Oscillation behaviour in the photon-number distribution of squeezed coherent states*

Wang Shuai(王 帅)[a)b)†], Zhang Xiao-Yan(张晓燕)[b)], and Fan Hong-Yi(范洪义)[a)]

[a)] *Department of Physics, Shanghai Jiao Tong University, Shanghai* 200240, *China*
[b)] *Department of Physics, Heze University, Heze* 274015, *China*

(Received 20 October 2011; revised manuscript received 24 November 2011)

From the normally ordered form of the density operator of a squeezed coherent state (SCS), we directly derive the compact expression of the SCS's photon-number distribution (PND). Besides the known oscillation characteristics, we find that the PND is a periodic function with a period of π and extremely sensitive to phase. If the squeezing is strong enough, and the compound phase which is relevant to the complex squeezing and displacement parameters are assigned appropriate values, different oscillation behaviours in PND for even and odd photon numbers appear, respectively.

Keywords: photon-number distribution, squeezed coherent states, normal ordering

PACS: 42.50.–p **DOI:** 10.1088/1674-1056/21/5/054206

1. Introduction

Photon-number distribution (PND) is a key parameter of optical fields. Particularly, oscillations in the PND has been regarded as one of the most impressive manifestations of the quantum nature of squeezed states, and also a signal of nonclassical behaviour of squeezed states.[1−4] Therefore, the PND for various single-mode Gaussian states of optical fields, such as squeezed vacuum states (SVSs) and squeezed coherent states (SCSs), has been studied.[2−11] Schleich *et al.* firstly studied the oscillation in the PND of SCSs and interpreted it as an interference effect in phase space.[2−4] Dutta *et al.*[10] have conducted a further study on the oscillation in the PND of an SCS and found that photon-number oscillations are extremely sensitive to phase. They also found different oscillation behaviours of the PND of an SCS for even and odd photon numbers (called giant oscillation) for complex displacement parameters.

It is well-known that there will be oscillation in the PND of strongly squeezed states. Recently, strong odd–even photon-number oscillations in the PND of SVS have been observed for strong squeezing in experiments.[12−13] It is convenient to divide nonclassical oscillations into two classes. According to Zhu and Caves,[14] for a single-mode SVS, the PND is always zero for an odd photon number since the squeezing operator creates and annihilates photons in pairs. One can refer to these simple even–odd oscillations as squeezed vacuum oscillations, which disappear quickly as the photon number increases. Nonetheless, there are more complicated nonclassical oscillations even for large displacements, i.e., oscillations for even and odd photon numbers, which can be called Schleich–Wheeler oscillations. It should be pointed out that the boundary between squeezed vacuum oscillations and Schleich–Wheeler oscillations is not clear.

Although the PND of squeezed states has been studied for many years, variation of the PND with the compound phase that is relevant to the complex squeezing and displacement parameters has not been investigated thoroughly and extensively. In addition, the main method of deriving the PND is based on quasi-probability functions in phase space. Calculation of the PND becomes complicated in the case of complex squeezing and displacement parameters. In this work, we present a new way of deriving the PND of SCSs and further study PND's oscillation behaviours that are sensitive to the compound phase. Our method to derive the PND is more effective and

*Project supported by the National Natural Science Foundation of China (Grant No. 11175113), the Natural Science Foundation of Shandong Province, China (Grant No. ZR2010AQ024), and the Scientific Research Foundation of Heze University of Shandong Province, China (Grant No. XYJJKJ-1).

†Corresponding author. E-mail: wangshuai197903@sohu.com

© 2012 Chinese Physical Society and IOP Publishing Ltd

direct because it is based on the normally ordered form of an SCS's density operator. Such a form, in which all annihilation operators are on the right of creation operators, is a useful tool in quantum optics.[15−20]

This paper is organized as follows. In Section 2, we first derive the normally ordered form of an SCS's density operator by using the operator's Weyl ordering invariance under similarity transformation and the Weyl–Wigner correspondence scheme. In Section 3, we further derive the PND of an SCS and investigate the oscillation behaviour for different values of complex squeezing and displacement parameters in detail. We find that for strong squeezing and an appropriate compound phase, respective oscillation behaviours for even and odd photon numbers appear. The PND of an SCS is a periodic function of the compound phase with a period of π. Our main results are summarized in Section 4.

2. The normally ordered form of an SCS's density operator

Theoretically, the SCS can be obtained by first exerting the squeezing operator $S(\zeta)$ and then the displacement operator $D(\beta)$ on a vacuum state, i.e.,

$$\rho_{\rm SCS} = D(\beta) S(\zeta) |0\rangle \langle 0| S^{-1}(\zeta) D^{-1}(\beta), \quad (1)$$

where $|0\rangle$ is the vacuum state, $D(\beta) = \exp(\beta a^\dagger - \beta^* a)$, the complex displacement parameter $\beta = |\beta| \, {\rm e}^{{\rm i}\phi}$, a ($a^\dagger$) is the annihilation (creation) operator and $\left[a,\ a^\dagger\right] = 1$, and the squeezing operator[21]

$$S(\zeta) = \exp\left[\frac{\zeta}{2} a^{\dagger 2} - \frac{\zeta^*}{2} a^2\right], \quad (2)$$

with the complex squeezing parameter $\zeta = \lambda \, {\rm e}^{{\rm i}\theta}$. The phase θ of the squeezing parameter determines the squeezing direction. The squeezing operator provides a Bogoliubov transformation of a and $a^\dagger$

$$S(\zeta) a S^{-1}(\zeta) = a \cosh\lambda - {\rm e}^{{\rm i}\theta} a^\dagger \sinh\lambda, \quad (3)$$

$$S(\zeta) a^\dagger S^{-1}(\zeta) = a^\dagger \cosh\lambda - {\rm e}^{-{\rm i}\theta} a \sinh\lambda. \quad (4)$$

In order to study the property in the PND of an SCS, it is convenient to use the normally ordered expansion of $\rho_{\rm SCS}$. In what follows, by using operator's Weyl ordering invariance under similarity transformation and the Weyl correspondence of $|0\rangle \langle 0|$, we will derive the normally ordered form of $\rho_{\rm SCS}$. The Weyl–Wigner quantization[22] is a widely acknowledged scheme for connecting classical functions with quantum operators. In our view, the advantage of this scheme lies in the fact that the Wigner operator has a neat Gaussian form that is complete in normal order[23]

$$\Delta(\alpha, \alpha^*) = \frac{1}{\pi} : \, {\rm e}^{-2\left(a^\dagger - \alpha^*\right)(a-\alpha)} : , \quad (5)$$

$$2 \int {\rm d}^2\alpha \Delta(\alpha, \alpha^*) = 1. \quad (6)$$

Thus, an arbitrary operator $F\left(a, a^\dagger\right)$ in the single-mode case can be expanded in terms of $\Delta(\alpha, \alpha^*)$

$$F\left(a, a^\dagger\right) = 2 \int {\rm d}^2\alpha h(\alpha, \alpha^*) \Delta(\alpha, \alpha^*), \quad (7)$$

where $h(\alpha, \alpha^*)$ is the classical Weyl correspondence of the operator $F\left(a, a^\dagger\right)$, i.e., $h(\alpha, \alpha^*) = {\rm Tr}\left\{F\left(a, a^\dagger\right) \Delta(\alpha, \alpha^*)\right\}$. Therefore, if the classical Weyl correspondence of a operator is known, its normally ordered form can be derived from Eq. (7).

In practice it may be difficult to directly calculate $h(\alpha, \alpha^*)$ for complex $F\left(a, a^\dagger\right)$. In what follows, we shall firstly derive the normally ordered form of SVS's density operator

$$\rho_{\rm SVS} = S(\zeta) |0\rangle \langle 0| S^{-1}(\zeta) \quad (8)$$

by using the Weyl–Wigner correspondence and the technique of integration within an ordered product of operators.[24] In Refs. [25] and [26], the Weyl ordering symbol $\vdots \ \vdots$ was introduced. Bose operators are permuted within the symbol. The technique of integration within the Weyl-ordered product of operators is invented. Using the operator formula for $F\left(a, a^\dagger\right)$ in its Weyl ordering

$$F\left(a, a^\dagger\right) = 2 \vdots \int \frac{{\rm d}^2\beta}{\pi} \langle -\beta| F\left(a, a^\dagger\right) |\beta\rangle$$

$$\exp\left[2\left(a^\dagger a + a\beta^* - a^\dagger\beta\right)\right] \vdots , \quad (9)$$

noticing the normal product form of vacuum projector

$$|0\rangle \langle 0| =: \, \exp[-a^\dagger a] : , \quad (10)$$

we obtain the Weyl ordering form of $|0\rangle \langle 0|$, that is,

$$|0\rangle \langle 0| = 2 \vdots \exp\left[-2a^\dagger a\right] \vdots . \quad (11)$$

Next, by virtue of the Weyl ordering invariance under similarity transformation

$$S \vdots (\circ\circ\circ) \vdots S^{-1} = \vdots S(\circ\circ\circ) S^{-1} \vdots , \quad (12)$$

substituting Eq. (11) into Eq. (12) and noticing Eqs. (3) and (4), we obtain the Weyl ordering form of $\rho_{\rm SVS}$

$$\rho_{\rm SVS} = 2 \vdots \exp\left[-2\left(a^\dagger \cosh\lambda - {\rm e}^{-{\rm i}\theta} a \sinh\lambda\right)\right. \times \left.\left(a\cosh\lambda - {\rm e}^{{\rm i}\theta} a^\dagger \sinh\lambda\right)\right] \vdots . \quad (13)$$

According to the definition of the Wely correspondence rule, the classical Weyl function $h(\alpha,\alpha^*)$ of operator $F\left(a,a^\dagger\right)$ can be given by replacing a by α, $a^\dagger$ by α^* in its Weyl ordering form, so the classical corresponding function is

$$h(\alpha,\alpha^*) = 2\exp\left[-2\left|\alpha^*\cosh\lambda - {\rm e}^{-{\rm i}\theta}\alpha\sinh\lambda\right|^2\right]. \quad (14)$$

Substituting Eq. (14) into Eq. (7), and using the integration formula[27]

$$\int \frac{{\rm d}^2 z}{\pi} \exp\left[\zeta|z|^2 + \xi z + \eta z^* + f z^2 + g z^{*2}\right] = \frac{1}{\sqrt{\zeta^2 - 4fg}} \exp\left[\frac{-\zeta\xi\eta + \xi^2 g + \eta^2 f}{\zeta^2 - 4fg}\right], \quad (15)$$

whose convergent condition is that ${\rm Re}\,(\zeta \pm f \pm g) < 0$ and ${\rm Re}[(\zeta^2 - 4fg)/(\zeta \pm f \pm g)] < 0$, we obtain the normally ordered form of SVS's density operator

$$\begin{aligned}\rho_{\rm SVS} &= 4\int_{-\infty}^{\infty} \frac{{\rm d}^2\alpha}{\pi} \colon \exp[-2|\alpha|^2\cosh 2\lambda - 2|\alpha|^2 \\ &\quad + \sinh 2\lambda\left({\rm e}^{{\rm i}\theta}\alpha^{*2} + {\rm e}^{-{\rm i}\theta}\alpha^2\right) \\ &\quad + 2a^\dagger\alpha + 2a\alpha^* - 2a^\dagger a]\colon \\ &= \frac{1}{\cosh\lambda} \colon \exp\left[\frac{1}{2}\tanh\lambda\,{\rm e}^{-{\rm i}\theta}a^2 \right. \\ &\quad \left. + \frac{1}{2}\tanh\lambda\,{\rm e}^{{\rm i}\theta}a^{\dagger 2} - a^\dagger a\right]\colon . \end{aligned} \quad (16)$$

The displacement operator satisfies the following transformation:

$$\begin{aligned} D(\beta)\, a D^\dagger(\beta) &= a - \beta, \\ D(\beta)\, a^\dagger D^\dagger(\beta) &= a^\dagger - \beta^*. \end{aligned} \quad (17)$$

Finally, we obtain the normally ordered form of an SCS's density operator

$$\begin{aligned}\rho_{\rm SCS} &= \frac{1}{\cosh\lambda} \colon \exp\left\{\frac{1}{2}\tanh\lambda\left[{\rm e}^{-{\rm i}\theta}(a-\beta)^2 \right.\right. \\ &\quad \left. + {\rm e}^{{\rm i}\theta}\left(a^\dagger - \beta^*\right)^2\right] \\ &\quad \left. - \left(a^\dagger - \beta^*\right)(a-\beta)\right\}\colon . \end{aligned} \quad (18)$$

In summary, we have derived the normally ordered form of an SCS's density operator by virtue of operator's Weyl ordering invariance under similarity transformation and the Weyl–Wigner correspondence scheme.

Considering the normally ordered form of a vacuum projector in Eq. (11), we split the exponential, and then obtain the explicit form of the SCS

$$D(\beta)\, S(\zeta)\,|0\rangle = \frac{1}{\sqrt{\cosh\lambda}} \exp\left[-\frac{|\beta|^2}{2} + a^\dagger\beta + \frac{\tanh\lambda}{2}\,{\rm e}^{{\rm i}\theta}\left(a^\dagger - \beta^*\right)^2\right]|0\rangle . \quad (19)$$

When $\beta = 0$, Eq. (19) becomes

$$S(\zeta)\,|0\rangle = \frac{1}{\sqrt{\cosh\lambda}} \exp\left[\frac{\tanh\lambda}{2}\,{\rm e}^{{\rm i}\theta}a^{\dagger 2}\right]|0\rangle , \quad (20)$$

which is exactly the explicit form of SVS.[28]

3. Oscillation behaviour in the PND of an SCS

We now demonstrate that the compact expression of the PND of an SCS can be directly obtained based on the normally ordered form of $\rho_{\rm SCS}$. The PND, i.e., the probability of finding n photons in a quantum state described by the density operator ρ, is defined as follows:

$$\mathcal{P}(n) = {\rm tr}\left[\rho\,|n\rangle\langle n|\right], \quad (21)$$

where $|n\rangle$ is the eigenstate of the number operator $a^\dagger a$. Obviously, $\mathcal{P}(n)$ is nothing but the diagonal matrix element ρ_{nn} in Fock basis. We use the un-normalized coherent state $|z\rangle = \exp[za^\dagger]\,|0\rangle$,[29,30] which satisfies $|n\rangle = (n!)^{-1/2}\frac{{\rm d}^n}{{\rm d}z^n}|z\rangle|_{z=0}$, $\langle z_2|z_1\rangle = \exp(z_1 z_2^*)$. Based on the normally ordered form of $\rho_{\rm SCS}$ in Eq. (18), the probability of finding n photons in the SCS is

$$\begin{aligned}\mathcal{P}(n) &= \frac{1}{n!}\frac{{\rm d}^n}{{\rm d}z_2^{*n}}\frac{{\rm d}^n}{{\rm d}z_1^n}\langle z_2|\,\rho_{\rm SCS}\,|z_1\rangle_{z_1=z_2=0} \\ &= \frac{\exp(B)}{n!\cosh\lambda}\frac{{\rm d}^n}{{\rm d}z_2^{*n}}\frac{{\rm d}^n}{{\rm d}z_1^n}\exp\left[Az_1 + A^* z_2^* \right. \\ &\quad \left. + \frac{\tanh\lambda}{2}\left({\rm e}^{-{\rm i}\theta}z_1^2 + {\rm e}^{{\rm i}\theta}z_2^{*2}\right)\right]_{z_1=z_2=0} \\ &= \frac{\tanh^n\lambda\exp(B)}{2^n n!\cosh\lambda}\left|H_n\left(\frac{{\rm i}A\,{\rm e}^{{\rm i}\theta/2}}{\sqrt{2\tanh\lambda}}\right)\right|^2 , \end{aligned} \quad (22)$$

where

$$A = \beta^* - \tanh\lambda\,{\rm e}^{-{\rm i}\theta}\beta, \quad (23)$$

$$B = \frac{\tanh\lambda}{2}\left(\mathrm{e}^{-\mathrm{i}\theta}\beta^2 + \mathrm{e}^{\mathrm{i}\theta}\beta^{*2}\right) - |\beta|^2 . \quad (24)$$

In addition, the following generating function of Hermite polynomials

$$H_n(x) = \frac{\partial^n}{\partial t^n}\exp\left[2xt - t^2\right]_{t=0} \quad (25)$$

has been used to generate Eq. (22). Substituting Eqs. (23) and (24) into Eq. (22), we obtain the PND of an SCS

$$\mathcal{P}(n) = \frac{\tanh^n\lambda}{2^n n!\cosh\lambda} \times \exp\left[\frac{\tanh\lambda}{2}\left(\mathrm{e}^{-\mathrm{i}\theta}\beta^2 + \mathrm{e}^{\mathrm{i}\theta}\beta^{*2}\right) - |\beta|^2\right] \times \left|H_n\left(\frac{\mathrm{e}^{\mathrm{i}\theta/2}\beta^* - \mathrm{e}^{-\mathrm{i}\theta}\beta\tanh\lambda}{-\mathrm{i}\sqrt{2\tanh\lambda}}\right)\right|^2 . \quad (26)$$

Based on Eq. (26), we can investigate the oscillation behaviour of $\mathcal{P}(n)$. Due to $H_n(-z) = (-1)^n H_n(z)$ and $H_n(-z)H_n(-z^*) = H_n(z)H_n(z^*)$, the modulus square of Hermite polynomials in Eq. (26) is invariant under the replacement $(\phi - \theta/2) \to \pi + (\phi - \theta/2)$, where ϕ is the phase of displacement parameter $\beta = |\beta|\,\mathrm{e}^{\mathrm{i}\phi}$. Therefore, $\mathcal{P}(n)$ is a periodic function of the compound phase $(\phi - \theta/2)$ with a period of π. The following numerical investigation clearly exhibit this new result.

It is known that the SCS can be expressed in another form, i.e.,

$$\rho'_{\mathrm{SCS}} = S(\zeta)D(\alpha)|0\rangle\langle 0|D^{-1}(\alpha)S^{-1}(\zeta), \quad (27)$$

which is in fact equivalent to Eq. (1) with a modified displacement parameter determined by

$$S(\zeta)D(\alpha) = D\left(\alpha\cosh\lambda + \alpha^*\,\mathrm{e}^{\mathrm{i}\theta}\sinh\lambda\right)S(\zeta). \quad (28)$$

Let $\beta = \alpha\cosh\lambda + \alpha^*\,\mathrm{e}^{\mathrm{i}\theta}\sinh\lambda$ in Eq. (26), we obtain the PND of SCS expressed by Eq. (27)

$$\mathcal{P}'(n) = \frac{\tanh^n\lambda}{2^n n!\cosh\lambda} \times \exp\left[-\frac{\tanh\lambda}{2}\left(\mathrm{e}^{-\mathrm{i}\theta}\alpha^2 + \mathrm{e}^{\mathrm{i}\theta}\alpha^{*2}\right) - |\alpha|^2\right] \times \left|H_n\left(\frac{\alpha\,\mathrm{e}^{-\mathrm{i}\theta/2}}{\sqrt{-\sinh 2\lambda}}\right)\right|^2 , \quad (29)$$

which is exactly the same as Refs. [9] and [31].

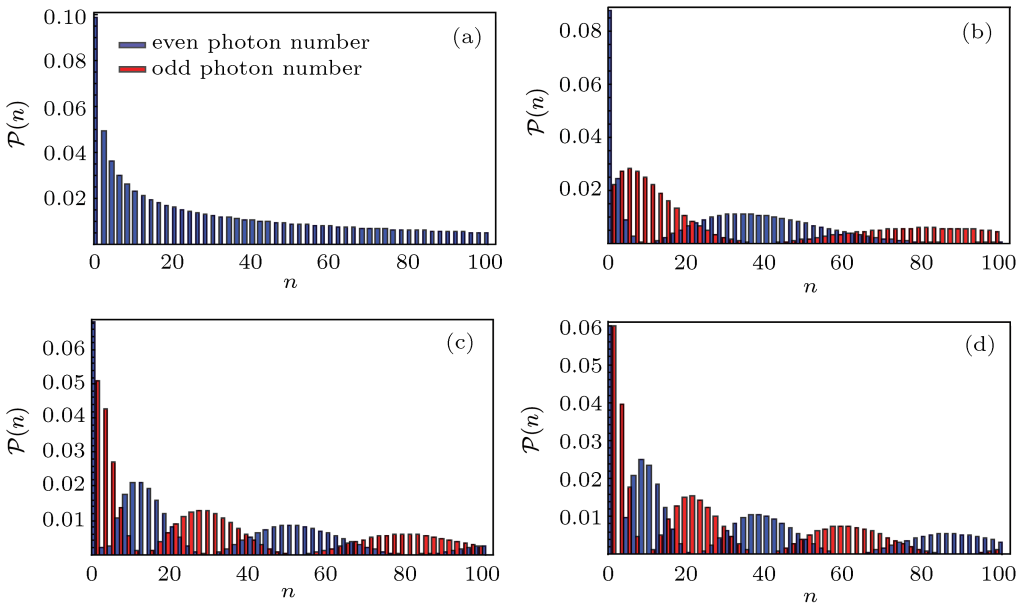


Fig. 1. (colour online) Photon-number distributions of an SCS with $|\beta| = 0.5$ and $\lambda = 3$ for (a) $\phi - \theta/2 = 0$, (b) $\pi/6$, (c) $\pi/3$, and (d) $\pi/2$.

Now we analyze the variation of $\mathcal{P}(n)$ with respect to the compound phase $(\phi - \theta/2)$ for given λ and $|\beta|$. According to Eq. (26), the variation of $\mathcal{P}(n)$ with $(\phi - \theta/2)$ is plotted in the range of $[0, \pi/2]$ in Fig. 1, from which one can see that $\mathcal{P}(n)$ is extremely sensitive to $(\phi - \theta/2)$. When $(\phi - \theta/2)$ is small (particularly equal to zero), $\mathcal{P}(n)$ exhibits squeezed vacuum oscillations (simple even–odd oscillations). As $(\phi - \theta/2)$ increases, $\mathcal{P}(n)$ quickly exhibits Schleich–Wheeler oscillations (respective oscillation behaviours

for even and odd photon numbers), particularly when $(\phi - \theta/2) = \pi/2$. These new oscillation behaviours are different from those in Ref. [10]. For example, they found that there are only squeezed vacuum oscillations when the phase angle of the complex displacement parameter is $\pi/2$ and the real squeezing parameter is considered.

Figure 2 is plotted to see how $\mathcal{P}(n)$ varies with the squeezing parameter λ for $(\phi - \theta/2) = \pi/2$ and $|\beta| = 1$. When $\lambda \ll 1$, the photon statistics resembles that of a coherent state. As the squeezing increases, the Schleich–Wheeler oscillations in PND quickly occur. In addition, comparing Fig. 1(d) with Fig. 2(d), we can see that when $|\beta|$ becomes larger, the Schleich–Wheeler oscillations become stronger. Of course when $|\beta| = 0$, there are only squeezed vacuum oscillations because the squeezing operator creates and annihilates photons in pairs.

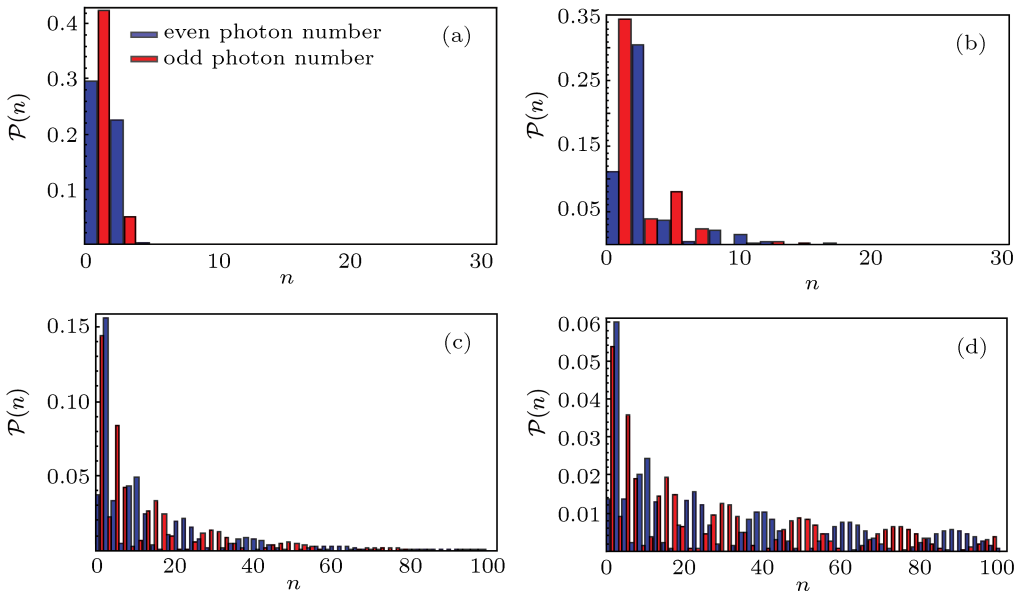


Fig. 2. (colour online) Photon-number distributions of an SCS with $(\phi - \theta/2) = \pi/2$ and $|\beta| = 1$ for (a) $\lambda = 0.2$, (b) 1, (c) 2, and (d) 3.

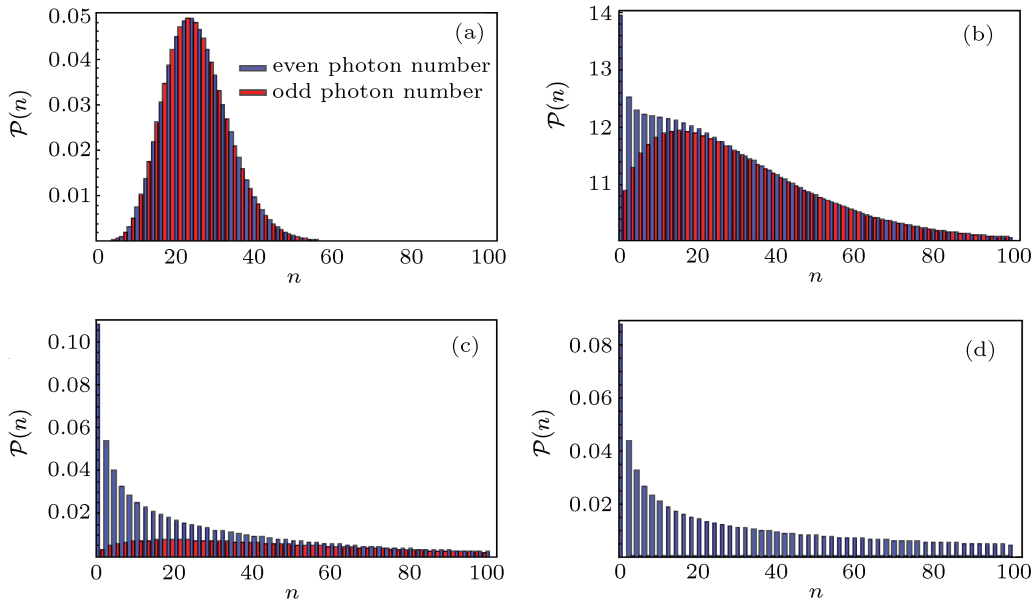


Fig. 3. (colour online) Photon-number distributions of an SCS with $\phi - \theta/2 = 0$, and $|\beta| = 5$ for (a) $\lambda = 0.5$, (b) 1.5, (c) 2, and (d) 3.

Finally, we consider the case of $(\phi - \theta/2) = 0$, and plot the picture of $\mathcal{P}(n)$ versus the squeezing parameter for $|\beta| = 1$ in Fig. 3. Figure 3 shows that when $\lambda \ll 1$, the photon statistics also resembles that of a coherent state. While the squeezing is strong enough, there are only squeezing vacuum oscillations in the case of $(\phi - \theta/2) = 0$. In addition, this character maintains regardless of the value of $|\beta|$ (see Figs. 1(a) and 3(d)). Therefore, only when strong squeezing and appropriate compound phase $(\phi - \theta/2)$ are both satisfied, the Schleich–Wheeler oscillations in the PND of an SCS will occur.

It should be noted that $\mathcal{P}'(n)$ is also a periodic function of the compound phase with a period of π and exhibits different oscillations for strong squeezing. However, Eq. (29) is a little different from Eq. (26) due to the transformation in Eq. (28). Thus, the variation of $\mathcal{P}'(n)$ with respect to the complex squeezing parameter and displacement parameter is somewhat different from that of $\mathcal{P}(n)$.

4. Conclusion

In summary, we present a new approach to derive the photon-number distribution of an SCS and further investigate the oscillation behaviour of its PND. Besides the known characters of the PND of an SCS, we find that the PND of an SCS is extremely sensitive to the phase angle θ of the complex squeezing parameter and the phase angle ϕ of the displacement parameter, and is a periodic function of the compound angle $(\phi - \theta/2)$ with a period of π. When the phase angle $(\phi - \theta/2) = 0$ (or π), the PND of an SCS obviously exhibits squeezed vacuum oscillations (even–odd oscillations) for strong squeezing. As the value of $(\phi - \theta/2)$ increases, the PND quickly shows Schleich–Wheeler oscillations (respective oscillation behaviours for even and odd photon numbers), particularly when $(\phi - \theta/2) = \pi/2$. Therefore, only when both strong squeezing and appropriate phases are satisfied, there will be Schleich–Wheeler oscillations in the PND of an SCS. In addition, compared with the method used to derive the PND of Gaussian states, our method is more effective and direct because it is based on the operator's normal ordering which can directly leads to the evaluation of coherent-state matrix elements.

References

[1] Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press)

[2] Schleich W, Walls D F and Wheeler J A 1988 *Phys. Rev.* A **38** 1177

[3] Schleich W and Wheeler J A 1987 *Nature* **326** 574

[4] Schleich W and Wheeler J A 1987 *J. Opt. Soc. Am.* B **4** 1715

[5] Agarwal G S and Adam G 1988 *Phys. Rev.* A **38** 750

[6] Agarwal G S and Adam G 1989 *Phys. Rev.* A **39** 6259

[7] Chaturvedi S and Srinivasan V 1989 *Phys. Rev.* A **40** 6095

[8] Kim M S, de Oliveira F A M and Knight P L 1989 *Phys. Rev.* A **40** 2494

[9] Peřina J and Bajer J 1990 *Phys. Rev.* A **41** 516

[10] Dutta B, Mukunda N, Simon R and Subramaniam A 1993 *J. Opt. Soc. Am.* B **10** 253

[11] Dodonov V V, Man'ko O V and Man'ko V I 1994 *Phys. Rev.* A **49** 2993

[12] Schiller S, Breitenbach G, Pereira S F, Müller T and Mlynek J 1996 *Phys. Rev. Lett.* **77** 2933

[13] Mehmet M, Vahlbruch H, Lastzka N, Danzmann K and Schnabel R 2010 *Phys. Rev.* A **81** 013814

[14] Zhu C and Caves C M 1990 *Phys. Rev.* A **42** 6794

[15] Xiang S H, Shao B and Song K H 2009 *Chin. Phys.* B **18** 418

[16] Xu X X, Hu L Y and Fan H Y 2009 *Chin. Phys.* B **18** 5139

[17] Jiang N Q 2005 *Opt. Commun.* **254** 256

[18] Jiang N Q and Zheng Y Z 2006 *Phys. Rev.* A **74** 012306

[19] Jiang N Q, Jin B Q, Zhang Y and Cai G C 2008 *Europhys. Lett.* **84** 14002

[20] Jiang N Q, Fan H Y and Hu L Y 2011 *J. Phys.* A **44** 195302

[21] Loudon R and Knight P L 1987 *J. Mod. Opt.* **34** 709

[22] Weyl H 1953 *The Classical Groups* (Princeton: Princeton University Press)

[23] Fan H Y and Zaidi H R 1987 *Phys. Lett.* A **124** 303

[24] Fan H Y 2006 *Ann. Phys.* **321** 480

[25] Fan H Y 2008 *Ann. Phys.* **323** 500

[26] Fan H Y and Fan Y 2002 *Int. J. Mod. Phys.* A **17** 701

[27] Puri R R 2001 *Mathematical Methods of Quantum Optics* (Berlin Heidelberg: Springer-Verlag)

[28] Schumaker B L and Caves C M 1985 *Phys. Rev.* A **31** 3093

[29] Glauber R J 1963 *Phys. Rev.* **130** 2529

[30] Glauber R J 1963 *Phys. Rev.* **131** 2766

[31] Yuen H P 1976 *Phys. Rev.* A **13** 2226