# Quantum dynamic behaviour in a coupled cavities system*

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The dynamic behaviour of the two-site coupled cavities model which is doped with ta wo-level system is investigated. The exact dynamic solutions in the general condition are obtained via Laplace transform. The simple analytical solutions are obtained in several particular cases, which demonstrate the clear and simple physical picture for the quantum state transition of the system. In the large detuning or hoppling case, the quantum states transferring between qubits follow a slow periodic oscillation induced by the very weak excitation of the cavity mode. In the large coupling case, the system can be interpreted as two Jaynes–Cummings model subsystems which interact through photon hop between the two cavities. In the case of $\lambda \approx \Delta \gg g$, the quantum states transition of qubits is accompanied by the excitation of the cavity, and the cavity modes have the same dynamic behaviours and the amplitude of probability is equal to 0.25 which does not change with the variation of parameter.

**Keywords:** coupled cavities, quantum dynamics behaviour, detuning, two-level system

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## 1. Introduction

Cavity quantum electrodynamics (QED) concerns the interaction between atoms (or artificial atoms) and a quantized electromagnetic field inside an optical cavity, which is one of the experimentally realizable systems for quantum information processing (QIP).[1−6] Recently, theoretical and experimental studies have focused mainly on the coherent interaction between distant cavities.[7−10] Coupled-cavity models have potential applications in QIP as well, since they can control and measure individual lattice sites and provide almost lossless guiding and coupling of light pulses at slow group velocities. The coupling between two cavities can be controlled in several ways, which offers great freedom and flexibility to engineer the transport of quantum states via photonic processes.

Considerable theoretical effort has been devoted to the coupled cavities model. The strong coupled interaction between coupled cavities in the system, each with embedded quantum dots (QDs),[11] atoms,[12] and superconducting qubits,[13] has been demonstrated, and these systems have been proposed for implementing quantum logic gates and constructing quantum networks.[14−20] A strong photon-blockade regime and photonic Mott insulator state and quantum communication between the ends of an array of coupled cavities have been demonstrated.[21,22] Nohama and Roversi[23] discussed the behaviour of atomic state transfer only in the case of the detuning between the atom and the cavity field with a strength equal to the photon hopping strength. Ogden *et al.*[24] examined the dynamics of the coupled cavities model, and derived an effective Hamiltonian in several limiting cases. Recently, Zhang and Li[25] have discussed the dynamics via the eigenproblem of an ordinary coupled linear system without detuning.

In this paper, the dynamics of a system consisting of two coupled microcavities, each doped with a single two-level system, is investigated. The rigorous dynamics of the system is solved directly via Laplace transform. Although the exact dynamics is complicated, there are several particular cases which can be simplified. In the cases of large detuning and large hopping, an excitation initially in one of the qubits will be periodically transferred in a qubit subsystem, which is induced by the very weak excitation of cavity mode subsystem. The difference between the large detuning and the large hopping is that the qubit excitation is synchronous with the excitation of the localized cavity in the former but with the delocalized cavity mode in the latter. The time-evolution of each

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state is characterized by the slowly varying amplitude envelope superimposed with two oscillations in the large coupling case. The system can be seen as two Jaynes–Cummings (JC) model subsystems which interact with each other through coupling between two cavities. The energy can be transferred between the two JC-model subsystems with the period $2\pi/\lambda$. In the case of $\lambda \approx \Delta \gg g$, the most prominent feature is that the cavity modes have the same dynamic behaviours and the probability amplitude is always equal to 0.25 which will not change with the variation of parameter.

## 2. Coupled cavity system and dynamics solution

The system under consideration is depicted in Fig. 1. The system consists of two coupled single-mode cavities with the same frequency $\omega_{\rm c}$, each cavity is doped with a single two-level system (QDs, atoms, or superconducting qubits) with transition frequency $\omega_{\rm a}$, the ground state and the excited state are denoted by $|0\rangle_j$ and $|1\rangle_j$, respectively. The two cavities are coupled so that photons may hop between them. In the rotating-wave approximation, the Hamiltonian for the whole system is ($\hbar = 1$)[24]

$$H = H_0 + H_1, \tag{1}$$

where

$$H_0 = \sum_{j=1}^{2} \left(\omega_{\rm a} |1\rangle_j \langle 1| + \omega_{\rm c} a_j^\dagger a_j\right), \tag{2}$$

$$H_1 = g \sum_{j=1}^{2} \left(a_j \sigma_{j+} + a_j^\dagger \sigma_{j-}\right) + \lambda \left(a_1^\dagger a_2 + a_2^\dagger a_1\right), \tag{3}$$

with operator $a_j$ ($a_j^\dagger$) being the lowering (raising) operator for the cavity field $j$, and operator $\sigma_{j-}$ ($\sigma_{j+}$) the lowering (raising) operator of the qubit in cavity $j$. The same coupling strength $g$ between the qubit and the cavity is obtained by changing the location of the qubit embedded in the cavity. In Eq. (3), $\lambda$ is the cavity–cavity hopping strength.

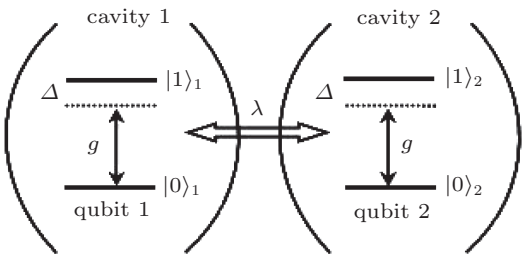



**Fig. 1.** Schematic diagram of the coupled cavities system, in which each cavity contains a single two-level system, which is coupled to their local cavity mode with strength $g$. The two cavities are coupled so that photons may hop between them at rate $\lambda$. $\Delta$ is the detuning between the cavity and the qubit dwelling in the cavity.

The interactions of cavity–qubit resonance and cavity–cavity hop are expressed in Eq. (3). Performing a unitary transformation, the interaction Hamiltonian in the interaction picture is obtained as

$$H_{\rm I} = \sum_{j=1}^{2} g \left(a_j^\dagger \sigma_{j-} \, {\rm e}^{-{\rm i}\Delta t} + a_j \sigma_{j+} \, {\rm e}^{{\rm i}\Delta t}\right) + \lambda \left(a_1^\dagger a_2 + a_2^\dagger a_1\right), \tag{4}$$

where $\Delta = \omega_{\rm a} - \omega_{\rm c}$ is the detuning between the level transition of qubit and cavity. Here, we discuss the dynamics of this Hamiltonian in the subspace containing zero and one excitations. The zero-excitation $|\psi_0(t)\rangle = |00\rangle\,|00\rangle_{\rm c}$ is invariant under the action of the Hamiltonian (4), thus, the general state vector (not necessarily normalized to 1) of the system is written as[25]

$$|\Psi_1(t)\rangle = a(t)\,|00\rangle\,|10\rangle_{\rm c} + b(t)\,|00\rangle\,|01\rangle_{\rm c} + c(t)\,|10\rangle\,|00\rangle_{\rm c} + d(t)\,|01\rangle\,|00\rangle_{\rm c}, \tag{5}$$

where $|\alpha\beta\rangle\,|\chi\gamma\rangle_{\rm c}$ is the abbreviation of the state $|\alpha\rangle_1\,|\beta\rangle_2\,|\chi\rangle_{\rm c1}\,|\gamma\rangle_{\rm c2}$ of qubits (1, 2) and cavity modes (c1, c2). The time evolution of the general state can be calculated exactly from the Schrödinger equation, and the solutions are given by

$$\begin{aligned} {\rm i}\dot{a}(t) &= g\,{\rm e}^{-{\rm i}\Delta t} c(t) + \lambda b(t), \\ {\rm i}\dot{b}(t) &= g\,{\rm e}^{-{\rm i}\Delta t} d(t) + \lambda a(t), \\ {\rm i}\dot{c}(t) &= g\,{\rm e}^{{\rm i}\Delta t} a(t), \\ {\rm i}\dot{d}(t) &= g\,{\rm e}^{{\rm i}\Delta t} b(t). \end{aligned} \tag{6}$$

The first term on the right-hand side of each equation in expression (6) involves a time-varying coefficient ${\rm e}^{{\rm i}\Delta t}$ or ${\rm e}^{-{\rm i}\Delta t}$. In order to obtain a constant coefficient differential equation set, we assume a transformation to be as follows:

$$\begin{aligned} a(t) &= (\zeta_1(t) + \zeta_2(t))\,{\rm e}^{-{\rm i}\Delta t/2}, \\ b(t) &= (\zeta_2(t) - \zeta_1(t))\,{\rm e}^{-{\rm i}\Delta t/2}, \\ c(t) &= (\beta_1(t) + \beta_2(t))\,{\rm e}^{{\rm i}\Delta t/2}, \\ d(t) &= (\beta_2(t) - \beta_1(t))\,{\rm e}^{{\rm i}\Delta t/2}. \end{aligned} \tag{7}$$

A constant coefficient equation set is obtained as follows:

$$\begin{aligned} {\rm i}\dot{\zeta}_1(t) &= g\beta_1(t) - (\lambda + \Delta/2)\zeta_1(t), \\ i\dot{\zeta}_2(t) &= g\beta_2(t) + (\lambda - \Delta/2)\zeta_2(t), \end{aligned}$$

$$\begin{aligned}
\mathrm{i}\dot{\beta}_1(t) &= g\zeta_1(t) + \Delta/2)\beta_1(t),\\
\mathrm{i}\dot{\beta}_2(t) &= g\zeta_2(t) + \Delta/2)\beta_2(t).
\end{aligned} \tag{8}$$

Using Laplace transform and solving Eq. (8), the solutions can be written as

$$\begin{aligned}
\zeta_1(t) &= \mathrm{e}^{\mathrm{i}\lambda t/2}\Big[\zeta_1(0)\cos(v_1 t) \\
&\quad + \frac{\mathrm{i}}{v_1}\left(\frac{\Delta+\lambda}{2}\zeta_1(0) - g\beta_1(0)\right)\sin(v_1 t)\Big],\\
\zeta_2(t) &= \mathrm{e}^{-\mathrm{i}\lambda t/2}\Big[\zeta_2(0)\cos(v_2 t) \\
&\quad + \frac{\mathrm{i}}{v_2}\left(\frac{\Delta-\lambda}{2}\zeta_2(0) - g\beta_2(0)\right)\sin(v_2 t)\Big],\\
\beta_1(t) &= \mathrm{e}^{\mathrm{i}\lambda t/2}\Big[\beta_1(0)\cos(v_1 t) \\
&\quad - \frac{\mathrm{i}}{v_1}\left(\frac{\Delta+\lambda}{2}\beta_1(0) + g\zeta_1(0)\right)\sin(v_1 t)\Big],\\
\beta_2(t) &= \mathrm{e}^{-\mathrm{i}\lambda t/2}\Big[\beta_2(0)\cos(v_2 t) \\
&\quad - \frac{\mathrm{i}}{v_2}\left(\frac{\Delta-\lambda}{2}\beta_2(0) + g\zeta_2(0)\right)\sin(v_2 t)\Big],
\end{aligned} \tag{9}$$

where

$$v_j = \sqrt{[\Delta + (-1)^{j+1}\lambda]^2/2 + g^2}, \quad (j = 1, 2)\,. \tag{10}$$

Therefore, the four coefficients of the general state can be obtained under arbitrary initial conditions by substituting Eq. (9) into Eq. (7). To simplify our presentation, we will not give a complete expression for the general solution due to its complexity. In this paper, we consider a particular initial state condition under which the qubit 1 is prepared in the excited state, qubit 2 is in the ground state, and both cavity modes are in the vacuum state, i.e. $c(0) = 1$, $a(0) = b(0) = d(0) = 0$. In this condition, the solutions are given by

$$\begin{aligned}
a(t) &= \frac{-\mathrm{i}g}{2v_1}\mathrm{e}^{(-\mathrm{i}(\Delta-\lambda)t/2)}\sin(v_1 t) \\
&\quad - \frac{\mathrm{i}g}{2v_2}\mathrm{e}^{(-\mathrm{i}(\Delta+\lambda)t/2)}\sin(v_2 t),\\
b(t) &= \frac{\mathrm{i}g}{2v_1}\mathrm{e}^{(-\mathrm{i}(\Delta-\lambda)t/2)}\sin(v_1 t) \\
&\quad - \frac{\mathrm{i}g}{2v_2}\mathrm{e}^{(-\mathrm{i}(\Delta+\lambda)t/2)}\sin(v_2 t),\\
c(t) &= \mathrm{e}^{\mathrm{i}(\Delta+\lambda)t/2}\left[\frac{1}{2}\cos(v_1 t) - \frac{\mathrm{i}(\Delta+\lambda)}{4v_1}\sin(v_1 t)\right] \\
&\quad + \mathrm{e}^{\mathrm{i}(\Delta-\lambda)t/2}\left[\frac{1}{2}\cos(v_2 t) - \frac{\mathrm{i}(\Delta-\lambda)}{4v_2}\sin(v_2 t)\right],\\
d(t) &= \mathrm{e}^{\mathrm{i}(\Delta+\lambda)t/2}\left[\frac{\mathrm{i}(\Delta+\lambda)}{4v_1}\sin(v_1 t) - \frac{1}{2}\cos(v_1 t)\right] \\
&\quad + \mathrm{e}^{\mathrm{i}(\Delta-\lambda)t/2}\left[\frac{1}{2}\cos(v_2 t) - \frac{\mathrm{i}(\Delta-\lambda)}{4v_2}\sin(v_2 t)\right].
\end{aligned} \tag{11}$$

From Eq. (11), the amplitude of random initial state is solved, and we give the solutions in initial condition as mentioned above. Similarly, we can investigate the dynamical behaviour when one-excitation is prepared in the cavity mode using the same method.

# 3. Discussion of dynamical behaviour in particular cases

The main interactions of the system involve the cavity–cavity hopping and the qubit–cavity coupling, which are controlled together by three variables $g$, $\lambda$, and $\Delta$. For a general case where the three parameters take arbitrary values, the physics process of the excited state transform becomes complicated as shown in Eq. (11). When one parameter is larger than other two, the competition between the two interactions will not be so intense, the solutions can be simplified. Indeed, in the large detuning, the large hopping and the large coupling cases, the simple solutions are obtained through numerical approximation. In addition, the cases of two random parameters which are approximately equal and much larger than the third one, are discussed, and the approximate solutions are obtained in the case of $\lambda \approx \Delta \gg g$. In each case mentioned above, the solutions clearly reflect the physical process of excited state transform and the influence of parameters on dynamic behaviour of the system.

## 3.1. The case of large detuning

The first case to be investigated is the large detuning $\Delta \gg \{g, \lambda\}$. The qubit is off-resonant with the cavity mode due to the presence of large detuning, which acts like the atom–cavity interaction of JC model, and the hopping of photons between the two cavities has a very small probability. The initial state of system is $c(0) = 1$, $a(0) = b(0) = d(0) = 0$. To find a more simplified description of the system dynamic behaviour, the following numerical approximation:

$$\begin{aligned}
&\frac{\Delta + (-1)^j\lambda}{2} + (-1)^i v_j \\
&= (-1)^{\delta_{i,j}}\Delta^{1-\mathrm{i}}\lambda^i + (-1)^{\delta_{i,2j-2}}\frac{g^2}{\Delta^{1-\mathrm{i}}\lambda^i}
\end{aligned} \tag{12}$$

is used, where $i = 0, 1$. From Eq. (11), we obtain the following solutions:

$$a(t) = \frac{g}{\Delta}\left[\mathrm{e}^{-\mathrm{i}\Delta t}\cos\left(g^2 t/\lambda\right) - \mathrm{e}^{\mathrm{i}g^2 t/\Delta}\cos(\lambda t)\right],$$

$$
\begin{aligned}
b(t) &= \mathrm{i}\frac{g}{\Delta}\left[\mathrm{e}^{-\mathrm{i}\Delta t}\sin\left(g^2t/\lambda\right) + \mathrm{e}^{\mathrm{i}g^2t/\Delta}\sin(\lambda t)\right],\\
c(t) &= \mathrm{e}^{-\mathrm{i}g^2t/\Delta}\cos(g^2\lambda t/\Delta^2),\\
d(t) &= -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}g^2t/\Delta}\sin(g^2\lambda t/\Delta^2). \qquad (13)
\end{aligned}
$$

The solutions of cavity modes are the linear superposition of two terms, which just correspond to two cavity interactions: the couple between the cavity and qubit within it with the evolution rate $g^2/\lambda$; the couple of both delocalized cavity modes with the evolution rate $\lambda$. The weak amplitude of cavity mode is proportional to $g/\Delta$. The behaviour of qubits at rate $g^2\lambda/\Delta^2$, which resembles the behaviour of two atoms coupled to a single detuning cavity mode with the shift proportional to $g^2/\Delta$.[26]

The evolution of the system is shown in Fig. 2, which is calculated numerically from Eq. (13) in this condition. The probabilities of the cavity modes are displayed in Figs. 2(a), 2(c), and 2(d), which exhibit the characteristics of fast and slow oscillation superposition. The fast period is $T_{\mathrm{f}} = 0.2\pi$ and the slow period is $T_{\mathrm{s}} = \pi$ for the parameters of $\Delta = 10$ and $g = \lambda = 1$ (all units are arbitrary) as shown in Fig. 2(a). When the parameters change to $\Delta = 20$ and $g = \lambda = 1$, the slow period does not change whereas the fast period is shortened to $T_{\mathrm{f}} = 0.1\pi$ as shown in Figs. 2(c) and 2(d) respectively. Moreover, the probabilities of the qubit are displayed in Figs. 2(b) and 2(e), which exhibit perfect sinusoidal oscillations with period $T = 100\pi$ ($\Delta = 10$) and $T = 400\pi$ ($\Delta = 20$) respectively. The discussion shows that the detuning mainly affects both the probability amplitude of cavity mode and the qubit oscillation period. On the other hand, the parameters $g$ and $\lambda$ also influence the dynamics of the system in different ways. The amplitude of cavity mode merely reaching a very small value $g^2/\Delta^2$ demonstrates that the cavity mode is in very weak excitation. It is the weak excitation that allows the quantum state to transfer between the two qubits which are placed in separate cavities. The transformation process for the quantum state is that the energy initially in qubit 1 first transfers to cavity mode 1, then passes through the cavity–cavity coupling to cavity mode 2, finally transfers from cavity mode 2 to qubit 2. When the time $t = (m + 1/2)\pi\Delta^2/(g^2\lambda)$ ($m$ is an integer), the excited state will be completely transformed to qubit 2.

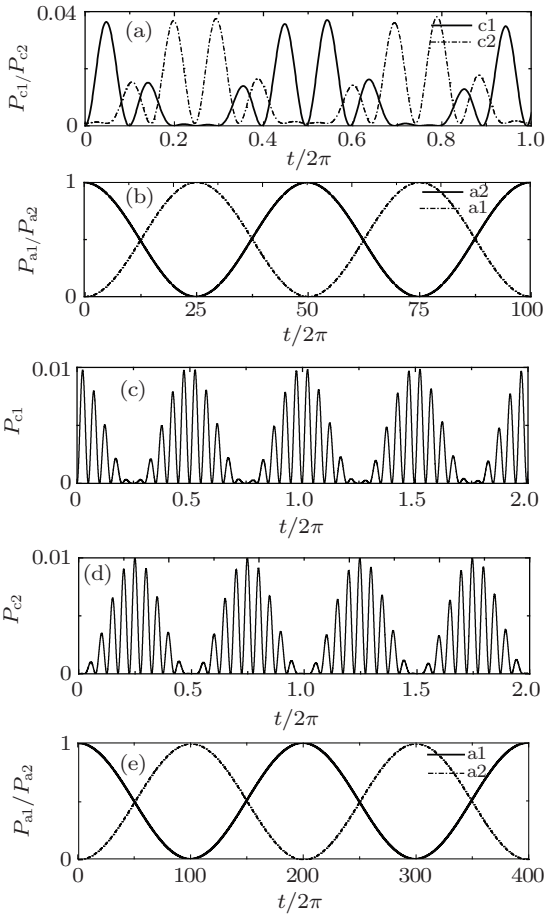



**Fig. 2.** Dynamics of system in the case of $\Delta \gg \{g, \lambda\}$. For the parameters $\Delta = 10$ and $g = \lambda = 1$, the solid (dashed) curve in panel (a) gives the probability $P_{\mathrm{c1}}$ ($P_{\mathrm{c2}}$) of the cavity mode 1 (2) as a function of time; the solid (dashed) curve in panel (b) corresponds to the probability $P_{\mathrm{a1}}$ ($P_{\mathrm{a2}}$) of the excited state of qubit 1 (2). For $\Delta = 20$ and $g = \lambda = 1$, the curves in panels (c) and (d) give the probabilities of field modes 1 and 2 respectively; the solid (dashed) curve in panel (e) gives the probability of the excited state of qubit 1 (2).

## 3.2. The case of large hopping

The second case to be investigated is the large hopping $\lambda \gg \{g, \Delta\}$. Similarly, using the method mentioned above, the approximation solutions are obtained as follows:

$$
\begin{aligned}
a(t) &= -\mathrm{i}\frac{g}{\lambda}\left[\mathrm{e}^{-\mathrm{i}\Delta t}\sin(g^2t/\lambda) + \mathrm{e}^{\mathrm{i}g^2t/\Delta}\sin(\lambda t)\right],\\
b(t) &= -\frac{g}{\lambda}\left[\mathrm{e}^{-\mathrm{i}\Delta t}\cos(g^2t/\lambda) - \mathrm{e}^{\mathrm{i}g^2t/\Delta}\cos(\lambda t)\right],\\
c(t) &= \mathrm{e}^{\mathrm{i}g^2\Delta t/\lambda^2}\cos(g^2t/\lambda),\\
d(t) &= \mathrm{i}\,\mathrm{e}^{\mathrm{i}g^2\Delta t/\lambda^2}\sin(g^2t/\lambda). \qquad (14)
\end{aligned}
$$

The evolutions of the system in this case and in the large detuning case have similar features: (i) the probabilities of the qubits exhibit perfect sinusoidal oscillations, but their periods change to $T = \pi\lambda/g^2$;

(ii) the probabilities of cavity mode 1 (2) and cavity 2 (1) have the same forms of oscillation, their amplitudes $g^2/\Delta^2$ and $g^2/\lambda^2$ are small. In this condition, the parameters $g$ and $\lambda$ also influence the dynamics of the system in different ways, while $\Delta$ mainly affects the phase factor of the quantum state.

Figure 3 shows the same periods of oscillation that inhabit the cavity modes and the qubits. Cavity mode 2 is first excited, showing a slow cosine style, which is different from the results of the large detuning case and the large hopping case which will be discussed below. The physical picture of the system is that the excitation initially stored in qubit 1 first excites cavity mode 2 via cavity mode 1 due to intense cavity–cavity coupling, then the excitation transfer to the qubit 2 takes place, then the inverse process begin to take action, which leads to the excitation of cavity mode 1 via cavity mode 2. At time $t = \pi m\lambda/g^2$, the excitation fully transfers from qubit 1 to qubit 2. To sum up, that the quantum state will fully transfer between two qubits placed in separate cavities results only in very weak excitation of cavity mode.

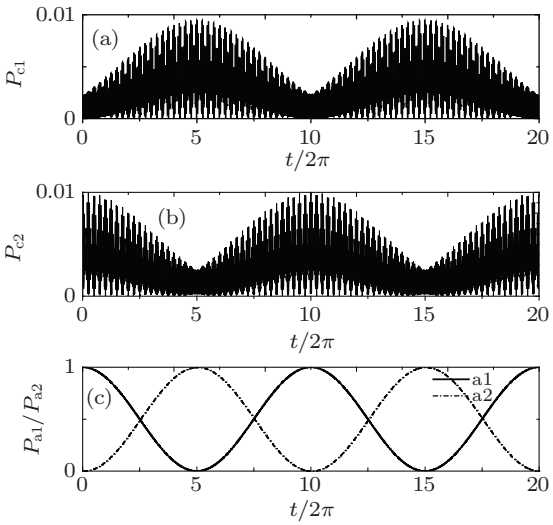



**Fig. 3.** Dynamics of system in the case of $\lambda \gg \{g, \Delta\}$. For $\lambda = 20$, $g = 1$, and $\Delta = 1$, the curves in panels (a) and (b) give the probabilities of field modes 1 and 2. The solid (dashed) curve in panel (c) shows the probability of the excitation of the excited state of qubit 1 (2).

## 3.3. The case of large coupling

The two extreme scenarios of the large detuning and the large hopping have been discussed above. The one-excitation dynamics shows the very weak excitation of the cavity mode. However, if the coupling strength is larger than other relevant parameters, i.e., inequality $g \gg \{\Delta, \lambda\}$ holds, the cavity mode is excited. The interaction between the qubit and the delocalized cavity mode can be neglected due to the small hopping strength. In an approximation $v_1 \approx v_2 \approx g$, the solutions are obtained as

$$\begin{aligned} a(t) &= -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\Delta t/2}\sin(gt)\cos(\lambda t/2),\\ b(t) &= -\mathrm{e}^{-\mathrm{i}\Delta t/2}\sin(gt)\sin(\lambda t/2),\\ c(t) &= \mathrm{e}^{\mathrm{i}\Delta t/2}\cos(gt)\cos(\lambda t/2),\\ d(t) &= -\mathrm{i}\,\mathrm{e}^{\mathrm{i}\Delta t/2}\cos(gt)\sin(\lambda t/2). \end{aligned} \tag{15}$$

According to Eq. (15), the rapid transfer of excited state takes place between the qubit and the localized cavity as the strong coupling interaction of JC model due to the large coupling strength $g$. Photon exchange between the two cavities is very weak due to the small hopping strength $\lambda$. In addition, the detuning $\Delta$ mainly affects the phase factor of quantum state. The excitation of each state displays the slowly varying amplitude envelope superimposed with some fast and small oscillations.

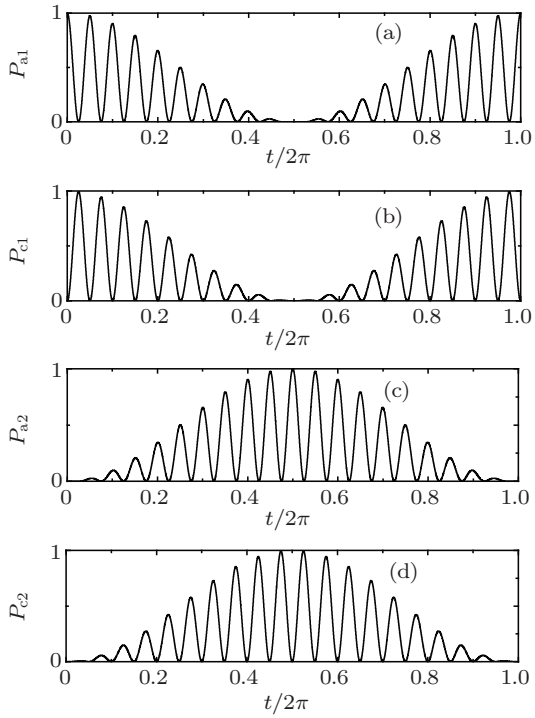



**Fig. 4.** Dynamics of system in the case of $g \gg \{\Delta, \lambda\}$. The parameters are $g = 10$ and $\Delta = \lambda = 1$. The curves in panels (a), (c), (b), and (d) correspond to the probabilities of the excited states of qubits 1, 2, and field modes 1, 2.

The evolution of the system for the parameters $g = 10$ and $\Delta = \lambda = 1$ is displayed in Fig. 4. The probability of each state is that of superposition of two oscillations, where the fast period is $T_\mathrm{f} = 0.1\pi$ and the slow period is $T_\mathrm{s} = 2\pi$. The amplitude of the cavity modes reaches 1, i.e., $A = 1$, which demonstrates that the cavity mode is intensely excited in this condition. Both the numerical calculation and the analytical model show that the large coupling strength determines the feature of the fast oscillation at $T_\mathrm{f} = 2\pi/g$,

while the coupling strength is correlated entirely with the fast oscillation feature at $T_{\rm s} = 4\pi/\lambda$. The physical picture for the system displayed in Fig. 4 clearly shows the energy transfer between qubit 1 and cavity mode 1. This process is accompanied by the transfer of energy from cavity mode 1 to cavity mode 2 due to cavity–cavity hopping; meanwhile, the energy which has been transferred to cavity mode 2 will transfer between cavity mode 2 and qubit 2. The energy will fully transfer from JC-model subsystem 1 to JC-model subsystem 2 at time $\tau = /\lambda$. As the time goes on, the inverse process starts to take action, in which the energy will transfer from qubit 2 to qubit 1. If the time satisfies both $\tau = (2n+1)\pi/\lambda$ and $\tau = m\pi/g$, the excitation would be fully transferred to qubit 2. Therefore, the quantum state can be transferred between the two qubits placed in separate cavities in this condition.

### 3.4. The case of approximate detuning and hopping

Next, we consider the case where two parameters are approximately equal and much larger than another one. Two types of interaction influence the dynamics of the system in different ways. The case of comparable parameters between the hopping and the coupling without the detuning has been studied in Ref. [23]. In the present section, we investigate the conditions $\lambda \approx \Delta \gg g$ and $|\lambda - \Delta| \ll g$. Similarly, the following approximate solutions are obtained:

$$
\begin{aligned}
a(t) &= b(t) = -\frac{\mathrm{i}}{2}\,\mathrm{e}^{-\mathrm{i}(\lambda+\Delta)t/2}\sin(gt)\,,\\
c(t) &= \frac{1}{2}\,\mathrm{e}^{-\mathrm{i}g^2 t/(\lambda+\Delta)} + \frac{1}{2}\,\mathrm{e}^{-\mathrm{i}(\lambda-\Delta)t/2}\cos(gt)\,,\\
d(t) &= -\frac{1}{2}\,\mathrm{e}^{-\mathrm{i}g^2 t/(\lambda+\Delta)} + \frac{1}{2}\,\mathrm{e}^{-\mathrm{i}(\lambda-\Delta)t/2}\cos(gt)\,. \qquad (16)
\end{aligned}
$$

The evolution of the cavity mode follows a uniform rule which is determined mainly by the interaction between the qubit and the localized cavity mode, where a very weak interaction between the qubit and the delocalized cavity mode is neglected due to small amplitude $g/(\lambda+\Delta)$. The first terms on the right-hand sides of $c(t)$ and $d(t)$ in expression (16) each represent a dispersive interaction between the qubit and the delocalized cavity mode, just when phase transfer takes place and no energy exchange between them appears. The second terms refer to the energy transfer between qubit and the localized cavity mode with the rate $g$.

The probabilities of finding an excitation in qubit or cavity are displayed in Fig. 5 for the parameters $\Delta = \lambda = 10$ and $g = 1$. The behaviours of cavity modes displaying the same sinusoidal oscillations with frequency $T_{\rm c} = \pi$ are shown in Fig. 5(a). Therefore, the two cavities can be seen as a single cavity because they display the same dynamical behaviours. The amplitudes of $a(t)$ and $b(t)$ in Eq. (16) each are a constant $(-1/2)$ without containing variables $g$, $\Delta$, and $\lambda$, showing that the amplitudes of the probabilities of cavity modes are always equal to $A = (-1/2)^2 = 0.25$. Figures 5 (b) and 5(c) illustrate that the probabilities of qubits are periodically oscillatory, accompanied with amplitude fluctuations, whose wave trough presents oscillation of a peak. The fast oscillation period is $T_{\rm f} = \pi$ and the slow one is $T_{\rm f} = 40\pi$. Figure 5(c) shows that the excited state is fully transferred to qubit 2 at time $\tau = (2m+1)(\Delta+\lambda)\pi/g^2$, demonstrating that the quantum state can be transferred between qubits placed in separate cavities. In general, the quantum state fully transfers between two qubits placed in separate cavities, and this transfer process is accompanied by the excitation of cavity modes which have the same features and the maximal probability is always equal to 0.25, and it does not change with the variation of parameter in this condition.

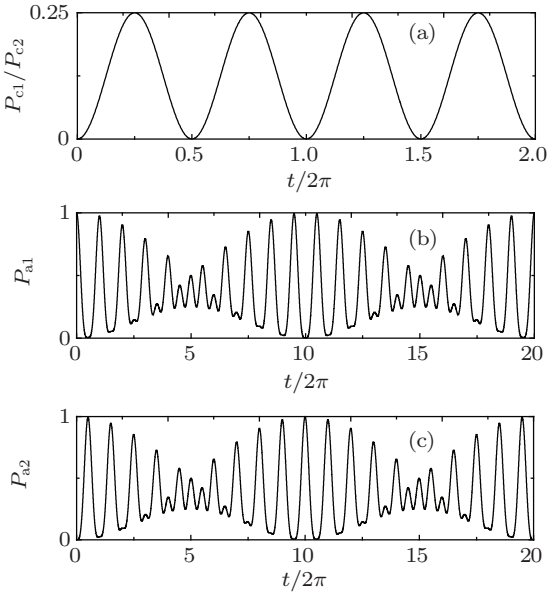



**Fig. 5.** Dynamics of system in the case of $\lambda \approx \Delta \gg g$. The parameters are $\Delta = \lambda = 10$ and $g = 1$. (a) The curves give the probabilities of field modes 1 and 2. The curves in panels (b) and (c) correspond to the probabilities of excited states of qubits 1 and 2 respectively.

## 4. Conclusions

In this paper, we investigate the one-excitation dynamics of a coupled cavities system, each cavity containing a two-level qubit with the coupling

strength $g$ and the detuning $\Delta$. The cavity–cavity interaction is controlled by the tunable hopping strength $\lambda$. The dynamics is strictly solved via Laplace transform and simple analytical solutions are obtained in several particular cases. We capture the clear physical process of quantum information transformation, and analyse the feature of dynamics and the influence of parameters on the dynamics behaviour. The present research is useful for implementing quantum logic gates and constructing quantum networks. Here, it should be pointed out that we consider only the dynamic behaviours of two coupled cavities and two-level qubits, and more unexpected phenomena should be found when more investigations are carried out on coupled cavity array and multilevel qubits.

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