Characteristics of a partially coherent Gaussian Schell-model beam propagating in slanted atmospheric turbulence*

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(Received 5 May 2011; revised manuscript received 10 October 2011)

On the basis of the extended Huygens–Fresnel principle and the model of the refractive-index structure constant in the atmospheric turbulence proposed by the International Telecommunication Union-Radio Communication Sector, the characteristics of the partially coherent Gaussian Schell-model (GSM) beams propagating in slanted atmospheric turbulence are studied. Using the cross- spectral density function (CSDF), we derive the expressions for the effective beam radius, the spreading angle, and the average intensity. The variance of the angle-of-arrival fluctuation and the wander effect of the GSM beam in the turbulence are calculated numerically. The influences of the coherence degree, the propagation distance, the propagation height, and the waist radius on the propagation characteristics of the partially coherent beams are discussed and compared with those of the fully coherent Gaussian beams.

Keywords: atmospheric turbulence, partially coherent beam, propagation in a slanted path

PACS: 42.25.Dd, 42.25.Kb, 42.68.Bz **DOI:** 10.1088/1674-1056/21/5/054203

1. Introduction

The wave front of the beam propagating in the atmospheric turbulence is distorted by the random fluctuation of the atmospheric refractive index. The radius expansion, the angle spread, the variance of the angle-of-arrival fluctuation, and the wander of the laser beam are produced, which affects the atmospheric channel transmission system severely. With the coherence of the beam propagating in the turbulence destroyed, the beam becomes partially coherent light. Therefore, study of the propagation problems of partially coherent beams in atmospheric turbulence is inevitable.

The theories of partially coherent beams propagating in atmospheric turbulence have been studied by many scholars. In 1967, Feizulin and Kravtsov[1] studied laser beam expansion in a turbulent medium. In 1978, Foley and Zubairy[2] discussed the directionality of the partially coherent Gaussian Schell-model (GSM) beam. Wu[3] and Wu and Boardman[4] proved that the influence of the atmospheric turbulence on the intensity distribution of a partially coherent GSM beam is smaller than that of a fully coherent beam. Gori *et al.*[5] analysed the intensity model of the partially coherent Hermite-Gaussian beam. Santarsiero *et al.*[6] studied the beam expansion characteristics of a partially coherent GSM beam emitted by a light source in 1999. Gbur and Wolf[7] studied the expansions of partially coherent beams in a turbulent medium analytically, and the analysis indicated under what conditions and to which extent the partially coherent beams are less affected by the medium. Dogariu and Amarande[8] verified Gbur and Wolf's conclusions experimentally. In addition, Shirai *et al.*[9] studied the beam expansions of partially coherent beams in atmospheric turbulence by using coherent-model representation. In recent years, some scholars have studied the theory of the optical communication and the polarization characteristic in atmospheric turbulence.[10−12] The coherence degree and the spectral change of the partially coherent beam propagating in the atmospheric turbulence have also been studied by some scholars.[13−16] Recently, Liu and Ji[17] derived the analytical formula for the beam propagation factor (M^2 factor) of a cosh-Gaussian (ChG) array beam propagating through atmospheric turbulence, and studied the influence of the turbulence on the M^2 factor by using the relative M^2 factor. The studies above are limited to a partially coherent GSM beam

*Project supported by the National Natural Science Foundation of China (Grant No. 61172031).

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propagating in a horizontal path. In satellite communication, laser-guided systems, laser warning systems, and other fields, problems of a laser beam propagating in a slanted path (i.e. the variation of the refractive index structural constant with the propagation height) are often involved.[18−23] In the research of a beam propagating in slanted turbulence, it is expensive and difficult to perform experiments because the atmospheric structural constant varies rapidly with the environmental characteristics. This requires a study of the beam propagating in the slanted atmospheric turbulence theoretically to provide a foundation for experiments.

In the present paper, according to the extended Huygens–Fresnel principle, we derive the expressions for the effective beam radius, the spreading angle, and the average intensity of the partially coherent GSM beam in slanted atmospheric turbulence by using the cross spectral density function (CSDF). The numerical results are presented, and compared with those of fully coherent Gaussian beams.

2. Theoretical formulations

In the $z = 0$ plane, the free-space electric field of a unit-amplitude paraxial Gaussian beam propagating along the z axis can be represented by[24]

$$U(\boldsymbol{r},0) = \exp\left[-\left(\frac{1}{w_0^2}+\frac{\mathrm{i}k}{2F_0}\right)\boldsymbol{r}^2\right], \tag{1}$$

where $r = (x^2+y^2)^{1/2}$, $\boldsymbol{r} = (x,y)$ denotes a two-dimensional transverse vector perpendicular to the direction of the beam propagation in the transmitter plane, w_0 and F_0 are the initial waist radius and the curvature radius of the phase front, respectively, and $k = 2\pi/\lambda$ is the optical wave number in the free space.

The expression of the extended Huygens–Fresnel principle can be expressed as[25]

$$\begin{aligned} U(\boldsymbol{\rho},L) =& \frac{-\mathrm{i}k}{2\pi L}\exp(\mathrm{i}kL)\int \mathrm{d}\boldsymbol{r}U(\boldsymbol{r},0) \\ &\times \exp\left[\frac{\mathrm{i}k}{2L}\left|\boldsymbol{\rho}-\boldsymbol{r}\right|^2+\psi(\boldsymbol{r},\boldsymbol{\rho},L)\right]. \end{aligned} \tag{2}$$

Using Eq. (2), we can write the CSDF of the beam through the turbulence reaching the receiving plane as[24]

$$\begin{aligned} & W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2;L) \\ =& \langle U(\boldsymbol{\rho}_1,L)U^*(\boldsymbol{\rho}_2,L)\rangle \\ =& \frac{1}{(\lambda L)^2}\iint \mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2 W_0(\boldsymbol{r}_1,\boldsymbol{r}_2) \\ &\times \langle\exp[\psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1)+\psi^*(\boldsymbol{r}_2,\boldsymbol{\rho}_2)]\rangle \\ &\times \exp\left\{\frac{\mathrm{i}k}{2L}[(\boldsymbol{\rho}_1-\boldsymbol{r}_1)^2-(\boldsymbol{\rho}_2-\boldsymbol{r}_2)^2]\right\}, \end{aligned} \tag{3}$$

where $W_0(\boldsymbol{r}_1,\boldsymbol{r}_2)$ is the CSDF at the transmitter, and $\langle\exp[\psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1)+\psi^*(\boldsymbol{r}_2,\boldsymbol{\rho}_2)]\rangle$ is the cross-correlation function of the complex phase of the spherical wave. Consider that a phase diffuser is placed over the laser transmitter aperture, then the emitted field can be modeled as

$$\tilde{U}(\boldsymbol{r},0) = U(\boldsymbol{r},0)\exp[\mathrm{i}\varphi_{\mathrm{d}}(\boldsymbol{r})], \tag{4}$$

where $U(\boldsymbol{r},0)$ is given by Eq. (1), and $\exp[\mathrm{i}\varphi_{\mathrm{d}}(\boldsymbol{r})]$ represents the small random perturbation introduced by the phase diffuser.

Assuming that the ensemble average of the spatially dependent random phases is Gaussian and depends only on the separation distance not on the actual location of the diffuser, we can express the CSDF at the transmitter in the form of a GSM beam

$$\begin{aligned} & W_0(\boldsymbol{r}_1,\boldsymbol{r}_2) \\ =& \langle\tilde{U}(\boldsymbol{r}_1,0)\tilde{U}^*(\boldsymbol{r}_2,0)\rangle \\ =& U(\boldsymbol{r}_1,0)U^*(\boldsymbol{r}_2,0) \\ &\times \langle\exp[\mathrm{i}\varphi_{\mathrm{d}1}(\boldsymbol{r}_1)]\exp[-\mathrm{i}\varphi_{\mathrm{d}2}(\boldsymbol{r}_2)]\rangle \\ =& U(\boldsymbol{r}_1,0)U^*(\boldsymbol{r}_2,0)\exp\left[-(\boldsymbol{r}_1-\boldsymbol{r}_2)^2/2\sigma_\mu^2\right], \end{aligned} \tag{5}$$

where σ_μ is the partial coherence length, and describes the partial coherence properties of the transmitter source.

Centre-of-mass coordinates $\boldsymbol{r}_{\mathrm{c}}$, $\boldsymbol{\rho}_{\mathrm{c}}$ and difference coordinates $\boldsymbol{r}_{\mathrm{d}}$, $\boldsymbol{\rho}_{\mathrm{d}}$ are given by

$$\begin{aligned} &\boldsymbol{r}_{\mathrm{c}} = (\boldsymbol{r}_1+\boldsymbol{r}_2)/2, \quad \boldsymbol{r}_{\mathrm{d}} = \boldsymbol{r}_1-\boldsymbol{r}_2, \\ &\boldsymbol{\rho}_{\mathrm{c}} = (\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)/2, \quad \boldsymbol{\rho}_{\mathrm{d}} = \boldsymbol{\rho}_1-\boldsymbol{\rho}_2. \end{aligned} \tag{6}$$

Substituting Eqs. (1) and (6) into Eq. (5), we can obtain

$$\begin{aligned} W_0(\boldsymbol{r}_{\mathrm{c}},\boldsymbol{r}_{\mathrm{d}}) =& \exp\left\{-\frac{1}{w_0^2}\left[\frac{1}{2}(\boldsymbol{r}_{\mathrm{d}}^2+4\boldsymbol{r}_{\mathrm{c}}^2)\right]\right. \\ &\left.-\frac{\mathrm{i}k}{2F_0}(2\boldsymbol{r}_{\mathrm{d}}\cdot\boldsymbol{r}_{\mathrm{c}})-\frac{\boldsymbol{r}_{\mathrm{d}}^2}{2\sigma_\mu^2}\right\}. \end{aligned} \tag{7}$$

The cross correlation function of the complex phase of the spherical wave can be expressed as[26]

$$\langle\exp[\psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1)+\psi^*(\boldsymbol{r}_2,\boldsymbol{\rho}_2)]\rangle = \exp\left(-D_\psi/2\right), \tag{8}$$

where D_ψ is the phase structure function, which can be written as

$$D_\psi = 2.92k^2L\int_0^1 \mathrm{d}tC_{\mathrm{n}}^2(tL)\left|\boldsymbol{r}_{\mathrm{d}}t+(1-t)\boldsymbol{\rho}_{\mathrm{d}}\right|^{5/3}$$

$$\cong 2\left[(\boldsymbol{r}_{\rm d}^2+\boldsymbol{r}_{\rm d}\cdot\boldsymbol{\rho}_{\rm d}+\boldsymbol{\rho}_{\rm d}^2)/\rho_{\rm T}^2\right]. \quad (9)$$

From Eqs. (8) and (9), we obtain

$$\begin{aligned}&\langle\exp[\psi(\boldsymbol{r}_1,\boldsymbol{\rho}_1)+\psi^*(\boldsymbol{r}_2,\boldsymbol{\rho}_2)]\rangle\\ &\cong \exp\left[-(\boldsymbol{r}_{\rm d}^2+\boldsymbol{r}_{\rm d}\cdot\boldsymbol{\rho}_{\rm d}+\boldsymbol{\rho}_{\rm d}^2)/\rho_{\rm T}^2\right],\end{aligned} \quad (10)$$

where $\rho_{\rm T}=[1.46k^2L\int_0^1 C_{\rm n}^2(\xi L)(1-\xi)^{5/3}{\rm d}\xi]^{-3/5}$ is the coherence length of the spherical wave propagating in the slanted atmospheric turbulence, and $C_{\rm n}^2(\xi L)$ is the model for the refractive-index structural constant in the atmosphere obtained by the International Telecommunication Union.[27]

Substituting Eqs. (6), (7), and (10) into Eq. (3), we obtain

$$\begin{aligned}&W(\boldsymbol{\rho}_{\rm c},\boldsymbol{\rho}_{\rm d},L)\\ &=\frac{1}{(\lambda L)^2}\int{\rm d}r_{\rm d}\int{\rm d}r_{\rm c}\exp\left(\frac{-2\boldsymbol{r}_{\rm c}^2}{w_0^2}\right)\\ &\times\exp\left[\frac{-{\rm i}k\boldsymbol{r}_{\rm c}\cdot\boldsymbol{r}_{\rm d}}{F_0}+\frac{{\rm i}k\boldsymbol{r}_{\rm c}\cdot(\boldsymbol{r}_{\rm d}-\boldsymbol{\rho}_{\rm d})}{L}\right]\\ &\times\exp\left[-\frac{\boldsymbol{r}_{\rm d}^2}{2w_0^2}-\frac{\boldsymbol{r}_{\rm d}^2}{2\sigma_\mu^2}-\frac{\boldsymbol{r}_{\rm d}^2+\boldsymbol{r}_{\rm d}\cdot\boldsymbol{\rho}_{\rm d}+\boldsymbol{\rho}_{\rm d}^2}{\rho_{\rm T}^2}\right.\\ &\left.-\frac{{\rm i}k\boldsymbol{\rho}_{\rm c}\cdot(\boldsymbol{r}_{\rm d}-\boldsymbol{\rho}_{\rm d})}{L}\right].\end{aligned} \quad (11)$$

Integrating Eq. (5) yields

$$\begin{aligned}&W(\boldsymbol{\rho}_{\rm c},\boldsymbol{\rho}_{\rm d},L)\\ &=\langle I(\boldsymbol{\rho}_{\rm c},\boldsymbol{\rho}_{\rm d},L)\rangle\\ &=\frac{w_0^2}{W_\zeta^2(L)}\exp\left\{-\boldsymbol{\rho}_{\rm d}^2\left(\frac{1}{\rho_{\rm T}^2}+\frac{1}{2w_0^2\Lambda_0^2}\right)\right.\\ &\left.-\frac{4\boldsymbol{\rho}_{\rm c}^2-\varphi^2\boldsymbol{\rho}_{\rm d}^2}{2W_\zeta^2(L)}-\frac{{\rm i}k\boldsymbol{\rho}_{\rm c}\cdot\boldsymbol{\rho}_{\rm d}}{F_\zeta^2(L)}\right\},\end{aligned} \quad (12)$$

$$\varphi\equiv\frac{\Theta_0}{\Lambda_0}-\Lambda_0\frac{w_0^2}{\rho_{\rm T}^2},$$

where $W_\zeta(L)$ and $F_\varsigma(L)$ are the effective wave radius and the curvature radius of the phase front in the atmospheric turbulence, respectively. They expressions are given by

$$W_\zeta(L)=w_0(\Theta_0^2+\zeta\Lambda_0^2)^{1/2},\quad \zeta=\zeta_{\rm S}+\frac{2w_0^2}{\rho_{\rm T}^2}, \quad (13)$$

$$F_\varsigma(L)=\frac{L(\Theta_0^2+\zeta\Lambda_0^2)}{\varphi\Lambda_0-\zeta\Lambda_0^2-\Theta_0^2}\quad \varphi=\frac{\Theta_0}{\Lambda_0}-\Lambda_0\frac{w_0^2}{\rho_{\rm T}^2}, \quad (14)$$

where $\Theta_0=(F_0-L)/F_0$ and $\Lambda_0=2L/kw_0^2$ are the effective beam parameters, and w_0 is the waist radius of the beam. The beam is collimated for $\Theta_0=1$, convergent for $\Theta_0<1$, and divergent for $\Theta_0>1$. The $\zeta=\zeta_{\rm S}+2w_0^2/\rho_{\rm T}^2$ is the global coherence parameter, where $\zeta_{\rm S}=1+w_0^2/\sigma_\mu^2$ is the source coherence parameter of the beam emitted by the transmitter. If $\zeta_{\rm S}=1$, the beam is fully coherent. If $\zeta_{\rm S}>1$, the beam is partially coherent.

3. Propagation properties of a partially coherent GSM beam

3.1. Effective beam radius

From Eq. (13), the effective beam radius of the partially coherent GSM beam at the receiver is

$$W_\zeta(L)=w_0(\Theta_0^2+\zeta\Lambda_0^2)^{1/2}. \quad (15)$$

In Ref. [28], the effective beam radius of the fully coherent Gaussian beam propagating in the slanted atmospheric turbulence is expressed as

$$W_{(z)}=w_0\left[1+L^2\left(\theta_{\rm D}^2+\theta_{\rm T}^2\right)/w_0^2\right]^{1/2}, \quad (16)$$

where $\theta_{\rm D}=\lambda/\pi w_0$ and $\theta_{\rm T}=\lambda/\pi\rho_{\rm T}$ are the beam spreading angles caused by the diffraction and the atmospheric turbulence, respectively.

As shown in Fig. 1, the beam radius increases with the increase of propagation distance L, and the beam radius in the turbulence is greater than that in the free space. However, with the decline of the coherence, the beam expansion caused by the turbulence decreases, and the influence of the turbulence on the radius expansion becomes small.

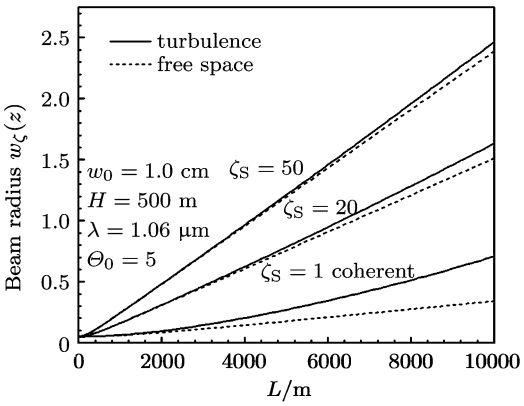


Fig. 1. Effective beam radii of the divergent GSM beams each as a function of propagation distance L for different coherence degrees.

Figure 2 shows that the beam radii of the partially coherent GSM beams and the fully coherent Gaussian beams decrease with the increase of propagation

height h. Under the same condition, the beam radius of the partially coherent GSM beam is larger than that of the fully coherent Gaussian beam. However, the influence of the propagation height on the radius expansion is smaller than that of the coherence.

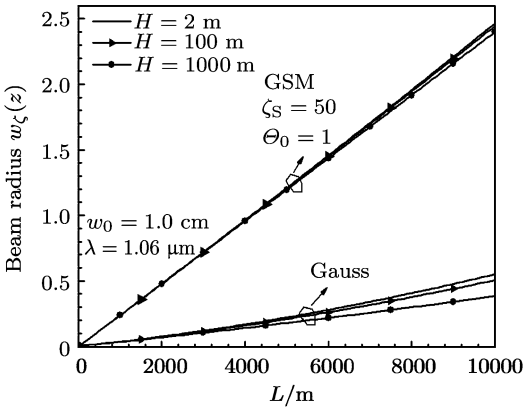


Fig. 2. Beam radii of the collimated GSM and Gaussian beams each as a function of propagation distance L for different receiving heights.

In Fig. 3, with the propagation distance increasing, the radius expansion effect becomes smaller as the waist radius increases. Under the conditions of the same wavelength and the same propagation height, the beam radius of the partially coherent GSM beam is greater than that of the fully coherent Gaussian beam. It is worth noting that when the waist radius is 10 cm, the radius expansion effect of the partially coherent GSM beam is smaller than that of the fully coherent Gaussian beam with the increase of the propagation distance.

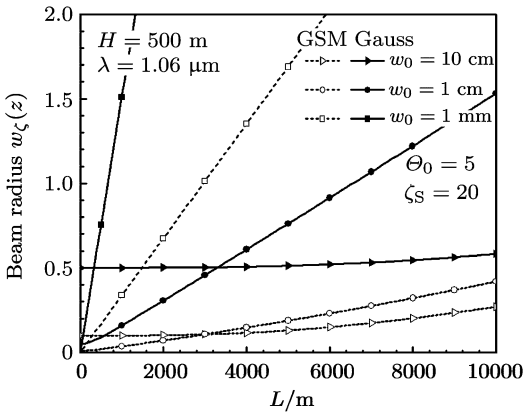


Fig. 3. Beam radii of the partially coherent GSM and the fully coherent Gaussian beams each as a function of propagation distance L for different waist radii.

3.2. Angle spread

By utilizing Eq. (13), the spreading angle of the convergent partially coherent GSM beam in the slanted atmospheric turbulence can be obtained as

$$\theta_{\rm sp}(L) = \left.\frac{W_\zeta(L)}{L}\right|_{L\to\infty} = \sqrt{\frac{Q_3}{k^2} + 8\left[1.46k^{1/3}\int_0^1 C_{\rm n}^2(\xi L)(1-\xi)^{5/3}{\rm d}\xi\right]^{6/5} L^{6/5}}, \tag{17}$$

where $Q_3 = 4\zeta_{\rm S}/w_0^2$.

In Ref. [28], the spreading angle of the fully coherent Gaussian beam in the slanted atmospheric turbulence is given by

$$\theta_{\rm SP} = \theta_{\rm D} + \theta_{\rm T} = \frac{\lambda}{\pi}\left(\frac{1}{w_0} + \frac{1}{\rho_{\rm T}}\right). \tag{18}$$

As shown in Fig. 4, the spreading angles of the partially coherent GSM and the fully coherent Gaussian beams increase with the increase of the propagation distance, and their beam angle spreads decrease with the increase of the receiving height. In addition, the angle spread of the partially coherent GSM beam is greater than that of the fully coherent Gaussian beam. But the influence of the receiving height on the angle spread of the partially coherent GSM beam is smaller than that of the fully coherent Gaussian beam.

Figure 5 shows that the spreading angle decreases with the increase of beam waist radius w_0. If the waist radius increases to a certain value (about 10 cm), the angle spread is not obvious.

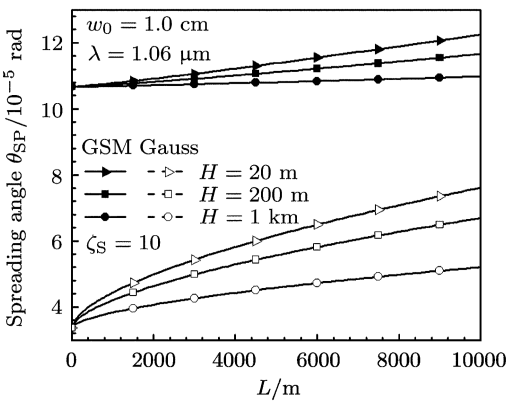


Fig. 4. Angle spreads of the partially coherent GSM and the fully coherent Gaussian beams each as a function of propagation distance L for different receiving heights.

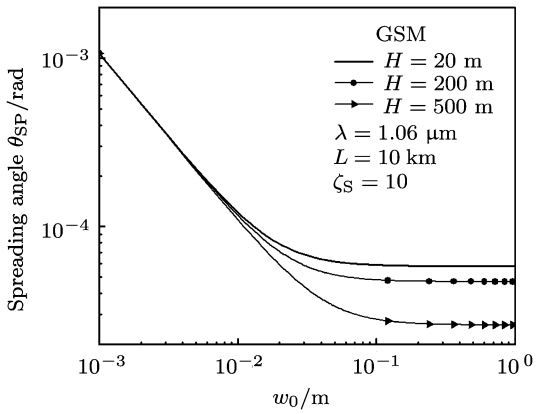


Fig. 5. Angle spreads of the partially coherent GSM beams each as a function of waist radius for different receiving heights.

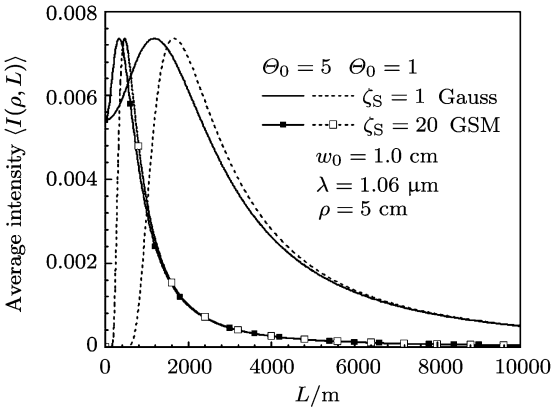


Fig. 6. Average intensities of the divergent and the collimated GSM beams each as a function of propagation distance L for different coherence degrees.

3.3. Average intensity

Using Eq. (12), let $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \boldsymbol{\rho}$, then the average intensity $\langle I(\boldsymbol{\rho})\rangle$ of the unit-amplitude partially coherent GSM beam will be obtained as

$$\langle I(\boldsymbol{\rho})\rangle = \frac{w_0^2}{W_\zeta^2(L)} \exp\left[\frac{-2\boldsymbol{\rho}^2}{W_\zeta^2(L)}\right]. \tag{19}$$

In Ref. [28] the average intensity of the fully coherent Gaussian beam at any point in the receiving plane is expressed as

$$\langle I(\boldsymbol{\rho})\rangle_{\text{Gauss}} = \frac{w_0^2}{2W_{(z)}^2} \exp\left(-\frac{\boldsymbol{\rho}^2}{W_{(z)}^2}\right), \tag{20}$$

where $W_{(z)}$ is the effective beam radius of the Gaussian beam propagating in the slanted atmospheric turbulence, which is given by Eq. (16).

As shown in Fig. 6, when the divergent and the convergent GSM beams propagate in the slanted atmospheric turbulence, the average intensities first increase and then decrease with the increase of the propagation distance. Within a short distance (500 m for the partially coherent beams and 1500 m for the fully coherent beams), the average intensity of the divergent beam is larger than that of the collimated beam, and the average intensity of the partially coherent beam is greater than that of the fully coherent beam. However, when the distance is large, the average intensity of the divergent beam is smaller than that of the collimated beam, and the average intensity of the partially coherent beam is less than that of the fully coherent beam.

Figure 7 shows that the variation trends of the average intensities for the collimated partially coherent GSM and fully coherent Gaussian beams are similar to those in Fig. 6. Within a short distance (400 m for the beam with 1.06 μm wavelength, 1300 m for the beam with 1.315 μm wavelength), the average intensity of the partially coherent GSM beam is greater than that of the fully coherent Gaussian beam, and the average intensity increases with the increase of the wavelength. However, when the distance is large, the average intensity of the partially coherent GSM beam is smaller than that of the fully coherent Gaussian beam, and the average intensity decreases with the increase of the wavelength. From Figs. 6 and 7, obviously, the average intensity of the partially coherent GSM beam propagating through the turbulence is larger than that of the fully coherent Gaussian beam with a short propagating range.

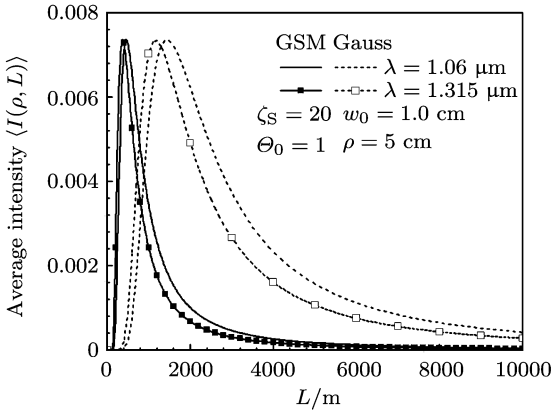


Fig. 7. Average intensities of the collimated partially coherent GSM and fully coherent Gaussian beams each as a function of propagation distance L for different wavelengths.

3.4. Beam wander effect

The beam wander is an important factor in the application of limiting the laser near the Earth, which reflects the time change of a light spot in a spatial location. The beam wander also plays an important role in the application of atmospheric engineering. Changes of the light spot centroid are often used to describe the condition of the light spot wander.[29]

By utilizing Eq. (19), the distribution of the light spot centroid for the partially coherent GSM beam is simulated numerically.

Comparing Fig. 8(a) with Fig. 8(b) shows that the larger the wavelength, the more obvious the beam wander effect. As shown in Figs. 8(b) and 8(c), the worse the beam coherence, the more obvious the beam wander effect. And the wander effect of the partially coherent GSM beam with a long-distance propagation is greater than that of the beam with a short-range propagation, which is shown by comparing Fig. 8(c) with Fig. 8(d). In practice, it is necessary for us to select a proper wavelength and a proper propagation distance to minimize the beam wander effect.

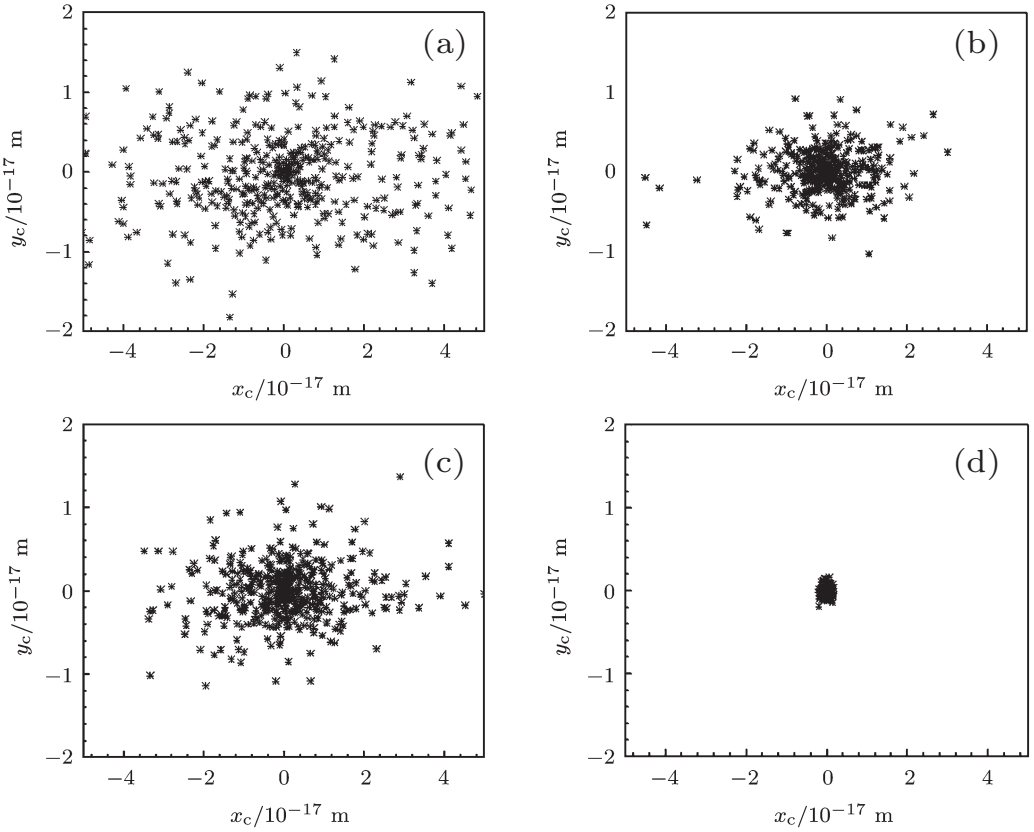


Fig. 8. Distributions of the light spot centroids for convergent partially coherent GSM Beams with $\theta_0 = 0.1$, $w_0 = 1$ cm, and (a) $\lambda = 3.8$ μm, $\zeta_S = 20$, $L = 5$ km; (b) $\lambda = 1.06$ μm, $\zeta_S = 20$, $L = 5$ km; (c) $\lambda = 1.06$ μm, $\zeta_S = 50$, $L = 5$ km; and (d) $\lambda = 1.06$ μm, $\zeta_S = 50$, $L = 500$ m.

3.5. Angle-of-arrival fluctuation

When a laser beam propagates in the atmospheric turbulence, different parts of the beam wave front have different phase shifts due to the fluctuation of the refractive index. The phase shift will result in a phase plane corresponding to the random fluctuation shape. And this phase deformation causes the angle-of-arrival fluctuation of the beam wave front, leading to the pixel jitter of the beam at the receiver.

In slanted atmospheric turbulence, the mutual coherence function of the beam can be obtained by using the CSDF function $W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, L)$[30]

$$\begin{aligned} \Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, L) &= \langle U(\boldsymbol{\rho}_1, L) U^*(\boldsymbol{\rho}_2, L) \rangle \\ &\approx W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, L). \end{aligned} \tag{21}$$

The relationship between the mutual coherence function and the wave structure function is expressed as[31]

$$\begin{aligned} &\exp\left[-\frac{1}{2} D(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, L)\right] \\ &= \frac{|\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, L)|}{[\Gamma(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, L)\Gamma(\boldsymbol{\rho}_2, \boldsymbol{\rho}_2, L)]^{1/2}}. \end{aligned} \tag{22}$$

Substituting Eq. (12) into Eq. (22), we obtain the wave structure function of the GSM beam propagating in slanted atmospheric turbulence as

$$D(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,L) = 2\boldsymbol{\rho}^2\left\{\left(\frac{1}{\rho_{\rm T}^2}+\frac{1}{2w_0^2\Lambda_0^2}\right)-\frac{2L^2}{W_\zeta^2(L)\rho_{\rm T}^4} - \frac{w_0^2}{W_\zeta^2(L)}\left[\frac{k\Theta_0^2}{2L\Lambda_0}-\frac{\Theta_0}{\rho_{\rm T}^2}\right]\right\}. \quad (23)$$

Given that phase structure function $D_{\rm s}$ plays a dominant role in the wave structure function $D = D_\chi + D_{\rm s}$, the influence of the log-amplitude structure function D_χ can be disregarded. Thus, $D \approx D_{\rm s}$ can be realized. Using the definition of the angle-of-arrival fluctuation[32] and Eq. (23), we can obtain the angle-of-arrival fluctuation of the partially coherent GSM beam in the slanted atmospheric turbulence at the receiver as

$$\langle\alpha^2\rangle = \frac{D\left(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,L\right)}{\left(kW_\zeta\right)^2} = \frac{2}{k^2}\left\{\left(\frac{1}{\rho_{\rm T}^2}+\frac{1}{2w_0^2\Lambda_0^2}\right)-\frac{\varphi^2}{2W_\zeta^2(L)}\right\}. \quad (24)$$

Figure 9 shows that the angle-of-arrival fluctuation of the fully coherent GSM beam ($\zeta_{\rm S}=1$) essentially does not vary with the propagation distance. However, the angle-of-arrival fluctuation of the partially coherent GSM beam decreases with the increase of the propagation distance. When the beams propagate a distance of about 1 km, the angle-of-arrival fluctuation of the beams with different coherence degrees reaches a minimum, which no longer varies with the increase of the propagation distance. Otherwise, within the range of 1 km, the worse the beam coherence, the larger the angle-of-arrival fluctuation of the beam.

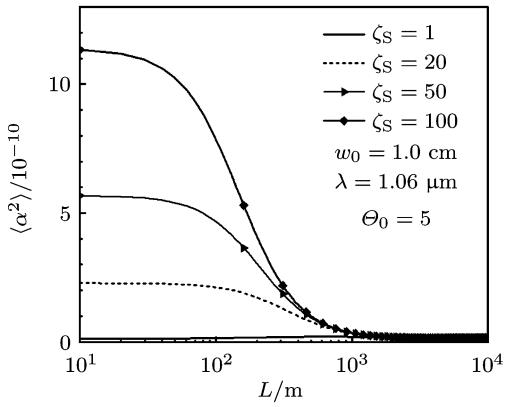


Fig. 9. Angle-of-arrival fluctuation of the divergent GSM beams as a function of propagation distance L for different coherence degrees.

4. Conclusion

According to the extended Huygens–Fresnel principle, we obtain expressions for the effective beam radius, the spreading angle, and the average intensity of a partially coherent GSM beam in slanted atmospheric turbulence by employing the CSDF. The numerical results are presented. Comparing the partially coherent GSM beams with the fully coherent Gaussian ones, the advantages of the partially coherent GSM beams over the fully coherent Gaussian ones are determined to be as follows. (i) The influences of the atmospheric turbulence and the propagation height on the radius expansion of the partially coherent GSM beams are smaller than those for the fully coherent Gaussian beams. (ii) The influences of the propagation height on the angle spreads of the partially coherent GSM beams are smaller than those of the fully coherent Gaussian beams. And the spreading angle of the partially coherent GSM beam decreases with the increase of beam waist radius w_0. (iii) The average intensity of the partially coherent GSM beam propagating in the turbulence is larger than that of the fully coherent Gaussian beam within a short propagation range.

In addition, in this paper, the angle-of-arrival fluctuation for a partially coherent GSM beam is obtained, and the wander effect of the GSM beam in slanted atmospheric turbulence is analysed numerically, therefore some conclusions are drawn. (i) The larger the wavelength, the more obvious the wander effect of the GSM beams. The worse the beam coherence, the more obvious the wander effect. And the wander effect of the partially coherent GSM beam in a short propagation range is smaller than that of the beam with a long-distance propagation. (ii) The worse the beam coherence, the larger the angle-of-arrival fluctuation of the beam. But the angle-of-arrival fluctuation of the partially coherent GSM beam decreases with the increase of the propagation distance. In practice, it is important to comprehensively consider the effects of the coherence degree, the propagation distance, the receiving height, the waist radius, etc. on the propagation characteristics of the beams and select the appropriate parameters to minimize the influence of the turbulence on the propagation characteristics of the beams.

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