Propagation of Airy beams from right-handed material to left-handed material*

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Based on the ABCD matrix formalism, the propagation property of an Airy beam from right-handed material (RHM) to left-handed material (LHM) is investigated. The result shows that when the Airy beam propagates in the LHM, the intensity self-bending due to its propagation in the RHM can be compensated. In particular, if the propagation distance in the RHM is equal to that in the LHM and the refractive index of the LHM is $n_{\rm L} = -1$, the transverse intensity distribution of the Airy beam can return to its original state.

Keywords: left-handed material, right-handed material, Airy beams, self-bending

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1. Introduction

About thirty years ago, Berry and Balazs[1] observed the nonspreading Airy wave packet. In recent years, this nonspreading Airy wave packet, which is generally called the Airy beam, has aroused much interest.[2−7] The remarkable feature of the Airy beam is its ability to freely accelerate, even in the absence of any external potential. It means that the intensity feature of the Airy beam can self-bend even in free space. Based on this novel feature, the self-bending of the Airy beam is shown to be advantageous for an optical-trapping system. The curved trajectory can be engineered to make it a versatile and powerful tool for optical manipulation. Moreover, as some experts pointed out, Airy beams may be used to convey nutrients or other chemicals around the curved membranes of living cells.[7] The Airy beam introduced by Berry and Balazs contains infinite energy, and its field distribution in the origin is $E(x,0) = \mathrm{Ai}(bx)$, where Ai denotes the Airy function, and b is a real and positive constant. While, the field distribution of the finite-energy Airy beam, which is introduced by Siviloglou and Christodoulides,[2] is $E(x,0) = \mathrm{Ai}(bx)\exp(ax)$, where a is a positive real quantity used to ensure the square integrability of the beam.

However, except for the anomalous refraction, which means the material is a negative refractive index dielectric, the left-handed material (LHM) has many other dramatically different phenomena compared with ordinary materials, viz. right-handed materials (RHMs), such as a reverse Doppler effect, an inverse Cerenkov radiation, and negative radiation pressure. Recently, research about materials with a negative index of refraction has received much attention.[8−19] Zhou *et al.*[12] revealed that an LHM slab can be used for focusing a Gaussian beam and for realizing the phase compensation when the light beam propagates inside and outside the LHM slab. Based on this, for further studying the focusing property of the LHM and investigating the self-bending of the Airy beam, in this paper, we investigate the propagation property of the Airy beam in the LHM, especially, the propagation from RHM to LHM.

The rest of this paper is organized as follows. First of all, based on the $ABCD$ matrix formalism and the Huygens diffraction integral, the field distribution of the Airy beam propagating from RHM to LHM is obtained. Then the evolutions of the intensity distributions in the RHM and the LHM are illustrated by detailed numerical calculations. Finally, some important conclusions are summarized.

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2. Theoretical analysis

The schematic diagram of the optical system is shown in Fig. 1.

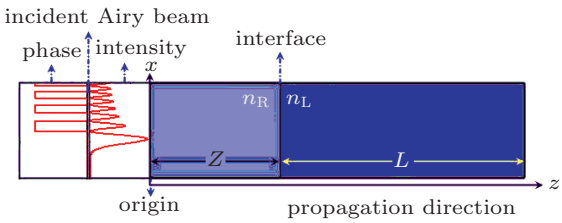


Fig. 1. (colour online) Schematic diagram of the propagation system.

It is assumed that the refractive index of the RHM is $n_R = 1$ (the Airy beam propagates in the free space first). The LHM is located at the right-hand side of the interface. In general, we take the incident Airy beam at the input plane as[3]

$$E(x_1, 0) = \mathrm{Ai}\left(\frac{x_1}{k_1}\right) \exp\left[\mathrm{i}s_1\left(\frac{x_1}{k_1}\right) + \mathrm{i}\frac{1}{3}s_1^3\right]. \quad (1)$$

When $s_1 = 0$, $E(x_1, 0)$ denotes the infinite-energy Airy beam, and when $s_1 \neq 0$, $E(x_1, 0)$ represents the finite-energy Airy beam. In Eq. (1), parameter k_1 controls the spatial frequency of the transverse field oscillation, and s_1 determines the possibility that the Airy beam has a transverse decay and tilt. In addition, we find that the value of k_1 has a strong influence on the self-bending when the Airy beam propagates forwards. The paraxial propagation of $E(x_1, 0)$ through the $ABCD$ system can be obtained by using the Huygens diffraction integral[20]

$$E(x_2, z) = \sqrt{\frac{k}{\mathrm{i}2\pi B}} \int_{-\infty}^{\infty} E(x_1, 0) \times \exp\left[\frac{\mathrm{i}k}{2B}(Ax_1^2 - 2x_1x_2 + Dx_2^2)\right] \mathrm{d}x_1, \quad (2)$$

where k is the wave number in free space. When the Airy beam propagates from RHM to LHM, the $ABCD$ matrix of the propagation system is given by[12]

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & n_R/n_L \end{bmatrix} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z + n_R L/n_L \\ 0 & n_R/n_L \end{bmatrix}, \quad (3)$$

where L is the propagation distance in the LHM, and Z denotes the position in the RHM. Substituting the transfer matrix into Eq. (2) and performing some integration in Eq. (2), we can obtain the transverse field of the Airy beam in the LHM as[3]

$$E(x_2, z) = \mathrm{Ai}\left(\frac{x_2 - \frac{\left(z+\frac{n_R}{n_L}L\right)}{kk_1}\left[s_1 + \frac{\left(z+\frac{n_R}{n_L}L\right)}{4kk_1^2}\right]}{k_1}\right) \times \exp\left(\mathrm{i}\left[s_1 + \frac{\left(z+\frac{n_R}{n_L}L\right)}{2kk_1^2}\right]\left\{\frac{x_2 - \frac{\left(z+\frac{n_R}{n_L}L\right)}{kk_1}\left(s_1 + \frac{\left(z+\frac{n_R}{n_L}L\right)}{4kk_1^2}\right)}{k_1}\right\} + \frac{\mathrm{i}}{3}\left(s_1 + \frac{z+\frac{n_R}{n_L}L}{2kk_1^2}\right)^3\right). \quad (4)$$

Thus, by using Eq. (4) we can obtain the propagation property of the Airy beam from RHM to LHM. In the next section, we will give some numerical calculation by using Eq. (4), and some important results will be illustrated.

3. Numerical results

In this section, we give some numerical results based on Eq. (4) to show how the Airy beam propagates from RHM to LHM. Some special properties of the Airy beam after the propagations in the two kinds of materials are illustrated.

Figure 2 shows the intensity evolution of the infinite-energy Airy beam during the propagation from RHM to LHM. From the intensity evolution we find that as the Airy beam propagates in the RHM, the intensity distribution experiences self-bending. However, when the Airy beam propagates through the interface and in the LHM, the self-bending caused by its propagation in the RHM can be compensated. In the numerical calculation we assume the refractive index of the LHM to be $n_L = -1$. Although $n_L = -1$ is special, it is convenient for revealing the propagation

property of the Airy beam.

The intensity evolution of the finite-energy Airy beam is presented in Fig. 3. We find that similar to the infinite-energy Airy beam propagating in the RHM and the LHM, the self-bending of the finite-energy Airy beam can also be compensated. It is found that when the Airy beam propagates in the RHM, the maximum intensity decreases as the propagation distance increases. However, after the beam passes through the interface of the LHM and continues to propagate in the LHM, the intensity increases as the propagation distance increases. This phenomenon is similar to the focusing behavior. As is well known, the LHM can act as a perfect lens; therefore the propagation of the Airy beam seems to be focused by the lens, the intensity becomes larger as it propagates in the LHM.

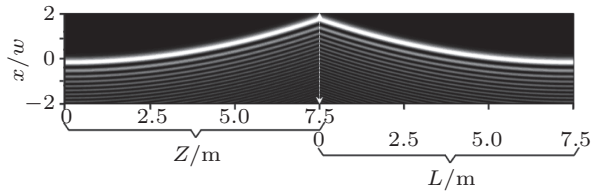


Fig. 2. Intensity distribution in the longitudinal section of the infinite-energy Airy beam propagating from RHM to LHM. The arrow indicates the interface. The parameters used are $k_1 = 1/6w$, $s_1 = 0$, and $w = 2$ mm. The wavelength of the beam is $\lambda = 780$ nm.

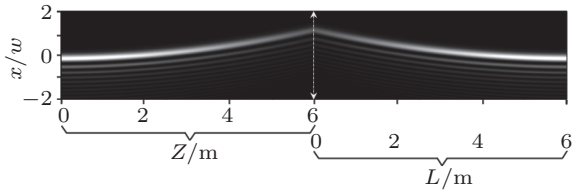


Fig. 3. Intensity distribution in the longitudinal section of the finite-energy Airy beam propagating from RHM onto LHM. The arrow indicates the interface. Here $s_1 = -(0.2k_1w)$i, and the other parameters are the same as those in Fig. 2.

The index of refraction of the LHM is assumed to be -1 in Figs. 2 and 3. We further consider the propagation of the Airy beam in the LHM that has a different refractive index. In Figs. 4(a1) and 4(a2), the refractive index is $n_{\rm L} = -0.8$; in Figs. 4(b1) and 4(b2), $n_{\rm L} = -1$; in Figs. 4(c1) and 4(c2), $n_{\rm L} = -1.2$. The Airy beam propagates 5 m in the free space, and then transmits through the interface into the LHM. Figure 4 shows the evolution of the Airy beam in the LHM. The calculation results reveal that when the Airy beam propagates through the interface and prop-

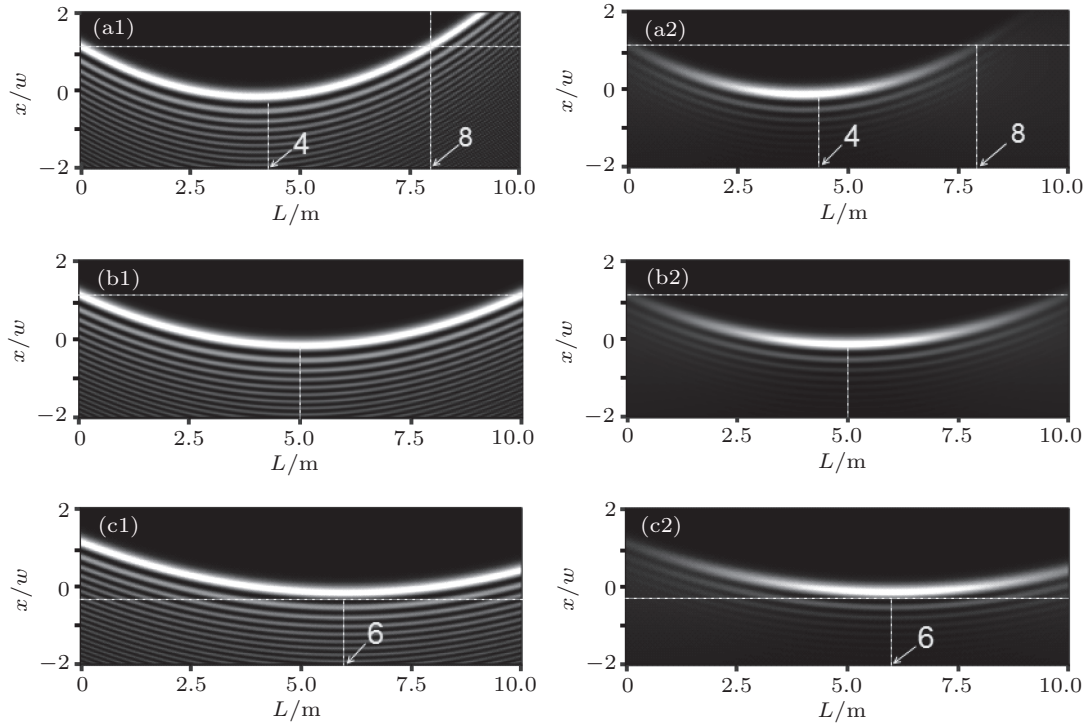


Fig. 4. Intensity distributions of the Airy beam during the propagation from RHM to LHM for different refractive indices of the LHM. The propagation distance in the RHM is $Z = 5$ m. In panels (a1), (b1), and (c1), $s_1 = 0$; in panels (a2), (b2), and (c2), $s_1 = -(0.2k_1w)$i. In panels (a1) and (a2), $n_{\rm L} = -0.8$; in panels (b1) and (b2), $n_{\rm L} = -1$; in panels (c1) and (c2), $n_{\rm L} = -1.2$. The other parameters are the same as those in Fig. 2. The label indicates the necessary distance for returning to the original state.

agates in the LHM, the self-bending caused by its propagation in the RHM can be compensated. Moreover, with the increase of the propagation distance the intensity self-bends again. In Fig. 4, the vertical dotted line indicates the necessary propagation distance for compensating the self-bending. For the case of $n_{\rm L} = -0.8$, the necessary propagation distance is $L = 4$ m. For $n_{\rm L} = -1.2$, the necessary propagation distance is $L = 6$ m. Especially, if the refractive indices of the RHM and the LHM are opposite numbers, as long as the propagation distance in the LHM is equal to that in the RHM, the transverse intensity distribution of the Airy beam returns to its original state. In addition, if the absolute value of the refractive index of the LHM is smaller than that of the RHM, the propagation distance in the LHM for compensating the self-bending is smaller than that in the RHM. Otherwise, the result is opposite.

For simplicity we take the refractive index of the LHM to be $n_{\rm L} = -1$ in the following discussion. In Fig. 5 we consider only the propagation of the Airy beam in the LHM. In Figs. 5(a1)–5(c1), we plot the intensity evolutions of the infinite-energy Airy beam in the LHM with different values of k_1. And Figures 5(a2)–5(c2) show the intensity evolutions of the finite-energy Airy beam in the LHM with different values of k_1. It is found from Fig. 5 that the value of k_1 not only controls the spatial frequency of the transverse field oscillation, but also influences the self-bending of the Airy beam. We find that the smaller the value of k_1, the more obvious the intensity self-bending of the Airy beam.

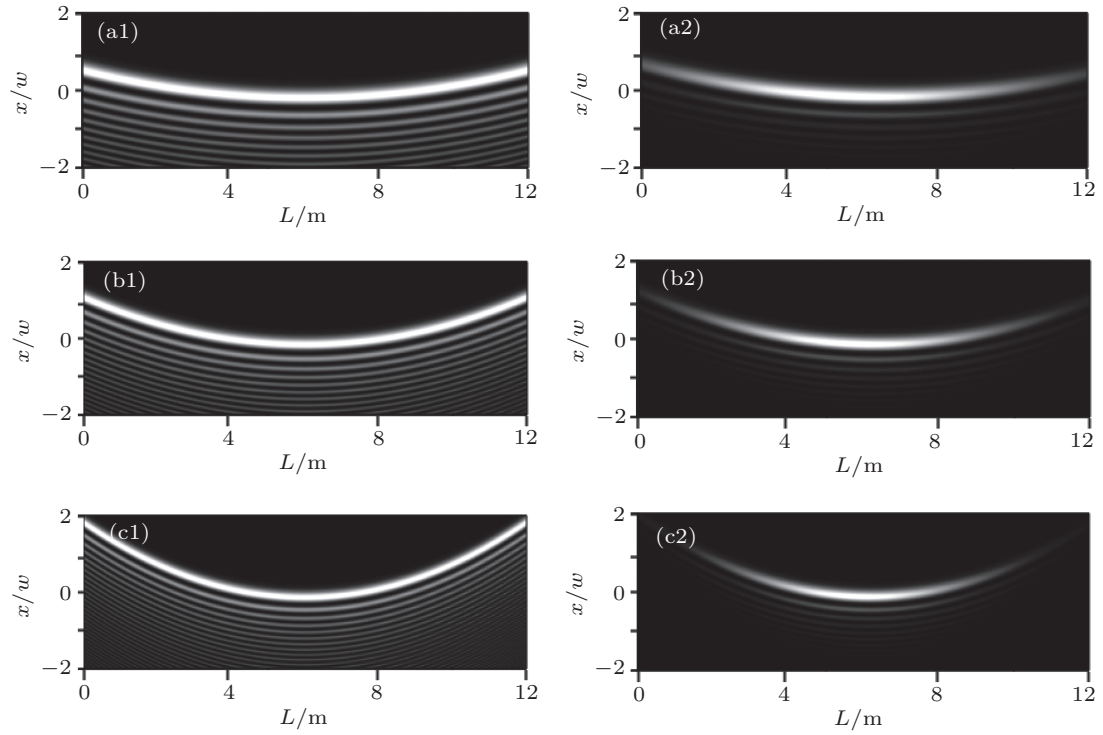


Fig. 5. Intensity distributions of the Airy beam during the propagation from RHM to LHM for different values of k_1, the propagation distance in the RHM is $Z = 6$ m. In panels (a1), (b1), and (c1), $s_1 = 0$; in panels (a2), (b2), and (c2), $s_1 = -(0.2k_1w)$i. In panels (a1) and (a2), $k_1 = 1/5w$; in panels (b1) and (b2), $k_1 = 1/6w$; in panels (c1) and (c2), $k_1 = 1/7w$. The wavelength is $\lambda = 632.8$ nm.

Figure 6 shows the intensity distributions of the finite-energy Airy beam in the LHM for different values of s_1. From Fig. 6, we can find that the value of s_1 modulates the transverse decay and tilt of the finite-energy Airy beam. By comparing Figs. 5(a)–5(c), we can see that the influence of s_1 on the self-bending is not obvious.

By using the following formula:

$$x_{\rm c}(z) = \langle x(z) \rangle = \frac{\int_{-\infty}^{\infty} x \left| E(x,z) \right|^2 {\rm d}x}{\int_{-\infty}^{\infty} \left| E(x,z) \right|^2 {\rm d}x}, \qquad (5)$$

we can obtain the centroid of the beam as a function of the propagation distance. Substituting Eq. (4) into Eq. (5), we can obtain the centroid of the Airy beam

as a function of the propagation distance. Figure 7 shows the centroids of the Airy beams each as a function of the propagation distance for different values of s_1 and k_1. As shown in Fig. 7, when the Airy beam propagates from RHM to LHM, the self-bending of the centroid can be compensated. Moreover, when the propagation distance in the RHM is equal to the one in the LHM, the self-bending of the centroid vanishes. Thus, we can expect that when the propagation distance in the RHM is equal to that in the LHM, the transverse centroid should return to the original position.

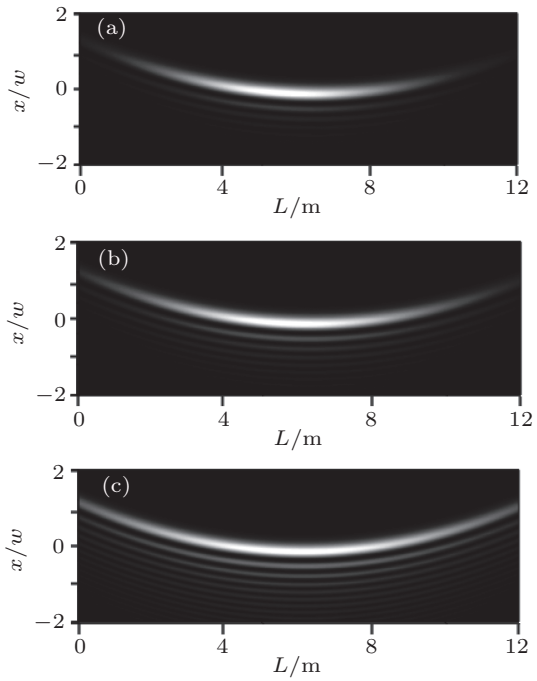


Fig. 6. Intensity distributions of the finite-energy Airy beam during the propagation from RHM to LHM for different values of s_1. The propagation distance in the RHM is $Z = 6$ m, $k_1 = 1/6w$, $\lambda = 632.8$ nm, and (a) $s_1 = -(0.3k_1w)$i, (b) $s_1 = -(0.2k_1w)$i, and (c) $s_1 = -(0.1k_1w)$i.

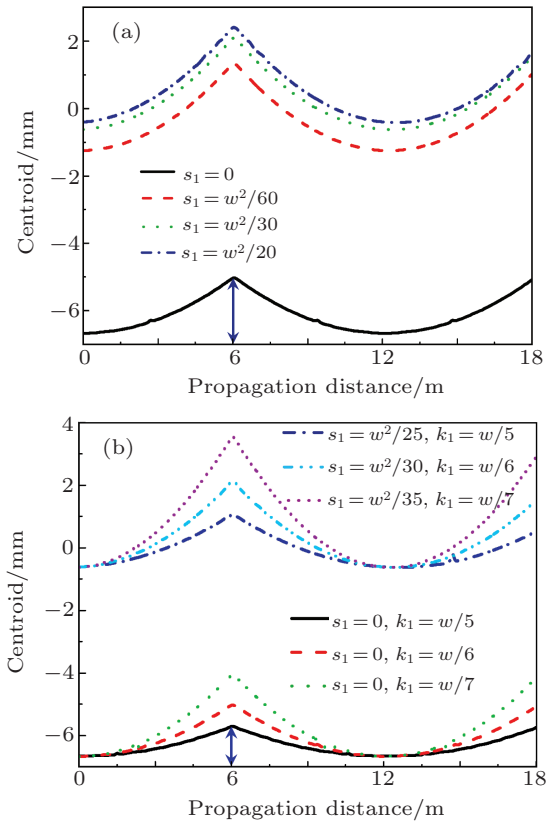


Fig. 7. (colour online) Expected values of x for the Airy beams with (a) different s_1 and (b) different s_1 and k_1 during the propagation from RHM to LHM. The arrow indicates the position of the interface.

4. Conclusion

Based on the Huygens diffraction integral and the formula of the $ABCD$ transfer matrix, we investigate the propagation properties of the Airy beam from RHM to LHM. It is shown that as the Airy beam propagates through the RHM, the transverse intensity of the Airy beam bends up. However, when the Airy beam propagates through the RHM and then propagates in the LHM, the self-bending caused by its propagation in the RHM can be compensated. Additionally, for $n_{\rm L} = -1$, when the propagation distance in the LHM is equal to that in the RHM, the transverse intensity distribution of the Airy beam returns to its original state. When the beam continues to propagate in the LHM, the transverse intensity of the Airy beam bends up again.

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