Interfacial spin Hall current in a Josephson junction with Rashba spin–orbit coupling*

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We theoretically investigate the spin transport properties of the Cooper pairs in a conventional Josephson junction with Rashba spin–orbit coupling considered in one of the superconducting leads. It is found that an angle-resolved spin supercurrent flows through the junction and a nonzero interfacial spin Hall current driven by the superconducting phase difference also appears at the interface. The physical origin of this is that the Rashba spin–orbit coupling can induce a triplet order parameter in the s-wave superconductor. The interfacial spin Hall current dependences on the system parameters are also discussed.

Keywords: Josephson junction, spin Hall current, Rashba spin–orbit coupling, interfacial current

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1. Introduction

In the last two decades, spin–orbit coupling (SOC) in crystals has played an important role in spintronics because researchers have been trying to control and manipulate the spin via a purely electrical method instead of the magnetic field or magnetic materials.[1−7] The latter can inevitably give rise to a spin dephasing effect, and moreover, the derivative field is difficult to control in reality. On the contrary, SOC can be effectively modulated by an electric field, e.g., the magnitude of the Rashba-type SOC (RSOC) was found to be changed as high as 50% in experiments.[8,9] Therefore, there are tremendous research works in the literature dedicated to utilizing SOC to generate, manipulate, and detect the spin. A celebrated example is the spin Hall effect,[4−7] which is proposed in two-dimensional electron gas or p-doped semiconductors with SOC. The intrinsic spin Hall effect is referred to as a pure transverse spin current generated by a longitudinal electric field in the RSOC medium, and is regarded as an effective tool to manipulate the spin.

The intrinsic spin Hall effect is observed experimentally[10] by measuring the spin accumulation at the lateral sides of the sample. Such a spin density induced by the spin Hall current is subject to dissipative processes of spin relaxation and diffusion. Therefore, it is very interesting to investigate the spin Hall current driven by the supercurrent in a superconductor (S) junction. Mal'shukov and Chu[11,12] studied a stationary Josephson S/N/S junction with RSOC in the normal metal (N) and concluded that there was no spin Hall current driven by the superconducting phase gradient. The reason was attributed to the zero spin of the Cooper pair. However, lots of studies[13−18] demonstrated that the singlet and the triplet order parameters could mix with each other when the RSOC was considered in a singlet superconductor, e.g., the noncentrosymmetric was believed to have a triplet component with the $\boldsymbol{d}$ vector along the pseudomagnetic field of the RSOC, and this situation favored a maximum critical temperature $T_{\rm c}$.[17,18] Actually, the intensively discussed Majorana fermions[19−21] are also believed to appear in the noncentrosymmetric superconductor system, which can be obtained using a two-dimensional electron gas semiconductor with the RSOC and a proximate s-wave superconductor. Generally, when the RSOC constant is much larger than the singlet pair potential, the triplet

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order parameter must appear in the s-wave superconductor system because the momentum is correlated to the spin of the particle due to RSOC.

In our recent work,[22,23] we demonstrated that there is a transverse interfacial spin Hall current in the hybrid s-wave/p-wave Josephson junction, since both the spin states and the orbital symmetries of the Cooper pairs are different in the leads, so transverse momentum-dependent (k_y) spin accumulation can form. Therefore, it is possible to obtain an interfacial spin Hall current flowing in the s-wave/noncentrosymmetric Josephson junction, which is worth studying in detail. Moreover, Yokoyama[24] recently investigated an S/RSOC/S system and reached a contrary conclusion to that of Ref. [11], i.e., there was a stationary transverse spin Hall current flowing in the system driven by the superconducting phase difference.

In this work, we study the interfacial spin Hall current flowing in a conventional Josephson junction, in which the RSOC is considered in one of the superconducting leads so that the system can be regarded as a hybrid s-wave/noncentrosymmetric superconductor junction. By using the quantum scattering method, we find that even the triplet order parameter is not explicitly taken into account, and there is still an angle-resolved spin supercurrent flowing in the system but with a vanishing net spin supercurrent. An interfacial spin Hall current is indeed flowing at the interface of the junction and is driven by the superconducting phase gradient. Moreover, it also exhibits a non-monotonous behavior with the increase in the RSOC constant.

The paper is organized as follows. In Section 2, an S/I/S(RSOC) junction model is given, and the current formula is presented. The numerical results are obtained in Section 3, and a discussion is presented, and a conclusion is drawn in the last section.

2. Model and method

Let us consider a Josephson junction composed of two singlet superconductors. The RSOC is considered in the right lead as shown in Fig. 1. The two superconductors are separated by a thin insulating barrier at $x = 0$, which is modeled by a δ function potential, $U = U_0\delta(x)$. The left (L) and the right (R) superconductors are assumed to be identical except for the difference in the macroscopic superconducting phase. The Bogoliubov–de Gennes (BdG) equation is employed to describe the Josephson junction

$$\begin{pmatrix} H(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ -\Delta^{\dagger}(-\boldsymbol{k}) & -H^{*}(-\boldsymbol{k}) \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} = E \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix}, \quad (1)$$

where $H(\boldsymbol{k}) = H_0(\boldsymbol{k}) + H_{\rm R}(\boldsymbol{k})$, $H_0(\boldsymbol{k}) = \varepsilon_{\boldsymbol{k}} - \mu + U$, and μ is the chemical potential of the system. Here $H_{\rm R} = \lambda(\sigma_x k_y - \sigma_y k_x)\theta(x)$ describes the RSOC term with λ being the RSOC constant and $\theta(x)$ being the Heaviside step function, $\Delta(\boldsymbol{k})$ is taken in the form of

$$\Delta_j(\boldsymbol{k}) = \mathrm{i}\Delta_0\sigma_y\,\mathrm{e}^{\mathrm{i}\phi_j}, \quad (2)$$

where Δ_0 is a constant, and ϕ_j (j = L, R) is the macroscopic phase of the L (R) superconducting lead. Without loss of generalization, $\phi_{\rm L} = 0$ and $\phi_{\rm R} = \phi$ are set in the following calculations.

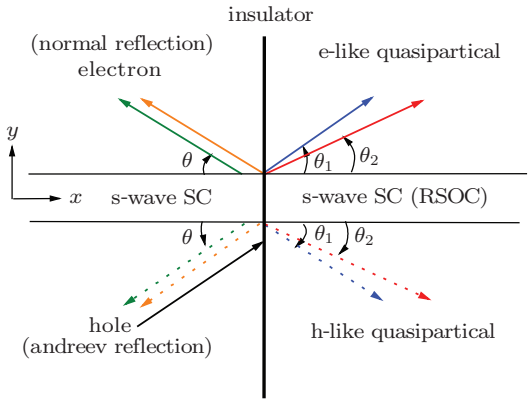


Fig. 1. (colour online) Schematic diagram of an S/I/S(RSOC) Josephson junction. A spin-up electron-like quasiparticle with incident angle θ is injected from the left superconducting lead and scattered at the interface. The scattered components include the spin-resolved normal reflections, the Andreev reflections, and the transmissions. In the left superconducting lead, the spin is degenerate, whereas the spin is split in the right superconducting lead with the RSOC, so the reflected angle is θ, and the transmitted angle is split into θ_1 and θ_2.

The σ_z-spin currents flowing along both the x and y axes can be evaluated using the Furusaki–Tsukada formalism,[23] and are respectively given by ($e = \hbar = m = 1$)

$$I_{{\rm L}x} = \frac{1}{w}\sum_{k_y}\int \frac{\mathrm{d}E}{2\pi}\frac{\Delta_0}{\Omega}[c_1 - d_2 - a_3 + b_4 + \text{c.c.}]f(E), \quad (3)$$

and

$$I_{{\rm L}y} = \frac{1}{w}\sum_{k_y}\int \frac{\mathrm{d}E}{2\pi}v_y\frac{\Delta_0}{k_x\Omega}[c_1 - d_2 + a_3 - b_4 + \text{c.c.}]f(E), \quad (4)$$

where c_1 and d_2 are, respectively, the spin-up and spin-down Andreev reflection (AR) coefficients when

an electron-like quasiparticle is incident from the left superconducting lead and scattered at the interface, a_3 and b_4 are the two spin-resolved AR coefficients for the incident hole-like quasiparticle, w denotes the width of the junction, v_y is the transverse velocity of the quasiparticle, k_x is the longitudinal momentum, and $\Omega = \sqrt{E^2 - \Delta_0^2}$. The system is assumed to have a translational symmetry along the y axis, and the transverse momentum k_y is conversed when the quasiparticle is scattered at the interface. It should be noted that only superconducting condensates are considered here to dominate the interfacial spin current $I_{\mathrm{L}y}$ in Eq. (4), while the normal reflections of electrons from the conduction band are neglected because the latter is negligibly small in the equilibrium case considered in this work.

To obtain the four AR coefficients, we need to determine the scattering wave functions in the different regions for each scattering process. For example, the process in which the spin-up electron-like quasiparticle is incident from the left superconducting lead and scattered at the interfaces is plotted in Fig. 1, and the corresponding wavefunctions in each region are given by

$$\begin{aligned}\boldsymbol{\psi}_{\mathrm{e}1}(x \le 0) = {} & \mathrm{e}^{\mathrm{i}k_y y}\left[\begin{pmatrix} u \\ 0 \\ 0 \\ v \end{pmatrix} \mathrm{e}^{\mathrm{i}k_{\mathrm{e}x}x} + a_1 \begin{pmatrix} u \\ 0 \\ 0 \\ v \end{pmatrix} \mathrm{e}^{-\mathrm{i}k_{\mathrm{e}x}x}\right. \\ & \left. + b_1 \begin{pmatrix} 0 \\ u \\ -v \\ 0 \end{pmatrix} \mathrm{e}^{-\mathrm{i}k_{\mathrm{e}x}x} + c_1 \begin{pmatrix} v \\ 0 \\ 0 \\ u \end{pmatrix} \mathrm{e}^{\mathrm{i}k_{\mathrm{h}x}x} + d_1 \begin{pmatrix} 0 \\ -v \\ u \\ 0 \end{pmatrix} \mathrm{e}^{\mathrm{i}k_{\mathrm{h}x}x}\right], \end{aligned} \tag{5}$$

$$\begin{aligned}\boldsymbol{\psi}_{\mathrm{e}1}(x \ge 0) = {} & \mathrm{e}^{\mathrm{i}k_y y}\left[e_1 \begin{pmatrix} u\,\mathrm{e}^{\mathrm{i}\phi} \\ -\mathrm{i}\,\mathrm{e}^{\mathrm{i}(\theta_1+\phi)}u \\ \mathrm{i}\,\mathrm{e}^{\mathrm{i}\theta_1}v \\ v \end{pmatrix} \mathrm{e}^{\mathrm{i}k_{1x}x} + f_1 \begin{pmatrix} u\,\mathrm{e}^{\mathrm{i}\phi} \\ \mathrm{i}\,\mathrm{e}^{\mathrm{i}(\theta_2+\phi)}u \\ -\mathrm{i}\,\mathrm{e}^{\mathrm{i}\theta_2}v \\ v \end{pmatrix} \mathrm{e}^{\mathrm{i}k_{2x}x}\right. \\ & \left. + g_1 \begin{pmatrix} v\,\mathrm{e}^{\mathrm{i}\phi} \\ -\mathrm{i}\,\mathrm{e}^{\mathrm{i}(\theta_{1\mathrm{f}}+\phi)}v \\ \mathrm{i}\,\mathrm{e}^{\mathrm{i}\theta_{1\mathrm{f}}}u \\ u \end{pmatrix} \mathrm{e}^{-\mathrm{i}k_{1x}x} + h_1 \begin{pmatrix} v\,\mathrm{e}^{\mathrm{i}\phi} \\ \mathrm{i}\,\mathrm{e}^{\mathrm{i}(\theta_{2\mathrm{f}}+\phi)}v \\ -\mathrm{i}\,\mathrm{e}^{\mathrm{i}\theta_{2\mathrm{f}}}u \\ u \end{pmatrix} \mathrm{e}^{-\mathrm{i}k_{2x}x}\right], \end{aligned} \tag{6}$$

where

$$u = \sqrt{\frac{1}{2} + \frac{\Omega}{2E}}, \quad v = \sqrt{\frac{1}{2} - \frac{\Omega}{2E}}, \tag{7}$$

$\theta_{j\mathrm{f}} = \pi - \theta_j$, scattering coefficients a_1 and c_1 denote the normal and the Andreev reflections, respectively, while b_1 and d_1 are the spin-crossed counterparts. The spin flip effect at the interface is induced by the RSOC in the right superconducting lead. In the above equations, k_{e} and k_{h} are the wavevectors of the electron-like and the hole-like quasiparticles in the left superconductor, respectively; k_1 are k_2 are the two spin resolved Fermi wavevectors of the quasiparticles in the right superconducting lead; and θ_j $(j = 1,\ 2)$ are the angles between the particle wavevectors and the x axis. The wavevectors of the particles in the two regions are expressed as $k_{\mathrm{e}} = \sqrt{k_{\mathrm{F}}^2 + 2m\Omega/\hbar^2}$, $k_{\mathrm{h}} = \sqrt{k_{\mathrm{F}}^2 - 2m\Omega/\hbar^2}$, $k_1 = \sqrt{k_{\mathrm{R}}^2 + k_{\mathrm{F}}^2} - k_{\mathrm{R}}$, and $k_2 = \sqrt{k_{\mathrm{R}}^2 + k_{\mathrm{F}}^2} + k_{\mathrm{R}}$, where $k_{\mathrm{F}} = \sqrt{2mE_{\mathrm{F}}/\hbar^2}$ and $k_{\mathrm{R}} = m\lambda/\hbar^2$ are the Fermi and the Rashba wavevectors, respectively, and E_{F} is the Fermi energy. Since k_y is conserved, the longitudinal momenta are given by $k_{\mathrm{e}x} = \sqrt{k_{\mathrm{e}}^2 - k_y^2}$, $k_{\mathrm{h}x} = \sqrt{k_{\mathrm{h}}^2 - k_y^2}$, $k_{1x} = \sqrt{k_1^2 - k_y^2}$, and $k_{2x} = \sqrt{k_2^2 - k_y^2}$. Dimensionless parameter $Z = 2mU_0/\hbar^2 k_{\mathrm{F}}$ is defined as a measurement of the insulator barrier strength, and $\beta = k_{\mathrm{R}}/k_{\mathrm{F}}$ represents the strength of the RSOC in the right superconducting re-

gion. The eight coefficients in Eqs. (5) and (6) can be determined by applying the boundary conditions[26,27] to the wave functions at $x = 0$ as follows

$$\begin{aligned}&\boldsymbol{\psi}(x)|_{x=+0} = \boldsymbol{\psi}(x)|_{x=-0},\\ &\boldsymbol{v}_x\boldsymbol{\psi}(x)|_{x=+0} - \boldsymbol{v}_x\boldsymbol{\psi}(x)|_{x=-0} = Z_0\boldsymbol{\tau}_3\boldsymbol{\psi}(0),\end{aligned} \quad (8)$$

with $Z_0 = Zk_{\rm F}/{\rm i}m$, and

$$\boldsymbol{\tau}_3 = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The velocity operator in the superconducting lead along the x direction in the boundary conditions is given by

$$\begin{aligned}\boldsymbol{v}_{jx} &= \frac{\partial H}{\hbar\partial k_x}\\ &= \begin{pmatrix} \frac{\hbar}{{\rm i}m}\frac{\partial}{\partial x} & \frac{{\rm i}\lambda}{\hbar}\theta_0(x) & 0 & 0\\ -\frac{{\rm i}\lambda}{\hbar}\theta_0(x) & \frac{\hbar}{{\rm i}m}\frac{\partial}{\partial x} & 0 & 0\\ 0 & 0 & -\frac{\hbar}{{\rm i}m}\frac{\partial}{\partial x} & -\frac{{\rm i}\lambda}{\hbar}\theta_0(x)\\ 0 & 0 & \frac{{\rm i}\lambda}{\hbar}\theta_0(x) & -\frac{\hbar}{{\rm i}m}\frac{\partial}{\partial x} \end{pmatrix}.\end{aligned} \quad (9)$$

By solving the eight linear equations, we can obtain the AR coefficient c_1 in Eq. (5). The other three scattering processes determine the corresponding AR coefficients d_2, a_3, and b_4, which together with c_1 are substituted into Eqs. (3) and (4) for computing the σ_z-spin current.

3. Numerical results and discussion

It is not wise to present the complicated expressions of the AR coefficients, so instead we present the numerical results in the following. The angle-resolved spin current $I_{{\rm L}x}$ is plotted as a function of the superconducting phase difference ϕ for different insulating barrier strengths in Figs. 2(a) and 2(c), and for different RSOC constants in Figs. 2(b) and 2(d). It is shown that a nonzero spin-resolved spin current is driven by the nonzero phase ϕ and flows in the junction. But by comparing the two opposite cases of $\theta = 0.2\pi$ and $\theta = -0.2\pi$, we find that there is no net spin supercurrent because they are exactly reversed with each other. This is also shown in Fig. 3, i.e., the angle-resolved spin current $I_{{\rm L}x}$ is an odd function of the incident angle θ, so the spin supercurrent is absent. As pointed out in Ref. [13], an applied external magnetic field can lead to a net spin supercurrent due to the Doppler shift.

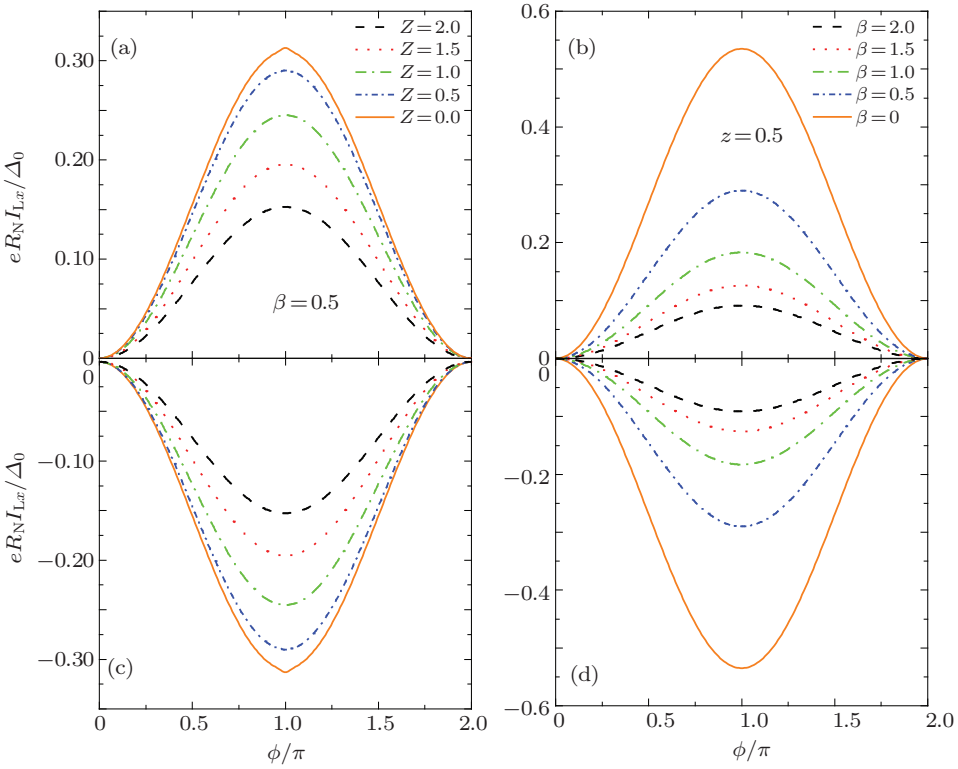


Fig. 2. (colour online) Resolved spin currents versus phase different ϕ for ((a), (b)) $\theta = 0.2\pi$ and ((c), (d)) $\theta = -0.2\pi$, where $R_{\rm N}$ is the normal resistance.

The angle-resolved spin supercurrents shown in Figs. 2 and 3 indicate that there should be a triplet superconducting component[13] in the right superconducting lead, though the triplet order parameter is not explicitly considered in the Hamiltonian of our model. As a matter of fact, the RSOC in the singlet superconductor must yield a triplet component when the Rashba energy is much larger than the singlet pair potential strength. A spin-up electron in the $\varepsilon_{+\boldsymbol{k}}$ band can not be scattered into the spin-down state in the other $\varepsilon_{-\boldsymbol{k}}$ band to form a Cooper pair, where the spin-split energy between the $\varepsilon_{+\boldsymbol{k}}$ and the $\varepsilon_{-\boldsymbol{k}}$ bands is induced by the RSOC. As a result, the pairing state in the $\varepsilon_{+\boldsymbol{k}}$ or $\varepsilon_{-\boldsymbol{k}}$ band should be a mixture of the spin-singlet and the spin-triplet states. This is the reason why there is a nonzero angle-resolved spin supercurrent even when only the RSOC is considered in the right superconducting lead.

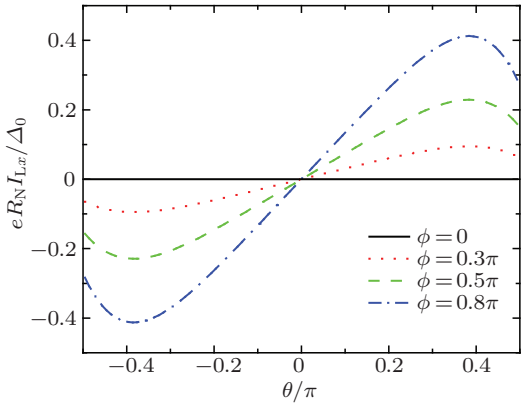


Fig. 3. (colour online) Resolved spin currents versus angle θ with $\beta = 0.5$ and $Z = 0.5$.

Since a triplet order parameter ingredient exists in the right superconducting lead, the S/I/S(RSOC) Josephson junction can be regarded as a hybrid s-wave/noncentrosymmetric superconducting junction. Therefore, it is very probable that a nonzero transverse spin Hall current flows at the interface of the junction. With Eq. (4), the calculated spin Hall current $I_{\rm Ly}$ is plotted against phase ϕ in Fig. 4, where $I_{\rm Ly}$ displays a sine function of ϕ like the supercurrent. This can be understood from the spin accumulation forming at the hybrid s-wave/p-wave junction found in Refs. [28] and [29]. Since the induced triplet order parameters in the right superconducting lead are transverse momentum k_y dependent, the spin polarizations of the spin densities are opposite for the k_y and $-k_y$ modes. Therefore, there is a transverse spin Hall current at the interface, and it should attenuate from the interface into the bulk in a superconducting coherence length.

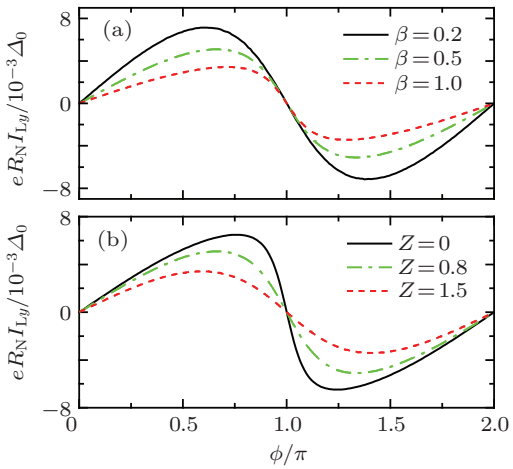


Fig. 4. (colour online) Spin currents versus macroscopic phase ϕ for (a) different β with $Z = 0.8$ and (b) different Z with $\beta = 0.5$.

In Fig. 5, the spin Hall current $I_{\rm Ly}$ is plotted as a function of the RSOC constant β, as well as the insulating barrier strength Z. It is shown that the barrier can definitely suppress the spin current $I_{\rm Ly}$ and make the $I_{\rm Ly}$–ϕ curve exhibit a perfect sine-function behavior, as shown in Fig. 4(a). In contrast to the barrier dependence, the RSOC β impact on $I_{\rm Ly}$ is not a monotonic enhancement behavior, as shown in Fig. 5(b). The $I_{\rm Ly}$ increases rapidly and then gradually decreases as β increases, so there is an optimal value of β for the interfacial spin Hall current. At very small β, the triplet component may not form in the right superconducting lead as discussed earlier unless the Rashba energy exceeds the singlet pair potential.

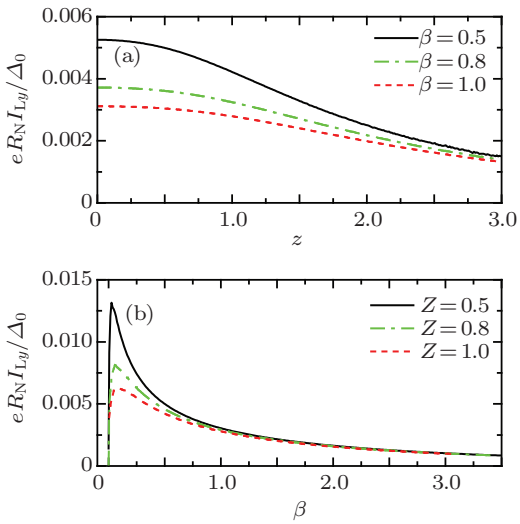


Fig. 5. (colour online) Spin currents versus (a) Z and (b) β with $\phi = 0.5\pi$.

When the triplet component appears, $I_{\mathrm{L}y}$ will significantly increase from zero, and with the further increase in β, the mismatch between the two superconducting leads will increase greatly, which will suppress the AR coefficients, so the spin Hall current will decrease significantly, as shown in Fig. 5(b).

4. Conclusion

In summary, we have investigated the spin transport of the Cooper pair in an s-wave Josephson junction with RSOC considered in one of the superconducting leads. It is found that though the triplet order parameter is not explicitly taken into account in the junction, an angle-resolved spin supercurrent is flowing through the junction, and a nonzero interfacial spin Hall current is also flowing transversely along the interface of the junction. The reason for this is attributed to the fact that the RSOC can induce a triplet order parameter in the s-wave superconductor, so the Josephson junction contains the interplay of the singlet and the triplet order parameters, which gives rise to the interfacial spin Hall current.

References

[1] Zütic J, Fabian J and Sarma S D 2004 *Rev. Mod. Phys.* **76** 323

[2] Wolf S A, Awschalom D D, Buhrman R A, Daughton J M, von Molnar S, Roukes M L, Chtchelkanova A Y and Treger D M 2001 *Science* **294** 1488

[3] Awschalom D D, Loss D and Samarth N 2002 *Semiconductor Spintronics and Quantum Computation* (Berlin: Springer)

[4] Murakami S, Nagaosa N and Zhang S C 2003 *Science* **301** 1348

[5] Sinova J, Culcer D, Niu Q, Sinitsyn N A, Jungwirth T and MacDonald A H 2004 *Phys. Rev. Lett.* **92** 126603

[6] Yan Y Z, Li H W and Hu L B 2009 *Chin. Phys.* B **18** 2981

[7] Li D F and Shi J R 2009 *Chin. Phys.* B **18** 282

[8] Nitta J, Akazaki T, Takayanagi H and Enoki T 1997 *Phys. Rev. Lett.* **78** 1335

[9] Koga T, Nitta J, Akazaki T and Takayanagi H 2002 *Phys. Rev. Lett.* **89** 046801

[10] Wunderlich J, Kaestner B, Sinova J and Jungwirth T 2005 *Phys. Rev. Lett.* **94** 047204

[11] Mal'shukov A G and Chu C S 2008 *Phys. Rev.* B **78** 104503

[12] Mal'shukov A G and Chu C S 2011 *Phys. Rev.* B **84** 054520

[13] Tanaka Y, Yokoyama Y, Balatsky A V and Nagaosa N 2009 *Phys. Rev.* B **79** 060505R

[14] Sato M 2006 *Phys. Rev.* B **73** 214502

[15] Sato M and Fujimoto S 2009 *Phys. Rev.* B **79** 094504

[16] Fujimoto S 2007 *J. Phys. Soc. Jpn.* **76** 051008

[17] Frigeri P A, Agterberg D F, Koga A and Sigrist M 2004 *Phys. Rev. Lett.* **92** 097001

[18] Frigeri P A, Agterberg D F and Sigrist M 2004 *New J. Phys.* **6** 115

[19] Sau J D, Tewari S, Lutchyn R M, Stanescu T D and Sarma S D 2010 *Phys. Rev.* B **82** 214509

[20] Sau J D, Lutchyn R M, Tewari S and Sarma S D 2010 *Phys. Rev.* B **82** 094522

[21] Alicea J 2010 *Phys. Rev.* B **81** 125318

[22] Wang J and Chan K S 2010 *J. Phys.*: *Condens. Matter* **22** 225701

[23] Yang Z H, Wang J and Chan K S 2011 *J. Phys.*: *Condens. Matter* **23** 085701

[24] Takehito Y 2011 arXiv: 1107.4202v2

[25] Furusaki A and Tsukada M 1991 *Sol. Sta. Com.* **78** 299

[26] Yokoyama T, Tanaka Y and Inoue J 2006 *Phys. Rev.* B **74** 035318

[27] Yang Z H and Yang Z H 2010 *Europhys. Lett.* **92** 17001

[28] Sengupta K and Yakovenko V M 2008 *Phys. Rev. Lett.* **101** 187003

[29] Lu C K and Yip S 2009 *Phys. Rev.* B **80** 024504