Gravitational collapse with standard and dark energy in the teleparallel equivalent of general relativity

Gamal G. L. Nashed[a)b)c)†]

[a)] *Mathematics Department, Faculty of Science, King Faisal University, P. O. Box* 380 *Al-Ahsaa* 31982, *the Kingdom of Saudi Arabia*

[b)] *Mathematics Department, Faculty of Science, Ain Shams University, Cairo,* 11566, *Egypt*

[c)] *Center for Theoretical Physics, British University of Egypt, Sherouk City* 11837, *P. O. Box* 43, *Egypt*

(Received 19 November 2011; revised manuscript received 6 January 2012)

A perfect fluid with self-similarity of the second kind is studied within the framework of the teleparallel equivalent of general relativity (TEGR). A spacetime which is not asymptotically flat is derived. The energy conditions of this spacetime are studied. It is shown that after some time the strong energy condition is not enough to satisfy showing a transition from standard matter to dark energy. The singularities of this solution are discussed.

Keywords: perfect fluid with self-similarity of second kind, teleparallel equivalent of general relativity, asymptotically nonflat spacetime, energy conditions

PACS: 04.20.Jb, 04.50.Kd, 04.30.–w **DOI:** 10.1088/1674-1056/21/6/060401

1. Introduction

Several recent astrophysical observations[1] have provided the astonishing result that around 95%–96% of the content in the universe consists of dark matter and dark energy, with only about 4%–5% being represented by baryonic matter. More intriguingly, around 70% of the energy-density may be in the form of so-called dark energy, which is responsible for the acceleration of the distant type Ia supernovae.[2,3] Therefore, today's models of astrophysics and cosmology face two severe problems: the dark energy problem and the dark matter problem, respectively. The nature of dark energy as well as dark matter up till now are not known, and many radically different models have been proposed, such as, a tiny positive cosmological constant, quintessence,[4−6] DGP branes,[7,8] the nonlinear F(R) models,[9−11] and dark energy in brane worlds, among many others (Refs. [12]–[35] see also the review articles,[36,37] and references therein). Although in recent years many different suggestions have been proposed to overcome these issues, a satisfactory answer has not yet been achieved.

The history of the observed universe is not known before the epoch of nucleosynthesis (i.e., temperature above 1 MeV). The assumption that our universe has to be homogeneous and isotropic is only true for a late universe. Therefore, one can consider the standard cosmology based on the Friedmann–Robertson–Walker (FRW) metric as an effective low energy theory for a more fundamental one at higher energy scale.

It seems that teleparallel theory to be popular again, and there is a trend of analyzing the basic solutions of general relativity (GR) with teleparallel theory and comparing the results. The teleparallel approach is considered as an essential part of generalized non-Riemannian theories such as the Poincaré gauge theory[38−43] or metric-affine gravity.[44] Physics relevant to teleparallel geometry may be related to the teleparallel description of gravity theory.[45−55] The teleparallel approach is used for positive-gravitational-energy proof.[56] A relation between spinor Lagrangian and teleparallel theory is established.[57]

Teleparallel theories are interesting for many reasons: first of all, general relativity (GR) can be viewed as a particular theory of teleparallelism and, thus, teleparallelism could be considered at the very least as a different point of view that can lead to the same results.[58] Second, in this framework, one can define an energy–momentum tensor for the gravitational field that is a true tensor under all general coordinate transformations. This is the reason why teleparallelism was reconsidered by Möller when he was studying the problem of defining an energy–momentum tensor for the gravitational field.[59−62] The idea was taken over by Pellegrini and Plebański, who constructed the gen-

†Corresponding author. E-mail: nashed@bue.edu.eg

© 2012 Chinese Physical Society and IOP Publishing Ltd

eral Lagrangian for these theories.[63] The third reason why these theories are interesting is that they can be seen as gauge theories of the translation group (not the full Poincaré group) and, thus, they gave an alternative interpretation of GR.[45−49,64,65]

Maluf *et al.*[66] have shown that the localization of the gravitational energy depends on the reference frame. Reference frames are described in terms of the tetrad fields that are the basic field quantities of the Weitzenböck geometry.[67−83] The total energy of a space–time is not expected to depend on a particular frame.[74−79,84−86] The dependence of the gravitational energy on the frame is a manifestation of the fact that the gravitational and inertial forces are of the same nature.[87]

The objective of the present work is to study the possibility of standard matter turning into dark energy considering TEGR only. In Section 2, we briefly review the TEGR theory of gravitational field. In Section 3, we study special tetrad field which contains two unknown functions of t and r, i.e., temporal and radial coordinates. In Section 3, the solution of the field equations of TEGR is derived and calculations of energy conditions are given. In Section 4, the singularity problem is discussed. The final section is devoted to the main results.

2. The TEGR of gravitation

In a spacetime with absolute parallelism, the parallel vector fields $e_a{}^\mu$ define the nonsymmetric affine connection

$$\Gamma^\lambda{}_{\mu\nu} \stackrel{\text{def}}{=\!=} e_a{}^\lambda e^a{}_{\mu,\nu}, \tag{1}$$

where $e_{a\mu,\nu} = \partial_\nu e_{a\mu}$.[1)] The curvature tensor defined by $\Gamma^\lambda{}_{\mu\nu}$, given by Eq. (1), is identically vanishing. The metric tensor $g_{\mu\nu}$ is defined by

$$g_{\mu\nu} \stackrel{\text{def}}{=\!=} \eta_{ab}\ e^a{}_\mu e^b{}_\nu \tag{2}$$

with $\eta_{ab} = (-1,+1,+1,+1)$ being the metric of Minkowski space–time.

The Lagrangian density for the gravitational field in the TEGR, in the presence of matter fields,[2)] is given by[74−79]

$$\begin{aligned}\mathscr{L}_{\rm G} &= eL_{\rm G}\\ &= -\frac{e}{16\pi}\left(\frac{T^{abc}T_{abc}}{4}+\frac{T^{abc}T_{bac}}{2}-T^aT_a\right)-L_{\rm m}\\ &= -\frac{e}{16\pi}\Sigma^{abc}T_{abc}-L_{\rm m},\end{aligned} \tag{3}$$

where $e = \det(e^a{}_\mu)$. The tensor Σ^{abc} is defined as

$$\begin{aligned}\Sigma^{abc} \stackrel{\text{def}}{=\!=}& \frac{1}{4}\left(T^{abc}+T^{bac}-T^{cab}\right)\\ &+\frac{1}{2}\left(\eta^{ac}T^b-\eta^{ab}T^c\right).\end{aligned} \tag{4}$$

T^{abc} and T^a are the torsion tensor and the basic vector field defined by

$$\begin{aligned}&T^a{}_{\mu\nu} \stackrel{\text{def}}{=\!=} e^a{}_\lambda T^\lambda{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu,\\ &T^a{}_{bc} \stackrel{\text{def}}{=\!=} e_b{}^\mu e_c{}^\nu T^a{}_{\mu\nu}, \quad T^a \stackrel{\text{def}}{=\!=} T^b{}_b{}^a.\end{aligned} \tag{5}$$

The quadratic combination $\Sigma^{abc}T_{abc}$ is proportional to the scalar curvature $R(e)$, except for a total divergence term.[74−79] $L_{\rm m}$ represents the Lagrangian density for matter fields.

The gravitational field equations described by $L_{\rm G}$ are the following

$$\begin{aligned}&e_{a\lambda}e_{b\mu}\partial_\nu\left(e\Sigma^{b\lambda\nu}\right)-e\left(\Sigma^{b\nu}{}_aT_{b\nu\mu}-\frac{1}{4}e_{a\mu}T_{bcd}\Sigma^{bcd}\right)\\ &=\frac{1}{2}\kappa eT_{a\mu},\end{aligned} \tag{6}$$

where

$$\frac{\delta L_{\rm m}}{\delta e^{a\mu}} \equiv eT_{a\mu}.$$

It is possible to prove by explicit calculations that the left-hand side of the symmetric field equations (6) is exactly given by[74−79]

$$\frac{e}{2}\left[R_{a\mu}(e)-\frac{1}{2}e_{a\mu}R(e)\right].$$

The definition of the gravitational energy–momentum, P^a, four-vector[74−79]

$$P^a = -\int_V {\rm d}^3x\partial_i\Pi^{ai}, \tag{7}$$

where V is an arbitrary volume of the three-dimensional space. Maluf *et al.*[74−79,87,88] defined

$$L^{ab} = \int_V {\rm d}^3x e_\mu{}^a e_\nu{}^b M^{\mu\nu}, \tag{8}$$

as the 4-angular–momentum of the gravitational field for an arbitrary volume V of the three-dimensional space.

[1)] space–time indices μ, ν, ... and $SO(3,1)$ indices a, b, ... run from 0 to 3. Time and space indices are indicated to $\mu = 0, i$, and $a = (0), (i)$.

[2)] Throughout this paper we use the relativistic units, $c = G = 1$ and $\kappa = 8\pi$.

3. Self-similar solution

In this section we are going to write the tetrad field in the coordinate $(t,\ r,\ \theta,\ \phi)$ as

$$(e_a{}^\mu) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \dfrac{\sin\theta\cos\phi}{\mathrm{e}^{\psi(r,t)}} & \dfrac{\cos\theta\cos\phi}{r\,S(r,t)} & \dfrac{-\sin\phi}{r\,S(r,t)\sin\theta} \\ 0 & \dfrac{\sin\theta\sin\phi}{\mathrm{e}^{\psi(r,t)}} & \dfrac{\cos\theta\sin\phi}{r\,S(r,t)} & \dfrac{\cos\phi}{r\,S(r,t)\sin\theta} \\ 0 & \dfrac{\cos\theta}{\mathrm{e}^{\psi(r,t)}} & \dfrac{-\sin\theta}{r\,S(r,t)} & 0 \end{pmatrix}, \quad (9)$$

where $\psi(r,t)$ and $S(r,t)$ are unknown functions of t and r. Self-similar solutions of the second kind are given by[89,90]

$$\psi(r,t) = \psi(x), \quad S(r,t) = S(x) \quad \text{with} \quad x = \ln\left[\frac{r}{(-t)^{1/\alpha}}\right],$$

where α is a dimensionless constant. Applying Eq. (9) to the field equations (6) we obtain a system of nonlinear partial differential equations having the following form

$$\begin{aligned}
&-\frac{1}{r^2S^2}\{1+r^2\dot{S}^2+2\,rS^2\,\psi'\,\mathrm{e}^{-2\,\psi}-6\,rS\,S'\,\mathrm{e}^{-2\,\psi}-S^2\,\mathrm{e}^{-2\,\psi}+2r^2S\;S'\,\psi'\,\mathrm{e}^{-2\,\psi}-r^2S'^2\,\mathrm{e}^{-2\,\psi} \\
&\quad +2\,r^2S\dot{S}\dot{\psi}\,\mathrm{e}^{-2\,\psi}-2\,r^2S\,S''\,\mathrm{e}^{-2\,\psi}\} = 4\pi\rho, \\
&\frac{r\dot{S}'+\dot{S}-S\,\dot{\psi}-r\dot{\psi}S'}{r\,S} = 0, \\
&\frac{\mathrm{e}^{-2\,\psi}\left\{\mathrm{e}^{2\,\psi}-2\,rSS'+r^2\dot{S}^2\,\mathrm{e}^{2\,\psi}-S^2+2\,r^2S\ddot{S}\,\mathrm{e}^{2\,\psi}-r^2S'^2\right\}}{r^2S^2} = 4\pi p, \\
&\frac{r\ddot{S}+S\psi'\,\mathrm{e}^{-2\,\psi}+rS'\psi'\,\mathrm{e}^{-2\,\psi}-2S'\,\mathrm{e}^{-2\,\psi}-rS''\,\mathrm{e}^{-2\,\psi}+r\dot{S}\dot{\psi}+rS\ddot{\psi}+rS\dot{\psi}^2}{rS} = 4\pi p, \quad (10)
\end{aligned}$$

where we have assumed that the matter filling the universe has the form of a prefect fluid, i.e.,

$$T^0{}_0 = \rho, \qquad T^1{}_1 = T^2{}_2 = T^3{}_3 = -p, \quad (11)$$

with ρ and p being the proper density and pressure of the fluid, respectively. We define

$$\dot{S} \overset{\text{def}}{=\!=} \frac{\partial S}{\partial t}, \quad \ddot{S} \overset{\text{def}}{=\!=} \frac{\partial^2 S}{\partial t^2}, \quad S' \overset{\text{def}}{=\!=} \frac{\partial S}{\partial r}, \quad S'' \overset{\text{def}}{=\!=} \frac{\partial^2 S}{\partial r^2}. \quad (12)$$

We try to find a solution to the above system of partial differential equations. From the second equation of Eqs. (10) one can obtain

$$\psi(x) = \ln\{S(x) + rS'(x)\}. \quad (13)$$

Substituting Eq. (13) into the rest of Eqs. (10) we finally obtain

$$S(x) = A_0\,U^{2/3}\,\mathrm{e}^U,$$

$$\mathrm{e}^{\psi(x)} = \frac{2A_0\,\mathrm{e}^U\{3U+1\}}{3\sqrt[3]{U}}, \quad (14)$$

where $U \equiv U(x) = \ln\{r\sqrt[3]{-t}\}$, A_0 is a constant of integration. Using Eq. (14) in Eq. (2) we find that the associated metric has the form[3)]

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2{}_0U^{4/3}\,\mathrm{e}^{2U}\left\{4\left(1+\frac{1}{3U}\right)^2\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right\}, \quad (15)$$

with $\mathrm{d}\Omega^2 = \mathrm{d}r^2 + \sin^2\theta\,\mathrm{d}\phi^2$.

The geometric radius is defined by

$$R_g \overset{\text{def}}{=\!=} rS = rA_0\,\sqrt[3]{U^2}\,\mathrm{e}^U = A_0r^2\sqrt[3]{-t}\sqrt[3]{\ln\{r\sqrt[3]{-t}\}^2}, \quad (16)$$

the first time derivative of R_g is given by

$$\dot{R}_g = -\frac{r^2A_0\,\left\{2+3\,\ln\left(r\sqrt[3]{-t}\right)\right\}}{9\,(-t)^{2/3}\,\sqrt[3]{\ln\left\{r\sqrt[3]{-t}\right\}}}, \quad (17)$$

showing that the fluid is collapsing and continue in this collapse until the first singularity happens, i.e.,

[3)] The FRW model has the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left\{\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\Omega^2\right\},$$

where $a(t)$ is the scale factor. From Eq. (15) one can find a direct relation between the scale factor $a(t)$ and U. When $k=0$, i.e., flat spacetime, $r = 2/3\,\sqrt[3]{t}$. The scale factor in this choice, i.e., $k=0$, is constant, i.e., $a(t)$ does not depend on time. The cosmological parameters related to this model are not consistent with the present observations.[91] The other choices, i.e., $k=\pm$ need intensive study to be able to compare the scale factor of FRW with Eq. (15).

$1/r^3$. Using Eq. (14) in Eq. (10) we obtain the energy density in the form

$$\rho = \frac{(U+1)(3U+2)}{3t^2U(3U+1)}. \tag{18}$$

From Eq. (18) we can observe two singularities, which are $U = 0$ and $U = -1/3$. The first singularity given by

$$U = 0 \to (-t) = \frac{1}{r^3} \tag{19}$$

is outer than the second one. Using Eq. (14) in Eq. (10), we obtain the pressure in the form

$$p = -\frac{1}{t^2}. \tag{20}$$

The asymptotic behavior of the geometric radius and the energy density are given in Figs. 1 and 2.

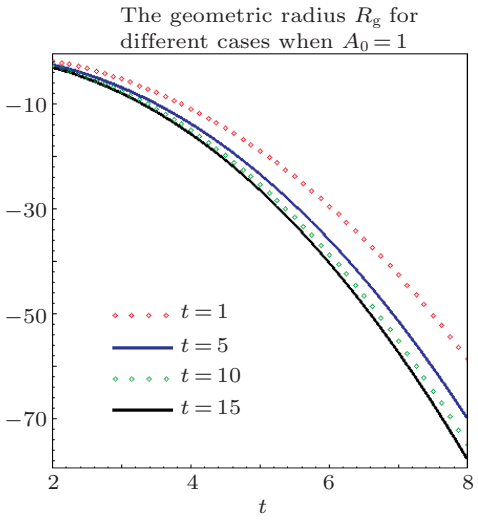


Fig. 1. (colour online) The geometric radius for different cases when $A_0 = 1$.

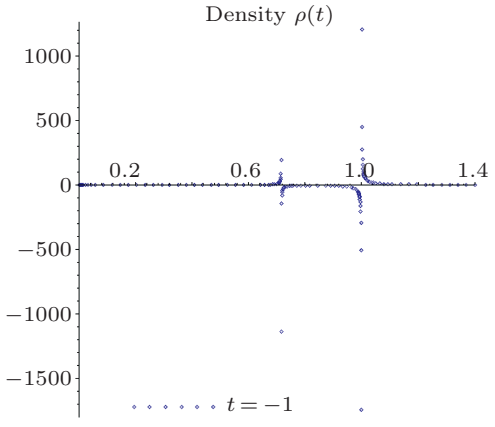


Fig. 2. (colour online) The density when $t = -1$ when $A_0 = 1$.

From Fig. 1, the geometric radius has always a negative value. The two singularities appear in Eq. (18) are shown in Fig. 2. It is clear from Fig. 2 that the density begins with a positive value and then becomes divergent at the point $r = 0.7$. And then it changes to negative infinity at $r = 0.7$ and becomes a negative quantity till it reaches the point $r = 1$. Then it becomes negative infinity and at $r = 1$ it changes to reach positive infinity and after that it becomes a positive quantity.

Now we are going to study the energy conditions which are given by:[92]

(i) Weak energy condition

$$\rho \geq 0, \quad \rho + p \geq 0; \tag{21}$$

(ii) Dominant energy condition

$$\rho + p \geq 0, \quad \rho - p \leq 0; \tag{22}$$

(iii) Strong energy condition

$$\rho + 3p \geq 0. \tag{23}$$

Using Eqs. (18) and (20) we obtain the following expressions

$$\begin{aligned} \rho + p &= 2\frac{2U+1}{3t^2U(3U+1)}, \\ \rho - p &= 2\frac{3U^2+3U+1}{3t^2U(3U+1)}, \\ \rho + 3p &= 2\frac{1+U-3U^2}{3t^2U(3U+1)}. \end{aligned} \tag{24}$$

The validity of $\rho + p$ is satisfied when

$$(-t) \geq \frac{\mathrm{e}^{-3/2}}{r^3} \approx \frac{0.2231301602}{r^3}, \tag{25}$$

while $\rho + 3p$ is satisfied when

$$(-t) \geq \frac{\left[\exp\left(\frac{1-\sqrt{13}}{2}\right)\right]^3}{r^3} \approx \frac{0.2717763924}{r^3}, \tag{26a}$$

$$(-t) \geq \frac{\left[\exp\left(\frac{1+\sqrt{13}}{2}\right)\right]^3}{r^3} \approx \frac{10.00190563}{r^3}. \tag{26b}$$

4. Singularities

In teleparallel theories we mean by the singularity of spacetime,[93] the singularity of the scalar concomitants of the torsion and curvature tensors.

Using the definitions of the Riemann–Christofell curvature tensor, Ricci tensor, Ricci scalar, torsion tensor, basic vector, traceless part, and the axial vector part given in Ref. [94] the scalars of the Riemann–Christofell curvature tensor, Ricci tensor, Ricci scalar,

torsion tensor, basic vector, traceless part, and the axial vector part of the spacetime given by solution (14) have the following form

$$
\begin{aligned}
R^{\mu\nu\lambda\sigma}R_{\mu\nu\lambda\sigma} &= \frac{\left[16+82254\ln\left(r\sqrt[3]{-t}\right)\right]}{2187t^4\ln\left(r\sqrt[3]{-t}\right)\left[3\ln\left(r\sqrt[3]{-t}\right)+1\right]^2},\\
R^{\mu\nu}R_{\mu\nu} &= \frac{\left[1+97\ln\left(r\sqrt[3]{-t}\right)\right]}{9t^4\ln\left(r\sqrt[3]{-t}\right)\left[3\ln\left(r\sqrt[3]{-t}\right)+1\right]^2},\\
R &= \frac{2\left[5\ln\left(r\sqrt[3]{-t}\right)-1\right]}{3t^2\ln\left(r\sqrt[3]{-t}\right)\left[3\ln\left(r\sqrt[3]{-t}\right)+1\right]},\\
T^{\mu\nu\lambda}T_{\mu\nu\lambda} &= -\frac{\left[1+342\ln\left(r\sqrt[3]{-t}\right)\right]}{9t^2\ln\left(r\sqrt[3]{-t}\right)\left[3\ln\left(r\sqrt[3]{-t}\right)+1\right]^2},\\
\Phi^{\mu}\Phi_{\mu} &= -\frac{\left[1+27\ln\left(r\sqrt[3]{-t}\right)\right]^2}{18t^2\ln\left(r\sqrt[3]{-t}\right)\left[3\ln\left(r\sqrt[3]{-t}\right)+1\right]^2},\\
t^{\mu\nu\lambda}t_{\mu\nu\lambda} &= -\frac{1}{18t^2\ln\left(r\sqrt[3]{-t}\right)\left[3\ln\left(r\sqrt[3]{-t}\right)+1\right]^2},\\
a^{\mu}a_{\mu} &= 0. \qquad (27)
\end{aligned}
$$

5. Main results and discussion

The main results of this paper can be summarized as follows.

(I) A special exact solution which satisfies the field equations (6) is derived. This solution is not asymptotically flat.

(II) The energy, using Eq. (7), associated with Eq. (9) has been calculated and it has the form

$$E = rS(r,t)\,\mathrm{e}^{-\psi(r,t)}\left(\mathrm{e}^{\psi(r,t)} - S(r,t) - r\dot{S}(r,t)\right). \qquad (28)$$

Equation (28) shows that the energy depends on r, on the functions $S(r,t)$, $\psi(r,t)$, and on $\dot{S}(r,t)$. This result is in contradiction with the previous results achieved in Ref. [66] "which assumed that the universe appeared from nowhere about 10^{10} years ago and the conventional laws of physics need not have been violated at the time of the creation of the universe". We are inclined to suggest that this contradiction is due to the following. i) The spacetime obtained is not asymptotically flat. ii) There is no symmetry assumed on the spacetime, i.e., the spacetime has not homogeneity or isotropy. (III) We have calculated the geometric radius and shown that its first time derivative is negative, i.e., the fluid is collapsing. This collapse occurs until the first singularity occurred, $1/r^3$, as is clear from Fig. 1. (IV) The density associated with this solution is found to have two singularities. These singularities are shown in Fig. 2. (V) The pressure has only one singularity occurs at $t = 0$. (VI) The singularity problem of solution (14) has been studied. It is clear from Eq. (27) that all the scalars have three singularities $t = 0$, $t = -1/r^3$, and $t = 20.08553692$. In addition to these singularities the Riemann–Christofell tensor, Ricci tensor, Ricci scalar torsion tensor, and basic vector tensor have another singularity at $t = 0$ and/or $r = 0$.

References

[1] Riess A G, Filippenko A V, Challis P, Clocchiatti A, Diercks A, Garnavich P M, Gilliland R L, Hogan C J, Jha S, Kirshner R P, Leibundgut B, Phillips M M, Reiss D, Schmidt B P, Schommer R A, Smith R C, Spyromilio J, Stubbs C, Suntzeff N B and Tonry J 1998 *Astron. J.* **116** 1009

[2] Perlmutter S, Aldering G, Deustua S, Fabbro S, Goldhaber G, Groom D E, Kim A G, Kim M Y, Knop R A, Nugent P, Pennypacker C R, Goobar A, Pain R, Hook I M, Lidman C, Ellis R S, Irwin M, McMahon R G, Ruiz-Lapuente P, Walton N, Schaefer B, Boyle B J, Filippenko A V, Matheson T, Fruchter A S, Panagia N, Newberg H J M and Couch W J 1999 *Astrophys. J.* **517** 565

[3] de Bernardis P, Ade P A R, Bock J J, Bond J R, Borrill J, Boscaleri A, Coble K, Crill B P, de Gasperis G, Farese P C, Ferreira P G, Ganga K, Giacometti M, Hivon E, Hristov V V, Iacoangeli A, Jaffe A H, Lange A E, Martinis L, Masi S, Mason P V, Mauskopf P D, Melchiorri A, Miglio L, Montroy T, Netterfield C B, Pascale E, Piacentini F, Pogosyan D, Prunet S, Rao S, Romeo G, Ruh J E, Scaramuzzi F, Sforna D and Vittorio N 2000 *Nature* **404** 955

[4] Hanany S, Ade P, Balbi A, Bock J, Borrill J, Boscaleri A, de Bernardis P, Ferreira P G, Hristov V V, Jaffe A H, Lange A E, Lee A T, Mauskopf P D, Netterfield C B, Oh S, Pascale E, Rabii B, Richards P L, Smoot G F, Stompor R, Winant C D and Wu J H P 2000 *Astrophys. J.* **545** L5

[5] Peebles P J E and Ratra B 2003 *Rev. Mod. Phys.* **75** 559

[6] Padmanabhan T 2003 *Phys. Rep.* **380** 235

[7] Caldwell R R, Dave R and Steinhardt P J 1998 *Phys. Rev. Lett.* **80** 1582

[8] Liddle A R and Scherrer R J 1999 *Phys. Rev.* D **59** 023509

[9] Steinhardt P J, Wang L M and Zlatev I 1999 *Phys. Rev.* D **59** 123504

[10] Dvali G R, Gabadadze G and Porrati M 2000 *Phys. Lett.* B **484** 112

[11] Deffayet C 2001 *Phys. Lett.* B **502** 199

[12] Capozziello S, Carloni S and Troisi A 2003 *arXiv*: astro-ph/0303041

[13] Carroll S M, Duvvuri V, Trodden M and Turner M S 2003 *Phys. Rev.* D **70** 043528

[14] Nojiri S and Odintsov S D 2003 *Phys. Rev.* D **68** 123512

[15] Townsend P K and Wohlfarth N R 2003 *Phys. Rev. Lett.* **91** 061302

[16] Gibbons G W 1985 *Supersymmetry, Supergravity and Related Topics*, eds. de Aguila F *et al.* (Singapore: World Scientific) p. 124

[17] Maldacena J M and Nuñez C 2001 *Int. J. Mod. Phys.* A **16** 822

[18] Ohta N 2003 *Phys. Rev. Lett.* **91** 061303
[19] Wohlfarth N R 2003 *Phys. Lett.* B **563** 1
[20] Roy S 2003 *Phys. Lett.* B **567** 322
[21] Webb J K, Murphy M T, Flambaum V V, Dzuba V A, Barrow J D, Churchill C W, Prochaska J X and Wolfe A M 2001 *Phys. Rev. Lett.* **87** 091301
[22] Cline J M and Vinet J 2003 *Phys. Rev.* D **68** 025015
[23] Ohta N 2003 *Prog. Theor. Phys.* **110** 269
[24] Ohta N 2005 *Int. J. Mod. Phys.* A **20** 1
[25] Chen C M, Ho P M, Neupane I P, Wang J E and Ohta N 2003 *JHEP* **10** 058
[26] Bergshoeff E 2004 *Class. Quantum Grav.* **21** 1947
[27] Gong Y and Wang A 2006 *Class. Quantum Grav.* **23** 3419
[28] Neupane I P and Wiltshire D L 2005 *Phys. Lett.* B **619** 201
[29] Neupane I P and Wiltshire D L 2005 *Phys. Rev.* D **72** 083509
[30] Maeda K and Ohta N 2005 *Phys. Rev.* D **71** 063520
[31] Neupane I P 2007 *Phys. Rev. Lett.* **98** 061301
[32] Gong Y, Wang A and Wu Q 2008 *Phys. Lett.* B **663** 147
[33] Pereira P R C T, da Silva M F A and Chan R 2006 *Int. J. Mod. Phys.* D **15** 991
[34] Pun C S J, Kovács Z and Harko T 2008 *Phys. Rev.* D **78** 024043
[35] Brandt C F C, Chan R, da Silva M F A and da Rocha J F V 2007 *Gen. Rel. Grav.* **39** 1675
[36] Copeland E J, Sami M and Tsujikawa S 2006 *Int. J. Mod. Phys.* D **15** 1753
[37] Padmanabhan T 2008 *Gen. Rel. Grav.* **40** 529
[38] Hehl F W, Neeman Y, Nitsch J and Von der Heyde P 1978 *Phys. Lett.* B **78** 102
[39] Hehl F W 1979 "Four Lectures on Poincaré Gauge Theory", in *Proceedings of the 6th Course on Spin, Torsion and Supergravity*, Eric, Italy, eds. Bergmann P G and de Sabbata V (New York: Plenum) p. 5
[40] Kawai T 1994 *Phys. Rev.* D **49** 2862
[41] Kawai T 2000 *Phys. Rev.* D **62** 104014
[42] Blagojević M and Vasilić M 2000 *Class. Quantum Grav.* **17** 3785
[43] Blagojević M and Nikolić I A 2000 *Phys. Rev.* D **62** 024021
[44] Hehl F W, MacCrea J D, Mielke E W and Ne'eman Y 1995 *Phys. Rep.* **258** 1
[45] Hayashi K 1977 *Phys. Lett.* B **69** 441
[46] Hayashi K and Shirafuji T 1979 *Phys. Rev.* D **19** 3524
[47] Hayashi K and Shirafuji T 1981 *Phys. Rev.* D **24** 3312
[48] Blagojević M and Vasilić M 1988 *Class. Quantum Grav.* **5** 1241
[49] Kawai T, Shibata K and Tanaka I 2000 *Prog. Theor. Phys.* **104** 505
[50] Nitsch J and Hehl F W 1980 *Phys. Lett.* B **90** 98
[51] Leclerc M 2005 *Phys. Rev.* D **71** 027503
[52] Mielke E W 2004 *Phys. Rev.* D **69** 128501
[53] Mielke E W 1980 *Phys. Rev.* D **42** 3388
[54] Obukhov Y N and Pereira J G 2004 *Phys. Rev.* D **69** 128502
[55] Obukhov Y N and Pereira J G 2003 *Phys. Rev.* D **67** 044016
[56] Meyer H 1982 *Gen. Rel. Grav.* **14** 531
[57] Tung R S and Nester J M 1999 *Phys. Rev.* D **60** 021501
[58] Ortín T 2004 *Gravity and Strings* (Cambridge: Cambridge University Press) p. 166
[59] Møller C 1961 *Ann. Phys.* **12** 118
[60] Møller C 1961 *Mat. Fys. Medd. Dan. Vid. Selsk.* **1** 10
[61] Møller C 1964 *Nucl. Phys.* **57** 330
[62] Møller C 1978 *Mat. Fys. Medd. Dan. Vid. Selsk.* **39** 13
[63] Pellegrini C and Plebański J 1963 *Mat. Fys. Scr. Dan. Vid. Selsk.* **2** No. 4
[64] Hehl F W 1980 *Proceedings of the 6th School of Cosmology and Gravitation on Spin, Torsion, Rotation and Supergravity*, Erice, Italy, eds. Bergmann P G and de Sabbata V (New York: Plenum)
[65] Hehl F W, McCrea J D, Mielke E W and Neeman Y 1995 *Phys. Rep.* **258** 1
[66] Maluf J W, Faria F F and Castello-Branco K H 2003 *Class. Quantum Grav.* **20** 4683
[67] Chang C C, Nester J M and Chen C M 1999 *Phys. Rev. Lett.* **83** 1897
[68] Tung R S and Nester J M 1999 *Phys. Rev.* D **60** 021501
[69] Nester J M and Yo Chin H J 1999 *J. Phys.* **37** 113
[70] Nester J M, Ho F H and Chen C M 2003 "Quasilocal Center-of-Mass for Teleparallel Gravity", *Proceeding of the 10th Marcel Grossman Meeting* Rio de Janeiro, 2003 gr-qc/0403101
[71] Nester J M 1989 *Phys. Lett.* A **139** 112
[72] Nester J M 1992 *J. Math. Phys.* **33** 910
[73] Nester J M 1988 *Class. Quantum Grav.* **5** 1003
[74] Maluf J W 1994 *J. Math. Phys.* **35** 335
[75] Maluf J W and Kneip A 1997 *J. Math. Phys.* **38** 458
[76] Maluf J W and da Rocha-Neto J F 1999 *J. Math. Phys.* **40** 1490
[77] Maluf J W and Goya A 2001 *Class. Quantum Grav.* **18** 5143
[78] Maluf J W, da Rocha-Neto J F, Toribio T M L and Castello-Branco K H 2002 *Phys. Rev.* D **65** 124001
[79] Sousa A A and Maluf J W 2002 *Prog. Theor. Phys.* **108** 457
[80] Szabados L B 2004 "Quasi-Local Energy–Momentum and Angular Momentum in General Relativity" (review article), *Living Rev. Relativity* **7** 4 http://www.livingreviews.org/lrr-2004-4
[81] de Andrade V C and Pereira J G 1997 *Phys. Rev.* D **56** 4689
[82] de Andrade V C, Guillen L C T and Pereira J G 2000 *Phys. Rev. Lett.* **84** 4533
[83] de Andrade V C, Guillen L C T and Pereira J G 2001 *Phys. Rev.* D **64** 027502
[84] Nashed G G L 2008 *Eur. Phys. J.* C **54** 291
[85] Nashed G G L 2007 *Eur. Phys. J.* C **51** 377
[86] Nashed G G L 2007 *Eur. Phys. J.* C **49** 851
[87] Maluf J W and Faria F F 2004 *Ann. Phys.* (Leipzig) **13** 604
[88] Maluf J W, Ulhoa S C, Faria F F and da Rocha-Neto J F 2006 *Calss. Quantum Grav.* **23** 6245
[89] Hirschmann E W, Wang A and Wu Y 2004 *Class. Quantum Grav.* **21** 1791
[90] Elvang H, Emparan R, Mateos D and Reall H S 2004 *Phys. Rev. Lett.* **93** 211302
[91] Hawking S W and Ellis G F R 1973 *The Large Scale Structure of Spacetime* (Cambridge: Cambridge University Press)
[92] Kawai T and Toma N 1990 *Prog. Theor. Phys.* **83** 1
[93] Nashed G G L 2003 *Chaos, Solitons and Fractals* **15** 841
[94] Tryon E P 1973 *Nature* **246** 396