Effect of multicomponent dust grains in a cold quantum dusty plasma*

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By employing the quantum hydrodynamic model for electron–ion–dust plasma, we derive a dispersion relation of the quantum dusty plasma. The effects of the dust size distribution on the dispersion relation in a cold quantum dusty plasma are studied. Both analytical and numerical results are given to compare the differences between the dusty plasma by considering the dust size distribution and the mono-sized dusty plasma. It is shown that many system parameters can significantly influence the dispersion relation of the quantum dusty plasma.

Keywords: quantum dusty plasma, dispersion relation, dust size distribution

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1. Introduction

Dusty plasma is an ionized gas containing small particles of solid matter, each of which acquires a large electric charge by collecting electrons and ions from the plasma, which has been observed widely in planetary rings, asteroid zones, cometary tails, the magnetosphere as well as the lower part of the earth's atmosphere.[1,2] In the laboratory the dust grains exist as impurities and significantly influence the behaviour of the surrounding plasma.[3,4] Rao *et al.*[5]have predicted the existence of dust acoustic waves (DAWs) in the unmagnetized dusty plasma, and the DAWs were observed in experiments.[6−8] Since then there has been much research on dusty plasmas both theoretically and experimentally.

However, most of the theories have focused on dusty plasma containing only monosized dust grains.[2] In a real case, the dust grains have many different species with different dust size distributions both in the space plasmas and in the laboratory plasmas.[9−13]

The values of dust size (r) are usually in a certain range ($r_{\min}, r_{\max}$), where $r_{\min}$ is its minimal value and $r_{\max}$ is its maximal value. The dust size distribution (DSD) can usually be described by a power law distribution (PLD) in space plasmas (e.g., F-ring and G-ring of Saturn)[9−11] or by a Gaussian distribution in laboratory plasmas (e.g., in dusty crystals and charging experiments).[12] The effects of dust size distribution on the dust acoustic wave have previously been studied by several researchers.[14−19]

Aslaksen and Havnes[20] studied the effect of DSD on the thickness of a planetary ring. Brattli *et al.*[14] investigated the effect of DSD on the damping rate of DAW. Meuris[21] found an increase of plasma frequency when DSD is considered in comparison with the monosized dust grain case. Duan and Parkes[18] considered the effect of DSD on DAW in a magnetized dusty plasma and found that both soliton velocity and its width are larger than those of the monosized dust grain case, but their magnitudes are smaller.

Recently a great deal of attention has been paid to quantum effects in dusty plasmas.[22−25] When a dusty plasma is cooled down to an extremely low temperature so that the de-Broglie wavelength of the charge carriers is comparable to the dimension of the systems, then the ultracold dusty plasma behaves like a Fermi gas and quantum mechanical effects are ex-

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pected to play a central role in the behaviour of charged dust particles. The thermal de-Broglie wavelength for the jth species is $\lambda_{\mathrm{B}j} = \hbar/m_j V_{\mathrm{T}j} = a_j/\lambda_{\mathrm{D}j}$, where $a_j = \hbar/2q_j\sqrt{\pi m_j n_{j0}}$, $V_{\mathrm{T}j}$ is the thermal velocity, $\lambda_{\mathrm{D}j}$ is the Debye length, and q_j is the charge $(j = 1, 2, \ldots, N)$, where we assume that there are N different species of dust particles. For the classical regime, we assume $\lambda_{\mathrm{B}j} \ll \lambda_{\mathrm{D}j}$. That is why the charged particles are assumed to be point-like. But in the quantum regime, when $\lambda_{\mathrm{B}j} \gg \lambda_{\mathrm{D}j}$, the quantum effects become more important and cannot be ignored any more.[22–26] The important role of quantum mechanical effects in plasmas has been recognized in microelectronic devices,[27] dense astrophysical systems,[28] metallic nanostructures,[29] and laser plasmas.[30] In microelectronic mechanical systems, the presence of charged dust impurities can affect the system. Mathematically, the quantum hydrodynamic (QHD)[31] model generalizes the fluid model for plasmas, owing to the inclusion of a quantum correction term, the so-called Bohm potential.

Shukla[26] presented a new dust mode in a cold quantum plasma and studied the dispersion relation of quantum DAWs, but the DSD has not been considered. In this paper, we study the dispersion relation by considering the different dust size distributions and adiabatic dust charge variation in an unmagnetized, collisionless, ultracold quantum dusty plasma presented by Shukla.[26] It is well studied in this paper that the DSD can significantly affect the dispersion relation of this system.

The rest of the present paper is organized as follows. In Section 2, the basic set of QHD equations are presented. In Section 3, the dispersion relation is studied. In Section 4, the conclusion is presented.

2. Basic equation

In this paper, we present a dust mode in a cold quantum plasma. Our study is based on the QHD model consisting of the inertialess electron and ions[32]

$$0 = e\frac{\partial \phi'}{\partial x'} + \frac{\hbar^2}{4m_{\mathrm{e}}n_{\mathrm{e}0}}\frac{\partial^3 n'_{\mathrm{e}1}}{\partial x'^3},$$

$$0 = -e\frac{\partial \phi'}{\partial x'} + \frac{\hbar^2}{4m_{\mathrm{i}}n_{\mathrm{i}0}}\frac{\partial^3 n'_{\mathrm{i}1}}{\partial x'^3},$$

continuity and momentum equations for negatively charged dust particulates[33]

$$\frac{\partial n'_{\mathrm{d}1j}}{\partial t'} + n'_{\mathrm{d}0j}\frac{\partial u'_{\mathrm{d}j}}{\partial x'} = 0,$$

$$m'_{\mathrm{d}j}\frac{\partial u'_{\mathrm{d}j}}{\partial t'} = Z'_{\mathrm{d}j}e\frac{\partial \phi'}{\partial x'},$$

and Poisson's equation

$$\frac{\partial^2 \phi'}{\partial x'^2} = 4\pi e\left(n'_{\mathrm{e}1} + \sum_{j=1}^{N} Z'_{\mathrm{d}j}n'_{\mathrm{d}1j} - n'_{\mathrm{i}1}\right),$$

where e is the magnitude of the electron charge; ϕ is the electric potential; $\hbar$ is the Planck constant divided by 2π; m_{e}, m_{i} and $m_{\mathrm{d}j}$ are the mass of elctron, ion, and the jth dust grain $(j = 1, 2, \ldots, N)$, respectively; $n_{\mathrm{e}1}$, $n_{\mathrm{i}1}$, and $n_{\mathrm{d}1j}$ are the small electron, ion, and the jth dust grain number density perturbations around their equilibrium values $n_{\mathrm{e}0}$, $n_{\mathrm{i}0}$, and $n_{\mathrm{d}0j}$; $u_{\mathrm{d}j}$ is the jth dust grain velocity; $Z_{\mathrm{d}j}$ is the number of the electrons residing on dust.

We now introduce the normalization: $u_{\mathrm{d}j} = u'_{\mathrm{d}j}/c_{\mathrm{d}}$, $n_{\mathrm{d}j0} = n'_{\mathrm{d}j0}/N_{\mathrm{tot}}$, $t = t'\omega_{\mathrm{pd}}$, $Z_{\mathrm{d}j} = Z'_{\mathrm{d}j}/\bar{z}_{\mathrm{d}0}$, $m_{\mathrm{d}j} = m'_{\mathrm{d}j}/\bar{m}_{\mathrm{d}}$, $\phi = e\phi'/B_0$, $n_j = n'_j/n_{j0}$, $(j = e, i)$, $B_0 = {c_{\mathrm{d}}}^2\overline{m}_{\mathrm{d}}/\overline{z}_{\mathrm{d}0}$, $c_{\mathrm{d}} = \sqrt{(2\bar{z}_{\mathrm{d}o}K_{\mathrm{B}}T_{\mathrm{Fi}})/\bar{m}_{\mathrm{d}}}$, $\omega_{\mathrm{pd}} = \sqrt{(4\pi\bar{z}^2_{\mathrm{d}o}e^2N_{\mathrm{tot}})/\bar{m}_{\mathrm{d}}}$, $\lambda_D = \sqrt{(2K_{\mathrm{B}}T_{\mathrm{Fi}}/4\pi\bar{z}_{\mathrm{d}0}e^2N_{\mathrm{tot}})}$ is the Debye length, where K_{B} is the Boltzmann constant and T_{Fi} is the Fermi temperature of the ion. Therefore we obtain the following dimensionless equations:

$$0 = \frac{\partial \phi}{\partial x} + \frac{1}{4}{H_{\mathrm{e}}}^2\frac{\partial^3 n_{\mathrm{e}1}}{\partial x^3}, \tag{1}$$

$$0 = -\frac{\partial \phi}{\partial x} + \frac{1}{4}{H_{\mathrm{i}}}^2\frac{\partial^3 n_{\mathrm{i}1}}{\partial x^3}, \tag{2}$$

$$\frac{\partial n_{\mathrm{d}1j}}{\partial t} + n_{\mathrm{d}0j}\frac{\partial u_{\mathrm{d}j}}{\partial x} = 0, \tag{3}$$

$$m_{\mathrm{d}j}\frac{\partial u_{\mathrm{d}j}}{\partial t} = Z_{\mathrm{d}j}\frac{\partial \phi}{\partial x}, \tag{4}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \sum_{j=1}^{N} Z_{\mathrm{d}j}n_{\mathrm{d}1j} + \mu_{\mathrm{e}}n_{\mathrm{e}1} - \nu_{\mathrm{i}}n_{\mathrm{i}1}, \tag{5}$$

where $\mu_{\mathrm{e}} = n_{\mathrm{e}0}/\overline{z}_{\mathrm{d}0}N_{\mathrm{tot}}$, $\nu_{\mathrm{i}} = n_{\mathrm{i}0}/\overline{z}_{\mathrm{d}0}N_{\mathrm{tot}}$, $\mu_{\mathrm{e}} = 1/(\mu - 1)$, $\nu_{\mathrm{i}} = \mu/(\mu - 1)$. $H_{\mathrm{e}} = \sqrt{(\overline{z}_{\mathrm{d}0}\hbar^2\omega^2_{\mathrm{pd}})/m_e\overline{m}_{\mathrm{d}}c^4_{\mathrm{d}}}$, and $H_{\mathrm{i}} = \sqrt{(\overline{z}_{\mathrm{d}0}\hbar^2\omega^2_{\mathrm{pd}})/m_{\mathrm{i}}\overline{m}_{\mathrm{d}}c^4_{\mathrm{d}}}$ are the quantum parameters for electron and ion, respectively.

Equations (1) and (2) can be rewritten into

$$\frac{\partial^2 n_{\mathrm{e}1}}{\partial x^2} = -\frac{4}{{H_{\mathrm{e}}}^2}\phi, \tag{6}$$

$$\frac{\partial^2 n_{\mathrm{i}1}}{\partial x^2} = \frac{4}{{H_{\mathrm{i}}}^2}\phi. \tag{7}$$

From Eqs. (3) and (4) we have

$$\frac{\partial^2 n_{\mathrm{d}j}}{\partial t^2} + \frac{Z_{\mathrm{d}0j} n_{\mathrm{d}0j}}{m_{\mathrm{d}j}} \frac{\partial^2 \phi}{\partial x^2} = 0. \tag{8}$$

From Eqs. (5)–(7) we have

$$\sum_{j=1}^{N} Z_{\mathrm{d}j} \frac{\partial^2 n_{\mathrm{d}j}}{\partial x^2} - \frac{\partial^4 \phi}{\partial x^4} - 4\left(\frac{\mu_{\mathrm{e}}}{{H_{\mathrm{e}}}^2} + \frac{\nu_{\mathrm{i}}}{{H_{\mathrm{i}}}^2}\right)\phi = 0. \tag{9}$$

Combing Eqs. (8), and (9) we have

$$\frac{\partial^6 \phi}{\partial x^4 \partial t^2} + 4\left(\frac{\mu_{\mathrm{e}}}{{H_{\mathrm{e}}}^2} + \frac{\nu_{\mathrm{i}}}{{H_{\mathrm{i}}}^2}\right)\frac{\partial^2 \phi}{\partial t^2} + \sum_{j=1}^{N} \frac{Z_{\mathrm{d}j}^2 n_{\mathrm{d}0j}}{m_{\mathrm{d}j}} \frac{\partial^4 \phi}{\partial x^4} = 0. \tag{10}$$

3. Dispersion relation

We now study the dispersion relation of the system by assuming that ϕ is proportional to $\exp(\mathrm{i}kx - \mathrm{i}\omega t)$, where k and ω are the wave number and the frequency, respectively. Substituting it into Eq. (10), we obtain

$$\omega^2 = \frac{k^4 \sum_{j=1}^{N} Z_{\mathrm{d}j}^2 n_{\mathrm{d}0j}/m_{\mathrm{d}j}}{k^4 + 4(\mu_{\mathrm{e}}/{H_{\mathrm{e}}}^2 + \nu_{\mathrm{i}}/{H_{\mathrm{i}}}^2)}. \tag{11}$$

The charge $Z_{\mathrm{d}j}$ and the mass $m_{\mathrm{d}j}$ of the jth dust grain can be given as follows:[21] $Z_{\mathrm{d}j} = k_{\mathrm{z}} r_j^{\gamma}$, $m_{\mathrm{d}j} = k_{\mathrm{m}} r_j^3$, where r_j is the radius of jth dust grain; $k_{\mathrm{z}} = 4\pi\epsilon_0 V_0/e$; $k_{\mathrm{m}} = 4\pi\rho_d/3$; γ is a constant that depends on the dust parameters and other conditions, and satisfies $1 \le \gamma \le 2$; ε_0 is the vacuum permittivity of free space; V_0 is the electric surface potential at equilibrium; ρ_{d} is the mass density of the dust grain, which is assumed to be constant and equal for all dust grains; $n_{\mathrm{d}0j}$ is the dust size distribution function (DSDF). We will study two typical DSDFs which are the polynomial expressed distribution and power law distribution. From the above we obtain

$$\omega^2 = \frac{k^4 {k_{\mathrm{z}}}^2}{[k^4 + 4(\mu_{\mathrm{e}}/{H_{\mathrm{e}}}^2 + \nu_{\mathrm{i}}/{H_{\mathrm{i}}}^2)]k_{\mathrm{m}}} \times \int_{r_{\min}}^{r_{\max}} \frac{n_{\mathrm{d}}(r)}{r^{3-2\gamma}}\,\mathrm{d}r, \tag{12}$$

where $n_{\mathrm{d}}(r)$ is the dust size distribution function. Equation (**??**) is an implicit function of relation between frequency ω and wave number k. We can find the dispersion relation of $\omega = \omega(k)$ by numerical simulations. In our numerical simulation, we cite the typical parameters reported previously:[24,34–37] $n_{\mathrm{i}0} = 2.0 \times 10^{30}$ m^{-3}, $n_{\mathrm{e}0} = 5 \times 10^{29}$ m^{-3}, $Z_{\mathrm{d}0} = 10^3$, $m_{\mathrm{i}} \doteq m_p$, $T_{\mathrm{Fe}} = 100$ K, $m_{\mathrm{d}} = 10^{-17}$ kg, $\rho_{\mathrm{d}} \sim 1$ g/cm^3, $a \sim 0.1 - 1$ μm, $K = 10^{-6}$, $k_{\mathrm{z}} = 1$, $k_{\mathrm{m}} = 4$, $\mu = 4$, and the corresponding values are $H_{\mathrm{e}} = 0.2782$ and $H_{\mathrm{i}} = 6.494 \times 10^{-3}$.

3.1. Polynomial expressed DSD

We consider a polynomial expressed distribution of dust particles in quantum dusty plasma. For dust grains with radius r in a given range $[r_{\min}, r_{\max}]$, the differential polynomial expressed distribution function is in the form of[38]

$$n_{\mathrm{d}}(r)\mathrm{d}r = [a_0 + a_1 r + a_2 r^2 + a_3 r^3 + \cdots]\mathrm{d}r, \tag{13}$$

which satisfies the following equation:

$$N_{\mathrm{tot}} = \int_{r_{\min}}^{r_{\max}} n_{\mathrm{d}}(r)\mathrm{d}r,$$

where r is the radius of the dust grain, $n_{\mathrm{d}}(r)\mathrm{d}r$ is the number density of the dust grains per unit volume with radii in the range from r to $r + \mathrm{d}r$, a_j $(j = 0, 1, \ldots)$ are all constants, N_{tot} is the total number density of dust grains. Outside the limits $r < r_{\min}$ and $r > r_{\max}$, we use $n_{\mathrm{d}}(r) = 0$.

If $\gamma \neq 1$, we obtain the frequency of quantum DAWs as follows:

$$\omega^2 = \frac{k^4 {k_{\mathrm{z}}}^2}{[k^4 + 4(\mu_{\mathrm{e}}/{H_{\mathrm{e}}}^2 + \nu_{\mathrm{i}}/{H_{\mathrm{i}}}^2)]k_{\mathrm{m}}} \times \left[\frac{a_0}{2\gamma - 2}(r_{\max}^{2\gamma-2} - r_{\min}^{2\gamma-2}) + \frac{a_1}{2\gamma - 1}(r_{\max}^{2\gamma-1} - r_{\min}^{2\gamma-1}) + \frac{a_2}{2\gamma}(r_{\max}^{2\gamma} - r_{\min}^{2\gamma}) + \frac{a_3}{2\gamma + 1}(r_{\max}^{2\gamma+1} - r_{\min}^{2\gamma+1}) + \cdots\right]. \tag{14}$$

The following are our numerical results. Figures 1 and 2 show how the polynomial expressed dust distribution function affects the dispersion relation of the system. For generality and simplicity, we first consider the simplest case of $n_{\mathrm{d}}(r) = a_0$, $a_1 = a_2 = a_3 = 0$. We note that ω depends on the parameters a_0, k, and γ. It is noted from Fig. 1 that as a_0 increases, ω increases. Since a_0 denotes the magnitude of the total number density of dust particles, which should

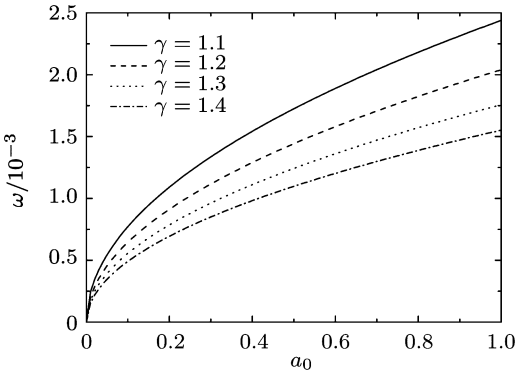


Fig. 1. Variation of ω with a_0 for different γ values. $H_{\rm e} = 0.2782$, $H_{\rm i} = 6.494 \times 10^{-3}$, $k_{\rm z} = 1$, $k_{\rm m} = 4$, $k = 1$, $\mu = 4$, $r_{\min} = 0.01$, $r_{\max} = 1$, $a_1 = a_2 = a_3 = \cdots = 0$.

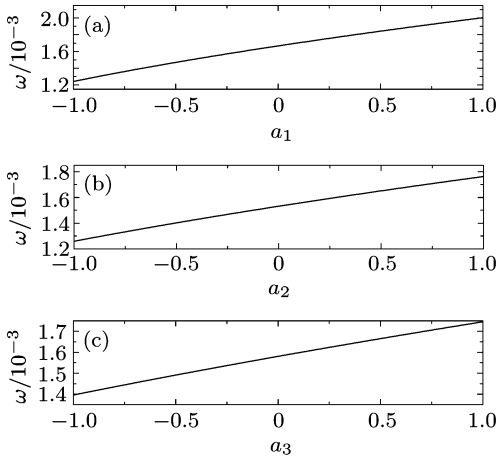


Fig. 2. (a) Variation of ω with a_1 for $k_{\rm z} = 1$, $k_{\rm m} = 4$, $k = 1$, $\gamma = 1.3$, $a_0 = 0.9$, $r_{\min} = 0.01$, and $r_{\max} = 1$; (b) variation of ω with a_2 for $k_{\rm z} = 1$, $k_{\rm m} = 4$, $k = 1$, $\gamma = 1.3$, $a_0 = 0.9$, $a_1 = -0.35$, $r_{\min} = 0.01$, and $r_{\max} = 1$; (c) variation of ω with respect to a_3 for $k_{\rm z} = 1$, $k_{\rm m} = 4$, $k = 1$, $\gamma = 1.3$, $a_0 = 0.9$, $a_1 = -0.35$, $a_2 = 0.2$, $r_{\min} = 0.01$, and $r_{\max} = 1$. All other parameters are the same as those in Fig. 1.

always be positive, we conclude that as the total number density of the dust particles increases the frequency increases. We also note that ω decreases as the parameter γ increases.

The effects of a_1, a_2, and a_3 on ω are shown in Fig. 2. First, we let $n_{\rm d}(r) = a_0 + a_1 r$, $a_2 = a_3 = a_4 = 0$. It is noted that the dispersion relation ω is a function of a_0 and a_1. It can be seen from Fig. 2(a) that as a_1 increases, ω increases. As is well known,[38] if $a_1 < 0$ (the usual case both in space plasmas and in laboratory plasmas), the number density of larger dust particles is smaller than that of smaller dust particles. On the other hand, if $a_1 > 0$, the number density of larger dust particles is greater than that of smaller dust particles. Therefore, we conclude that for the usual dusty plasma the frequency is smaller than that in the unusual case. This result is similar to that for classical dusty plasma when the quantum effects is negligible.[38]

Next, we let $n_{\rm d}(r) = a_0 + a_1 r + a_2 r^2$. We find from Eq. (14) that ω is a function of a_0, a_1, and a_2, with $a_0 = 0.9$ and $a_1 = -0.35$. The numerical results are shown in Fig. 2(b). It indicates that ω increases as a_2 increases. Similarly, we find from Fig. 2(c) that ω increases with a_3.

In Fig. 3, the dependences of ω on the system parameters $R = r_{\max}/r_{\min}$ and γ are given. We note that ω increases with R, but decreases with γ.

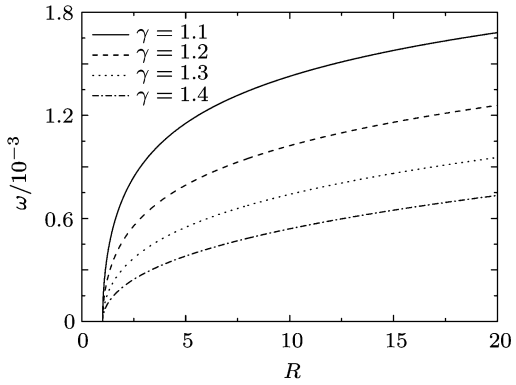


Fig. 3. Variation of ω against R for different values of γ. $H_{\rm e} = 0.2782$, $H_{\rm i} = 6.494 \times 10^{-3}$, $k_{\rm z} = 1$, $k_{\rm m} = 4$, $\mu = 4$, $r_{\min} = 0.01$, $a_0 = 0.9$, $a_1 = -0.35$, $a_2 = 0.2$, $a_3 = a_4 = \cdots = 0$.

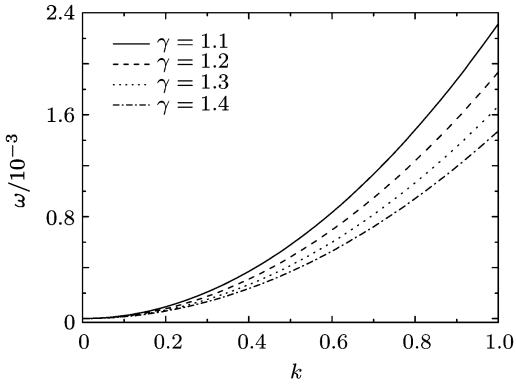


Fig. 4. Variation of ω against k for different values of γ. $H_{\rm e} = 0.2782$, $H_{\rm i} = 6.494 \times 10^{-3}$, $k_{\rm z} = 1$, $k_{\rm m} = 4$, $\mu = 4$, $r_{\min} = 0.01$, $r_{\max} = 1$, $a_0 = 0.9$, $a_1 = a_2 = \cdots = 0$.

Figures 4 and 5 show the curves of dispersion relation $\omega = \omega(k)$ of the system for different values of γ and quantum parameter $H_{\rm i}$, respectively. We find that ω increases with k if k is large enough. If k is small enough, ω is nearly independent of both γ and $H_{\rm i}$, while for large k it depends on both γ and $H_{\rm i}$. It decreases with γ, but increases with $H_{\rm i}$.

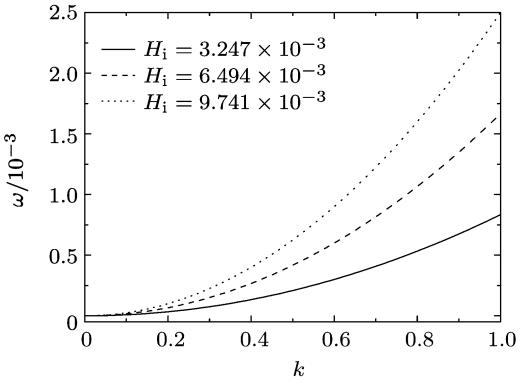


Fig. 5. Variation of ω against k for different values of H_i. $\gamma = 1.3$, $H_e = 0.2782$, $k_z = 1$, $k_m = 4$, $\mu = 4$, $r_{min} = 0.01$, $r_{max} = 1$, $a_0 = 0.9$, $a_1 = a_2 = \cdots = 0$.

3.2. Power law DSD

It has been widely accepted in space plasmas that the dust size distribution is given by a power law distribution. The differential form of the power law distribution can be written as [21]

$$n_d(r)\mathrm{d}r = Kr^{-\beta}\mathrm{d}r, \tag{15}$$

where $n_d(r)\mathrm{d}r$ is the number of the dust grains in a range from r to $r + \mathrm{d}r$, K is the normalization constant, and β is the power law index. The total number of all grains in a range of $[r_{min}, r_{max}]$ is given by

$$N_{tot} = \int_{r_{min}}^{r_{max}} n_d(r)\mathrm{d}r = \frac{K}{1-\beta}(r_{max}^{1-\beta} - r_{min}^{1-\beta}), \tag{16}$$

substituting Eq. (16) into Eq. (12), for the power law distribution, we obtain the dispersion relation of the system as follows:

$$\omega^2 = \frac{k^4 k_z^2 K}{k_m(2\gamma - \beta - 2)} \times \frac{r_{max}^{2\gamma-\beta-2} - r_{min}^{2\gamma-\beta-2}}{k^4 + 4(\mu_e/H_e^2 + \nu_i/H_i^2)}. \tag{17}$$

For the monosized dust grains case, all the grains have the same size, which equals to the mean radius $\overline{r}$, then the corresponding frequency is $\overline{\omega}$. $\overline{r}$ and $\overline{\omega}^2$ can be given as follows:

$$\overline{r} = \frac{\int r n_d(r)\mathrm{d}r}{N_{tot}} = \frac{1-\beta}{2-\beta}\left(\frac{r_{max}^{2-\beta} - r_{min}^{2-\beta}}{r_{max}^{1-\beta} - r_{min}^{1-\beta}}\right), \tag{18}$$

$$\overline{\omega}^2 = \frac{k^4 {k_z}^2}{[k^4 + 4(\mu_e/{H_e}^2 + \nu_i/{H_i}^2)]k_m} \frac{N_{tot}}{\overline{r}^{3-2\gamma}}. \tag{19}$$

we obtain from Eqs. (17) and (19) that

$$\frac{\omega^2}{\overline{\omega}^2} = \frac{(\beta-2)^{2\gamma-3}(1-R^{2\gamma-\beta-2})(1-R^{1-\beta})^{2\gamma-4}}{(\beta+2-2\gamma)(\beta-1)^{2\gamma-4}(1-R^{2-\beta})^{2\gamma-3}}, \tag{20}$$

where $R = r_{max}/r_{min}$. We can find from Eq. (17) that ω is a function of k, β, and γ. We choose the typical parameters reported previously [24,34–37] and study the effects of these parameters on ω.

The variation of ω with respect to R for different values of γ and β are given in Figs. 6 and 7, respectively. It is noted from Fig. 6 that as R increases, ω increases, while ω decreases as γ increases. This result is qualitatively the same as that in Fig. 3 in which the dust size distribution is the polynomial expressed DSD. We also note from Fig. 7 that ω increases as β increases.

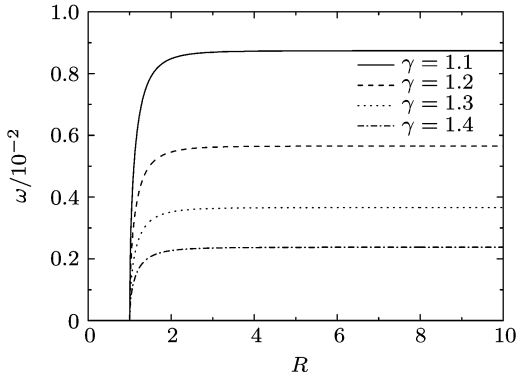


Fig. 6. Variation of ω with R for different values of γ. $K = 10^{-6}$, $H_e = 0.2782$, $H_i = 6.494 \times 10^{-3}$, $k_z = 1$, $k_m = 4$, $k = 1$, $\mu = 4$, $r_{min} = 0.01$, and $\beta = 4.3$.

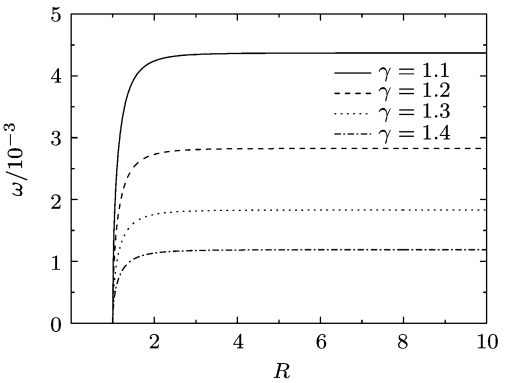


Fig. 7. Variation of ω with R for different values of β. $K = 10^{-6}$, $H_e = 0.2782$, $H_i = 6.494 \times 10^{-3}$, $k_z = 1$, $k_m = 4$, $k = 1$, $\mu = 4$, and $\gamma = 1.3$.

Figure 8 shows the dependences of $\omega^2/\overline{\omega}^2$ on the parameter $R = r_{max}/r_{min}$ for different values of β ($\gamma = 1.3$). As observed, the curves are remarkably similar to each other. We note that $\omega^2/\overline{\omega}^2$ increases with R, but decreases with β. Noting that $\omega^2/\overline{\omega}^2 > 1$, we can conclude that the frequency of the system is greater than that of the mono-sized dusty plasma, in view of the dust size distribution.

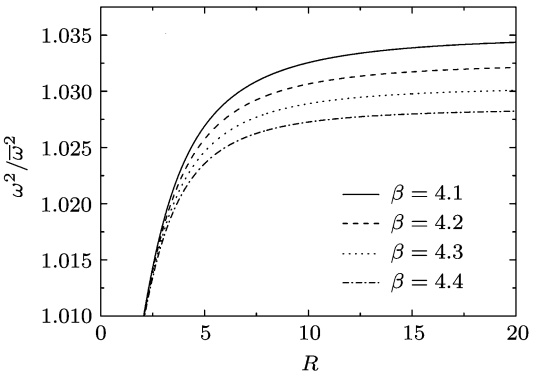


Fig. 8. Variation of $\omega^2/\overline{\omega}^2$ with R for different values of β. $\gamma = 1.3$.

Figure 9 also shows the dependence of $\omega^2/\overline{\omega}^2$ on the parameter $R = r_{\max}/r_{\min}$, with different values of γ ($\beta = 4.3$). It can be seen from Fig. 9 that if $\gamma < 1.5$, then $\omega^2/\overline{\omega}^2 > 1$. It means that the frequency of the system is larger than that of mono-sized dusty plasma if $\gamma < 1.5$. We also note that $\omega^2/\overline{\omega}^2$ increases with R, but decreases with γ. When $\gamma = 1.5$, then $\omega^2/\overline{\omega}^2 = 1$, this means at this point the frequency of the system with the dust size distribution considered is just the same as that of the mono-sized dusty plasma. However, if $\gamma > 1.5$, then $\omega^2/\overline{\omega}^2 < 1$, indicating that the frequency of the power law distribution dusty plasma in this case is smaller than that with the mono-sized dusty plasma.

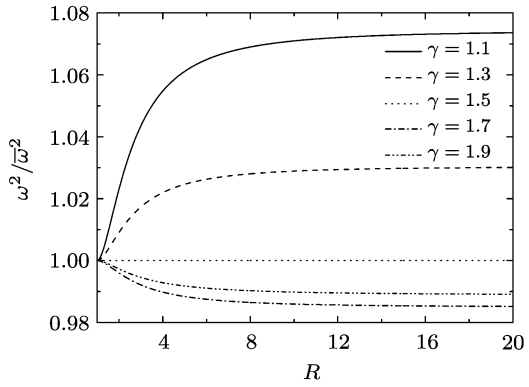


Fig. 9. Variation of $\omega^2/\overline{\omega}^2$ with R for different values of γ. $\beta = 4.3$.

Figures 10–13 show the variation of the frequency of each system with the wave number k for different values of β, γ, $k_{\rm m}$, and $H_{\rm i}$ respectively. We note that ω increases with k, β, and $H_{\rm i}$, but decreases with γ and $k_{\rm m}$.

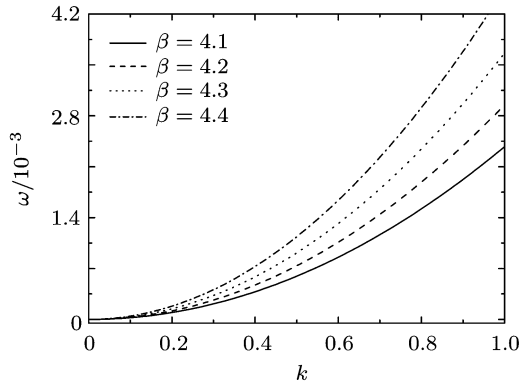


Fig. 10. Variation of ω with k for different values of β. $K = 10^{-6}$, $H_{\rm e} = 0.2782$, $H_{\rm i} = 6.494 \times 10^{-3}$, $k_{\rm z} = 1$, $k_{\rm m} = 4$, $k = 1$, $\mu = 4$, $r_{\min} = 0.01$, $r_{\max} = 1$, and $\gamma = 1.3$.

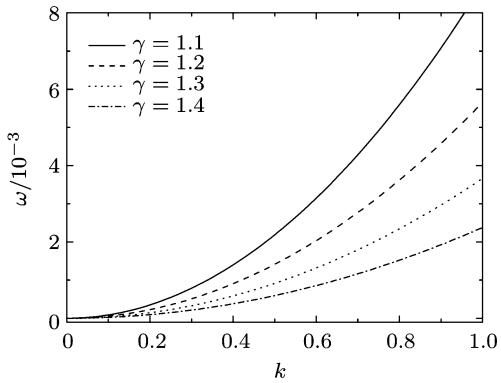


Fig. 11. Variation of ω with k for different values of γ. $K = 10^{-6}$, $H_{\rm e} = 0.2782$, $H_{\rm i} = 6.494 \times 10^{-3}$, $k_{\rm z} = 1$, $k_{\rm m} = 4$, $k = 1$, $\mu = 4$, $r_{\min} = 0.01$, $r_{\max} = 1$, and $\beta = 4.3$.

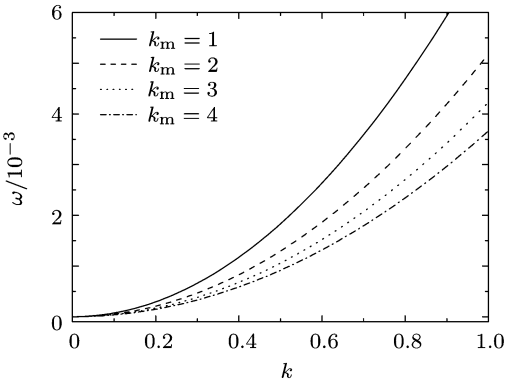


Fig. 12. Variation of ω with k for different values of $k_{\rm m}$. $K = 10^{-6}$, $H_{\rm e} = 0.2782$, $H_{\rm i} = 6.494 \times 10^{-3}$, $k_{\rm z} = 1$, $k = 1$, $\mu = 4$, $r_{\min} = 0.01$, $r_{\max} = 1$, $\beta = 4.3$, and $\gamma = 1.3$.

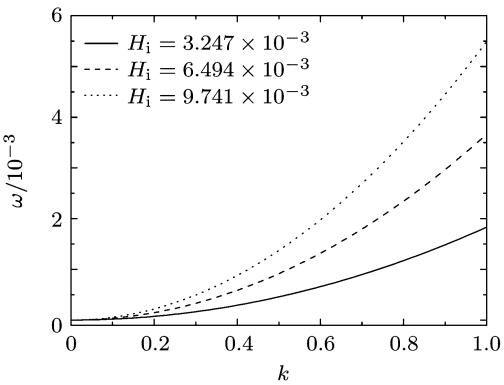


Fig. 13. Variation of ω with k for different values of $H_{\rm i}$. $K = 10^{-6}$, $H_{\rm e} = 0.2782$, $k_{\rm z} = 1$, $k_{\rm m} = 4$, $k = 1$, $\mu = 4$, $r_{\min} = 0.01$, $r_{\max} = 1$, $\gamma = 1.3$, and $\beta = 4.3$.

4. Conclusion

In this paper, the dispersion relation is studied by using the QHD model in an unmagnetized, collisionless, ultracold quantum dusty plasma. The effects of different dust size distributions and adiabatic dust charge variation are considered. Two different dust size distributions, polynomial DSD, and power law DSD, are studied. Numerical results indicate that the frequency ω increases with the total number density, and it is greater for unusual dusty plasma than that of the usual dusty plasma. This conclusion is similar to that in Ref. [38] for the classical dusty plasma in which the quantum effects are neglected. We also find that the frequency ω increases with the wave number k and $R = r_{\max}/r_{\min}$, but decreases with γ.

For the power law dust size distribution dusty plasma, ω increases with β. We also find that the frequency is larger than that of the mono-sized dusty plasma when $\gamma < 1.5$, but they are the same when $\gamma = 1.5$. It is smaller than that of the mono-sized dusty plasma when $\gamma > 1.5$. In addition, we find that the frequency increases with quantum parameter $H_{\rm i}$, and decreases with $k_{\rm m}$.

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