = x_2(k) - 0.5\hat{x}_2(k), \quad (20)$$

indicating that state $x_1(k)$ is synchronized with the linear combination of state variables, $(2\hat{x}_1(k) + \hat{x}_2(k))$, whereas the state $x_2(k)$ is in projective synchronization with the variable $\hat{x}_2(k)$. The error dynamics (Fig. 4) clearly shows that dead-beat

AFSHSP is achieved, given that the error is exactly zero after two steps.

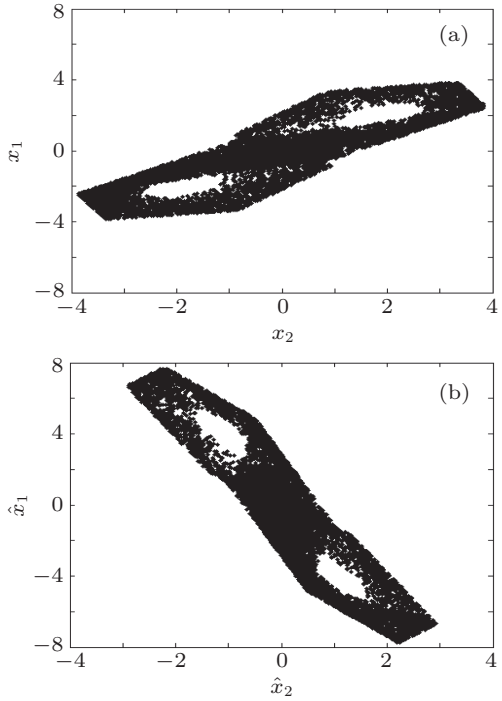


Fig. 3. Attractor of system (13) in the (x_1, x_2)-plane (a) and attractor of the observer (15) in the ($\hat{x}_1$, $\hat{x}_2$)-plane (b) for the scaling matrix (19). The two figures have the same scale, so that the two attractors can be compared. Note that the variable $\hat{x}_2(k)$ has been scaled by two with respect to $x_2(k)$.

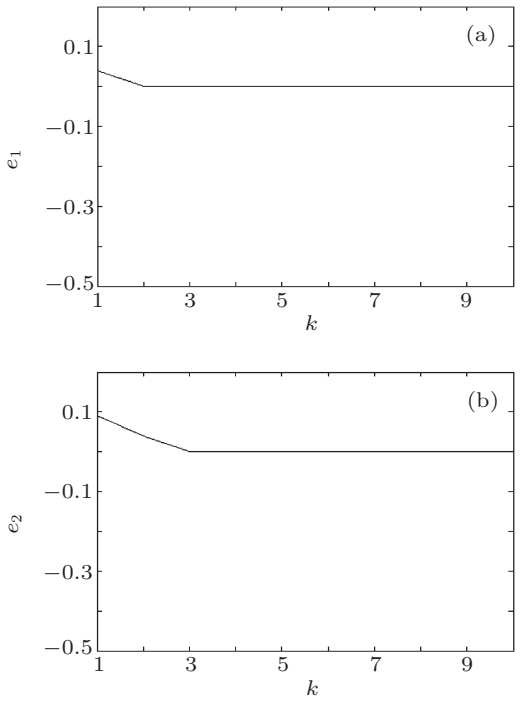


Fig. 4. Errors $e_1(k)$ (a) and $e_2(k)$ (b) each as a function of k, for the scaling matrix (19). The errors are zero after two steps, i.e., from step three on.

Note that the FSHPS methods in the literature (such as Refs. [24] and [27]) do not offer this capability, since the errors between drive and response systems are not exactly zero in finite time. Namely, in Refs. [24] and [27] only asymptotic synchronization ($k \to \infty$) is achieved. Moreover, note that in Ref. [27] FSHPS is achieved using several control inputs, while this approach enables AFSHPS to be achieved using a scalar synchronizing signal only. Additionally, note that most of the synchronization types defined so far can be derived from the proposed framework. For example, by choosing Eq. (17) as a diagonal matrix, the full state hybrid projective synchronization defined in Refs. [26]–[28] is achieved. By taking Eq. (17) as a scalar matrix (i.e., a diagonal matrix with all its main diagonal entries equal), the projective synchronization proposed in Refs. [16]–[21] is obtained. By selecting Eq. (17) as the identity matrix $\boldsymbol{I}$, identical (complete) synchronization is achieved. Finally, by taking Eq. (17) as $-I$, the anti-phase synchronization is obtained.

4. Conclusion

In this paper a new projective synchronization scheme is presented, where two chaotic (hyperchaotic) discrete-time systems synchronize for any arbitrary scaling matrix. It is shown that the approach, based on a simple and effective Theorem, presents some useful features: i) it enables dead-beat synchronization to be achieved; ii) it exploits a scalar synchronizing signal; iii) it can be applied to a wide class of chaotic (hyperchaotic) discrete-time systems; iv) it includes, as a particular case, most of the synchronization types defined so far. An example involving the hyperchaotic double scroll map has illustrated in detail the capability of the proposed observer-based technique.

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