Acoustic characteristics of bubble bursting at the surface of a high-viscosity liquid*

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An acoustic pressure model of bubble bursting is proposed. An experiment studying the acoustic characteristics of the bursting bubble at the surface of a high-viscosity liquid is reported. It is found that the sudden bursting of a bubble at the high-viscosity liquid surface generates N-shape wave at first, then it transforms into a jet wave. The fundamental frequency of the acoustic signal caused by the bursting bubble decreases linearly as the bubble size increases. The results of the investigation can be used to understand the acoustic characteristics of bubble bursting.

Keywords: bubble bursting, acoustic characteristics, high-viscosity liquid

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1. Introduction

The dynamic characteristic of bubbles in liquids has been intensively studied all over the world.[1−4] Bubbles bursting at liquid or solid surfaces show some acoustic characteristics,[5−10] which have been widely used in medical,[11−13] marine acoustics,[14−18] and volcanic prediction.[19−25] Many valuable research results focusing on dynamic bubbles under an external excitation in liquid have been obtained.

Bubble bursting is analogous to that of a balloon, which is considered as the process of an instantaneous removal of the membrane. When the balloon film ruptures, it shrinks before the pressurized gas contained within the balloon can expand due to its elasticity. Similarly, once the bubble bursts, the bubble film will rapidly retract. As the bubble cavity re-equilibrates, a jet of liquid is propelled upward,[1,3] and generates a jet wave.[19]

Experiments where the correlations between acoustic emission and the bursting of a gas bubble at the free surface of a non-Newtonian fluid were studied have been reported.[24−26] It is found that the acoustic wave generated during the bubble bursting is mainly associated with the fluid viscoelastic properties. During bursting, the bubble walls remain, and the bubble cavity acts as a resonator excited by the film bursting. The amplitude of the emitted acoustic wave in the bursting depends on various parameters: liquid concentration, volume of the bubble, and film rupture time. The acoustical signal exhibits a well defined fundamental frequency, which is principally governed by the cavity volume.[26] The acoustical characteristics of the dynamical bubble are related to many conditions, such as the physical properties of the liquid, the ambient pressure of the bubbles, the temperature, and the bubble shape.

In this paper, we theoretical analyze the acoustic pressure of a bursting bubble. In order to obtain the fundamental frequencies of the acoustic signals, we design an experiment to analyze the acoustic characteristics of the bubble bursting at a high viscosity liquid surface. It is found that the fundamental frequency of the acoustic signal caused by the bursting bubble decreases linearly with the increasing bubble size. The results of the theoretical calculation are basically consistent with that of the experiment. This investigation can be used to understand the acoustic characteristics of bubble bursting.

2. Acoustic pressure of bubble bursting

For a spherical sound source, the equation for pressure $P(r,t)$ in the spherical coordinates can be

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written as

$$\frac{1}{c^2}\frac{\partial^2 P(r,t)}{\partial t^2} = \frac{\partial P^2(r,t)}{\partial r^2} + \frac{\partial P(r,t)}{\partial r}\frac{\partial \ln S}{\partial r}, \quad (1)$$

where c is the velocity of sound in air; r stands for the distance from the space point to the bubble aperture; and S is the area of wave-front, $S = 4\pi r^2$. Wave equation (1) can be written as

$$\frac{1}{c^2}\frac{\partial^2 P(r,t)}{\partial t^2} = \frac{2}{r}\frac{\partial P(r,t)}{\partial r} + \frac{\partial^2 P(r,t)}{\partial r^2}. \quad (2)$$

The radiation pressure meeting with wave equation (2) can be expressed as

$$P(r,t) = \frac{|A|}{r}\,\mathrm{e}^{\mathrm{i}(\omega t - kr + \theta)}, \quad (3)$$

where ω is the angular velocity, k is the wave number, $k = \omega/c$, $|A| = \rho_{\mathrm{g}} c k r_{\mathrm{c}}^2 u_0/\sqrt{1+(kr_{\mathrm{c}})^2}$, u_0 is the amplitude of the bubble-wall velocity, $\theta = \arctan(1/kr_{\mathrm{c}})$, and ρ_{g} is the gas density. For a low frequency sound wave, $kr_{\mathrm{c}} = \omega r_{\mathrm{c}} \ll 1$, we have $\sqrt{1+(kr_{\mathrm{c}})^2} \approx 1$, $\theta \approx \pi/2$, $\mathrm{e}^{\mathrm{i}\theta} \approx \mathrm{i}$. Equation (3) can be written as

$$p(t) \cong \frac{\mathrm{i}\rho_{\mathrm{g}} c k Q}{4\pi r}\,\mathrm{e}^{\mathrm{i}(\omega t - kr)}, \quad (4)$$

where Q is the volume velocity of the source, $Q = 4\pi\, r_{\mathrm{c}}^2 u_0$.

When a bubble bursts on a solid wall (in Fig. 1), the volume velocity source strength equals $Q/2$, the sound pressure, however, is still given by Eq. (4).

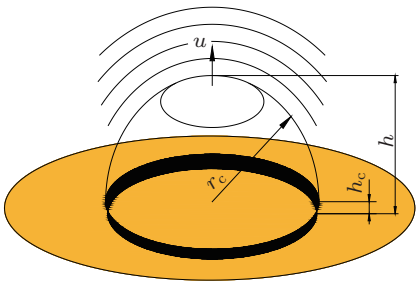


Fig. 1. (colour online) Diagram of acoustic radiation, where u is the velocity of the air flow, h_{c} is the film thickness, r_{c} is the bubble radius, and h is the bubble body height, $h \cong h_{\mathrm{c}} + r_{\mathrm{c}}$.

When the aperture appears, the overpressure air inside the bubble ($\Delta p = 2\sigma/r_{\mathrm{c}}$) will be released into the atmosphere. We here assume that the velocity of the air flow is u. Friction will be generated between the air flow and the bubble film around the aperture, which will cause bubble film vibration. The vibration amplitude of the bubble film is always dependent on the velocity of the air flow. According to Bernoulli's equation, the pressure fluctuation is related to the flow velocity, we can estimate the possible pressure fluctuation as $\Delta p = \rho_{\mathrm{g}} u^2/2$, then $u = 2\sqrt{\sigma/r_{\mathrm{c}}\rho_{\mathrm{g}}}$. The pulse of th ebubble wall will gradually shrink, and the bubble wall velocity amplitude will gradually decay due to the damping. The velocity amplitude is[6]

$$u_0 = u\,\mathrm{e}^{-\beta t}, \quad (5)$$

where $1/\beta$ is the time constant for the decay of amplitude, $\beta = \delta\omega/2$, and δ is a dimensionless damping constant, which depends on radiation damping coefficient δ_{rad} ($\delta_{\mathrm{rad}} = r_{\mathrm{c}}\omega/c$), thermal damping coefficient δ_{th}, and viscous damping coefficient δ_{vis} ($\delta_{\mathrm{vis}} = 4\mu/(r_{\mathrm{c}}^2\rho_{\mathrm{l}}\omega)$), $\delta = \delta_{\mathrm{rad}} + \delta_{\mathrm{vis}} + \delta_{\mathrm{th}}$. Under the condition of no heat energy loss, we have $\delta_{\mathrm{th}} = 0$.

The angular frequency ($\omega = 2\pi f$, where f is the fundamental frequency of the bubble bursting) depends on the shape and the size of the bubble cavity, and can be obtained from the first frequency peak of the acoustic power spectrum in bubble bursting experiments.

3. Experiment and analysis

In order to analyze the acoustic characteristics of bubble bursting and investigate the relations between the fundamental frequency and the size of the bubble, we design the experimental setup shown in Fig. 2, which consists of two microphones (ICP acoustic sensor), a laser sensor (OMETRON: VQ-400-A), a real-time signal acquisition and analysis system (LDS Focus (II)), and a thin copper plate (79 mm × 56 mm × 0.4 mm). The plate is supported by a soft cushion. A high-viscosity fluid (the viscosity is $\mu = 30$ Pa·s) is laid on the surface of the copper plate. Microphones (Mic 1# and Mic 2#) are located at different distances from the plate, so Mic 1# can monitor both the acoustic signal and the jet wave emitted in the bubble bursting, and Mic 2# can record the acoustic signals in further away. The laser sensor is set to measure the vibration of the thin plate.

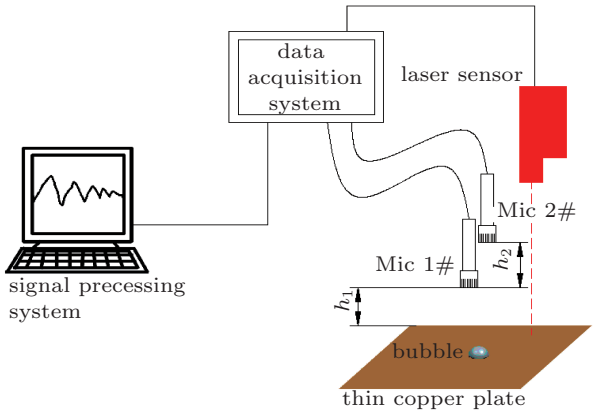


Fig. 2. (colour online) Schematic diagram of the experimental setup.

3.1. Acoustic-vibration signals of bubble bursting on a thin copper plate

Through many experiments, we find that the bubble wall spontaneously bursts after about 20 min due to gravity and the initial overpressure inside the bubble. Bursting events are systematically associated with the emission of acoustic waves and cause the vibration of the thin copper plate at the same time. From the acoustic-vibration signals, we find that the peaks of the vibration signals from the thin plate lag behind that of the acoustic signals (Fig. 3(b)).

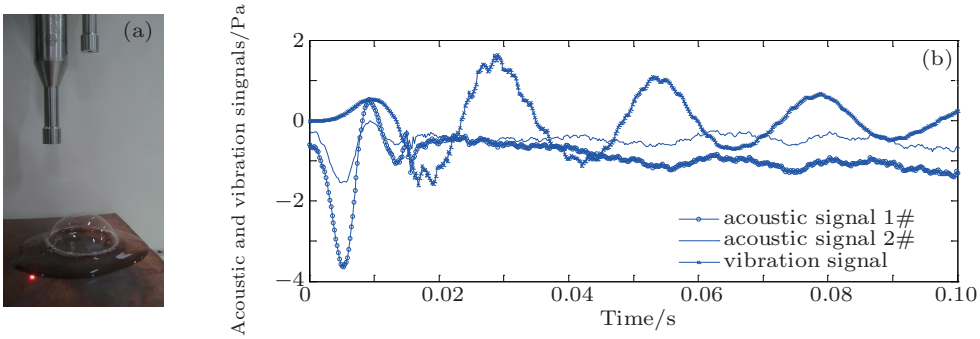


Fig. 3. (colour online) (a) Bubble at the surface of thin copper plate and (b) acoustic-vibration signal curves.

3.2. Fundamental frequency of bubble bursting

The experimental signals in Fig. 4(a) show that the sudden bursting of the bubble at the top excites a resonant pressure wave in the bubble cavity. This pressure is in an N-shape wave. As the cavity re-equilibrates, a jet of liquid is propelled upward,[1] and generates a jet wave. The waveforms recorded by two microphones are consistent except for the pressure amplitudes.

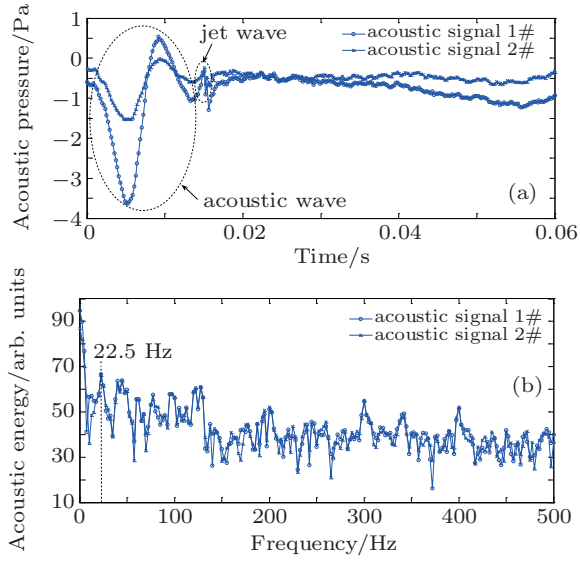


Fig. 4. (colour online) Acoustic signals of bubble (r_c = 22 mm) bursting: (a) the time-domain signal, and (b) the power spectrum.

The fundamental frequency f of the acoustic wave in air, which corresponds to the first frequency peak in the power spectrum, can be obtained (for instance, $f \cong 22.5$ Hz in Fig. 4(b)). Shortly afterwards, the bubble body vanishes, and the amplitude of the acoustic pressure also decays. In order to evaluate the effect of the bubble cavity size, the fundamental frequency is given in next section.

4. Experimental results and discussion

In this section, we investigate the relations between the fundamental frequency of the acoustic signal and the bubble size. The results of the theoretical calculation considering the background noise are compared with the experimental results.

4.1. Relations between fundamental frequency and bubble size

Triggered by the film bursting, the bubble cavity acts as a resonator, and the acoustic signal frequencies can be directly linked to the bubble height. Through many experiments, we find that the first frequency peak of the acoustic signal varies with the bubble size. Figure 5 shows the relation between fundamental frequency f and bubble cavity height h ($= h_c + r_c$, in this case, $h_c = 2.5$ mm). We can find that the fundamental frequency decreases linearly with the increasing bubble cavity size. The relation between the frequency and the bubble cavity height can be written as

$$f \cong k[f_0 - \xi(h_c + r_c)], \tag{6}$$

where f_0 is the offset, $f_0 \cong 40 \pm 1$ Hz, ξ is the slope, $\xi \cong 2 \pm 1$, and k is the correction factor defined by the experiments, $k = 1.5 \sim 2.0$.

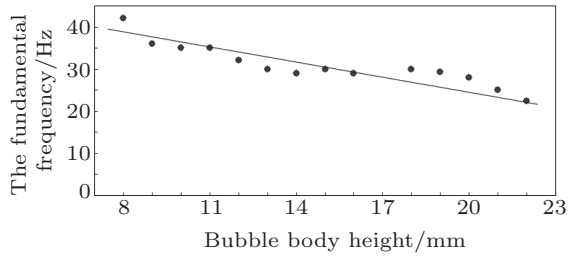


Fig. 5. Fundamental frequency f versus bubble body height h.

4.2. An example of the acoustic pressure of bubble bursting

An example of the calculation of the acoustic pressure emitted from the bubble bursting is given in this section. In this case, the liquid viscosity is $\mu = 30$ Pa·s, the liquid density is $\rho_l = 1200$ kg/m^3, the environmental pressure is 1 atm, the gas density is $\rho_g = 1.21$ kg/m^3, the work temperature is $T = 20$ °C, the bubble initial radius is $r_c = 22$ mm, the distance between the bubble and field point (Mic 2#) is $r = 80$ mm.

The noise measured is synthesized by two acoustic sources: one is the background noise; the other is the source noise. If the acoustic levels of the measured value and the background noise are known, the source acoustic level can be obtained by signal filtering.

Assume that $L_{p'(t)}$ is the acoustic pressure level measured for the noise source, the corresponding acoustic intensity is $I'(t)$, the acoustic pressure is $p'(t)$; $L_{p_2(t)}$ is the background acoustic pressure level, the corresponding acoustic intensity is $I_2(t)$, the acoustic pressure is $p_2(t)$; we can obtain $I'(t) = I_1(t) + I_2(t)$ and $p'(t)^2 = p_1(t)^2 + p_2(t)^2$ according to the principle of energy superposition, where $L_{p(t)}$ is the source acoustic level, $I_1(t)$ is the acoustic intensity , and $p_1(t)$ is the effective acoustic pressure obtained by Eq. (4). Then, acoustic pressure $p'(t)$ can be obtained as

$$p'(t) \cong \sqrt{p_1(t)^2 + p_2(t)^2}. \tag{7}$$

Substituting the property parameters of the liquid tested into Eq. (7), we can obtain the acoustic pressure of the bubble bursting.

Figure 6 shows the comparisons between the measurement and the calculated results. It shows a good agreement except for the jet wave.

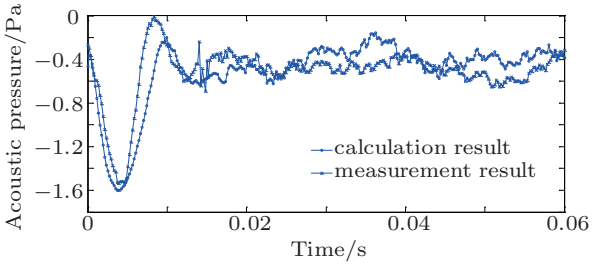


Fig. 6. (colour online) Comparisons of measurement and calculation results of the emitted acoustic pressure at Mic 2#.

5. Conclusion

A theoretical model for the acoustic pressure emitted by a bubble bursting on a liquid surface is proposed. Experiments of a bubble bursting at the surface of a high viscosity liquid on a thin copper plate are carried out to obtain the fundamental frequency of the acoustic signal. The following phenomena are observed. The sudden bursting of a bubble at the top excites a resonant pressure wave in the bubble cavity. The bursting generates an N-shape wave. As the cavity re-equilibrates, a jet of liquid is propelled upward, and generates a jet wave. The fundamental frequency of the acoustic signal varies with the bubble size, it decreases almost linearly with the increasing bubble cavity size. At the same time, we find that the bursting event at the liquid surface also causes the vibration of the thin plate, the peaks of the vibration signals from the thin plate lag behind that of the acoustic signals.

References

[1] Bird J C, de Ruiter R, Courbin L and Stone H A 2010 *Nature* **465** 759

[2] Ma Q Y, Qiu Y Y, Huang B, Zhang D and Gong X F 2010 *Chin. Phys.* B **19** 094302
[3] Zhang A M and Yao X L 2008 *Acta Phys. Sin.* **57** 1662 (in Chinese)
[4] Debrégeas G, de Gennes G P and Brochard-Wyart F 1998 *Science* **279** 1704
[5] Husin S and Mba D 2010 *Proceedings of the London U.K. World Congress on Engineering* 2010 Vol II WCE 2010
[6] Leighton T G 2004 *International Journal of Modern Physics* B **18** 3267
[7] Ida M 2002 *Phys. Lett.* A **297** 210
[8] Brennen C E 1995 *Cavitation and Bubble Dynamics* (Oxford: Oxford University Press)
[9] Michael S and Longuet-Higgins 1990 *J. Acoust. Soc. Am.* **87** 652
[10] Medwin H and Beaky M M 1989 *J. Acoust. Soc. Am.* **87** 1124
[11] Zheng H R, Mukdadi O and Shandas R 2006 *Physics in Medicine and Biology* **51** 557
[12] Chahine G L, Tanguay M and Loraine G 2009 *Journal of Hydrodynamics* **21** 47
[13] Zhang C B, Liu Z, Guo X S and Zhang D 2011 *Chin. Phys.* B **20** 024301
[14] Nie B S and Qiu R G 2007 *Ocean Technology* **26** 51
[15] Terrill E and Melville W K 1997 *J. Acoust. Soc. Am.* **102** 2607
[16] Deane G B 1997 *J. Acoust. Soc. Am.* **102** 2671
[17] Updegraff G E and Anderson V C 1991 *J. Acoust. Soc. Am.* **89** 2264
[18] Farmer D M and Vagle S J 1989 *Acoust. Soc. Am.* **86** 1897
[19] Vidal V, Ichihara M and Ripepe M 2009 *Phys. Rev.* E **80** 066314
[20] Parfitt E A 2004 *J. Volcanol Geoth. Res.* **134** 77
[21] Vergniolle S, Boichu M and Caplan-Auerbach J 2004 *J. Volcanol Geoth. Res.* **137** 109
[22] Vergniolle S and Caplan-Auerbach J 2004 *J. Volcanol Geoth. Res.* **137** 135
[23] Bouche E, Vergniolle S and Staudacher T 2010 *Earth and Planetary Science Letters* **295** 37
[24] Vidal V, Géminard J C, Divoux T and Melo F 2006 *Eur. Phys. J.* B **54** 321
[25] Divoux T, Vidal V and Melo F 2008 *Phys. Rev.* E **77** 056310
[26] Vidal V, Ichihara M and Ripepe M 2009 *Phys. Rev.* E **80** 066341