Nonautonomous bright solitons and soliton collisions in a nonlinear medium with an external potential*

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We present exact bright multi-soliton solutions of a generalized nonautonomous nonlinear Schrödinger equation with time- and space-dependent distributed coefficients and an external potential which describes a pulse propagating in nonlinear media when its transverse and longitudinal directions are nonuniformly distributed. Such solutions exist in certain constraint conditions on the coefficients depicting dispersion, nonlinearity, and gain (loss). Various shapes of bright solitons and interesting interactions between two solitons are observed. Physical applications of interest to the field and stability of the solitons are discussed.

Keywords: generalized nonautonomous nonlinear Schrödinger equation, multi-soliton solution, Fourier-synthesized optical lattice potential, flying bird potential

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1. Introduction

Nonautonomous systems have attracted much attention in recent years because of their interesting features and potential applications.[1−4] These systems support temporal and/or spatial solitons, soliton lasers, ultrafast soliton switches, and logic gates for experiments.[5−9] Nonautonomous general nonlinear Schrödinger (NLS) equations, appearing in the contexts of plasma physics, Bose–Einstein condensations (BECs), and nonlinear optics,[1,2,10−12] are the most typical ones among them. The soliton solutions to such models are often called nonautonomous solitons. Dynamics of solitons described by the generalized nonautonomous NLS equations with varying dispersion, nonlinearity, and dissipation or gain including soliton interaction scenarios among solitons have been studied in Refs. [1] and [13]. The management of dispersion, amplification, soliton pulse, and energy control of optical solitons was investigated in Refs. [10] and [11]. Nonautonomous solitons have been found in BECs[14] and investigated in nonautonomous systems with external potentials.[1,15] The dynamics of nonautonomous solitons with respect to NLS equations with variable coefficients has also been discussed via several methods.[16−21]

In addition, the similarity transformation technique has been intensively used to solve the nonautonomous NLS models and successfully applied to these models.[22−25] Recent studies in Refs. [26] and [27] provide a very useful similarity transformation for solving the NLS models with distributed coefficients and an external potential. Up to now, the generalized nonautonomous NLS equations with time- and space-dependent distributed coefficients (including dispersion, nonlinearity, and gain (loss) terms) and an external potential have been seldom reported,[28] but they also have very interesting properties. Gagnon and Winternitz[29] treated the variable coefficient NLS equations by using symmetry reduction and obtained a set of exact solutions. He and Li[30] studied the generalized nonautonomous cubic-quintic NLS equation with time- and space-dependent distributed coefficients and external potentials and gave the analytical solitary wave solutions to it. They[31] also dealt with the generalized nonautonomous NLS equation with time- and space-dependent distributed coefficients in a Fourier-synthesized optical lattice potential and gave the analytical bright multi-soliton solutions of it. In this paper, we aim at providing exact bright multi-soliton solutions of a nonautonomous NLS equation with different external potentials (including linear potentials, harmonic potentials, Fourier-synthesized optical lattice potentials, and flying bird potentials, which are important in the physical field and have been intensively studied[32−35]) by

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employing the similarity transformation technique.

2. General method and solutions

In this paper, we consider a generalized nonautonomous NLS equation with an external potential describing soliton management in nonlinear optics.[36] The equation takes the following one-dimensional form:

$$\mathrm{i}\frac{\partial\psi}{\partial t}+f(x,t)\frac{\partial^2\psi}{\partial x^2}+g(x,t)|\psi|^2\psi +V(x,t)\psi+\mathrm{i}\gamma(x,t)\psi=0, \tag{1}$$

where $\psi(x,t)$ is a complex function, x is the transverse variable, and t is the longitudinal variable. $f(x,t)$ and $g(x,t)$ are the dispersion and nonlinearity management parameters, respectively. $V(x,t)$ is the external potential and $\gamma(x,t)$ is the gain (loss) term. Equation (1) describes the pulse propagation in nonlinear media when its transverse and longitudinal directions are nonuniformly distributed. In the context of BEC, equation (1) can also describe the dynamics of matter-wave solitons, where t and x represent the time and spatial coordinate, respectively. The possibility of soliton management in BECs can be realized by adjusting the related control parameters via the technique of Feshbach resonance (FR).[37−39] The elimination of the loss term in Eq. (1) has been studied in Refs. [40] and [41], and the case of $f(x,t)=1$, $\gamma(x,t)=0$ in Eq. (1) has been studied in Refs. [22] and [23]. In Ref. [41], He *et al.* introduced the designable integrability of Eq. (1) and presented solutions for the optical super-lattice potentials and multi-well potentials. Recently, equation (1) was dealt with by using the Painlevé analysis, and the soliton solutions for linear and quadratic potentials under the integrability conditions were derived. More recently, the integrability conditions of Eq. (1) were derived by using the Lax pair and similarity transformation methods.[42] It should be emphasized that the present work is different from the above ones: by choosing different forms of the similarity transformations, we can not only obtain the harmonic trapping potential, but also obtain the Fourier-synthesized optical lattice potential and flying bird potential. Various shapes of the bright solitons and the interaction between two solitons are then observed under these potentials. This is also a significant difference between our work and Refs. [42] and [43].

Our idea is to reduce Eq. (1) to the following standard NLS equation:

$$\mathrm{i}\frac{\partial}{\partial T}\Phi+\frac{1}{2}\frac{\partial^2}{\partial X^2}\Phi+g_0|\Phi|^2\Phi=0, \tag{2}$$

where $\Phi(X,T)$, $\rho(t)$, $\phi(x,t)$, $X(x,t)$, and $T(t)$ are functions to be determined, and g_0 is a real constant.

The bright one-soliton solution in the bilinear form can be given by[43,44]

$$\psi(x,t)=k\rho\,\mathrm{sech}[k(X-vT)]\,\mathrm{e}^{\mathrm{i}\theta}, \tag{3}$$

where $\theta=vX+(k^2-v^2)/2T+\phi$ with $k,v\in\mathbb{R}$, and ρ and ϕ are functions of x and t to be determined below.

The bright two-soliton solution reads

$$\psi(x,t)=\rho\frac{A(X,T)}{B(X,T)}\,\mathrm{e}^{\mathrm{i}\phi}, \tag{4}$$

where $A(X,T)$ and $B(X,T)$ satisfy

$$A(X,T)=\sum_{m=1}^{2}\mathrm{e}^{\xi_m}+\sum_{m,n=1,m\neq n}^{2}\lambda_{mn}\,\mathrm{e}^{\xi_m+\xi_m^*+\xi_n},$$

$$B(X,T)=1+\sum_{m,n=1}^{2}\mu_{mn}\,\mathrm{e}^{\xi_m+\xi_n^*}+\kappa\,\mathrm{e}^{\xi_1+\xi_2+\xi_1^*+\xi_2^*},$$

where $\xi_m=k_mX+\mathrm{i}k_m^2T/2+\xi_{m0}$, $k_m=r_m+\mathrm{i}s_m$, $\lambda_{mn}=(k_m-k_n)^2[(k_m+k_m^*)(k_n+k_m^*)]^{-2}$, $\mu_{mn}=(k_m+k_n^*)^{-2}$, and $\kappa=[(k_1-k_2)(k_1^*-k_2^*)]^2[(k_1+k_1^*)(k_1+k_2^*)(k_2+k_1^*)(k_2+k_2^*)]^{-2}$ $(\xi_{m0},r_m,s_m\in\mathbb{R})$.

Next, we construct the exact solutions of Eq. (1) according to Eqs. (3) and (4).

2.1. Dynamics of the nonautonomous bright solitons and soliton collisions in linear and harmonic potentials

We introduce $X(x,t)=\alpha(t)x+\beta(t)$, with $\alpha(t)$ and $\beta(t)$ being positive definite functions of time. Applying the transformation

$$\psi(x,t)=\rho(x,t)\Phi(X(x,t),T(t))\,\mathrm{e}^{\mathrm{i}\phi(x,t)} \tag{5}$$

to Eq. (1) and supposing $\Phi(X,T)$ satisfies Eq. (2), we find a set of equations. By solving these equations, we find that the coefficients of Eq. (1) are time-dependent, i.e.,

$$f(x,t)=f(t),\ g(x,t)=g(t),\ \gamma(x,t)=\gamma(t), \tag{6}$$

and they satisfy some constraint conditions. For simplicity, we choose $g(t)=\alpha$, and then find the following solutions:

$$f(t)=\frac{a}{2g_0\alpha^2},\ \gamma(t)=\frac{\alpha_t}{\alpha}-\frac{a_t}{2a},\ \rho(x,t)=\sqrt{\frac{a}{\alpha}}, \tag{7a}$$

$$\phi(x,t)=-\frac{g_0\alpha}{2a}(\alpha_tx^2+2\beta_tx)+b, \tag{7b}$$

$$T(t) = g_0^{-1} \int_0^t a(t')\mathrm{d}t' + T_0, \tag{7c}$$

where $a = a(t)$ and $b = b(t)$ are arbitrary functions of time, and T_0 is an arbitrary constant.

Here the potential is given by

$$V(x,t) = V_1(t)x^2 + V_2(t)x + V_3(t), \tag{8}$$

where

$$\begin{aligned} V_1(t) &= \frac{g_0\alpha}{2a^2}(a_t\alpha_t - a\alpha_{tt}), \\ V_2(t) &= \frac{g_0\alpha}{a^2}(a_t\beta_t - a\beta_{tt}), \\ V_3(t) &= \frac{\alpha^2 b_t}{a^2} + \frac{g_0\beta_t^2}{2a}. \end{aligned}$$

Therefore, Eqs. (7) and (8) are the constraint conditions on the coefficients depicting dispersion, nonlinearity, gain, and external potential for exact solutions by the transformation (5). From Eq. (9) we can obtain the linear and harmonic potentials and a combination of them by selecting special values of the parameters α, β, a, and b. We will demonstrate this situation below. Here we will use

$$\alpha(t) = 1 + \epsilon\cos(\omega_0 t), \tag{9}$$

where $\epsilon \in (-1, 1)$ and $\omega_0 \in \mathbb{R}$. Note that it is always true that the parameters $\alpha = \beta = a = b = 1$ in our solutions when they are arbitrary constants.

Case 1 Vanishing potential. As can be seen from Eq. (8), when α, β, and b are real constants, one can set $V(x,t) = 0$. In this case, Eq. (1) describes the evolution of nonlinear optical pulses in nonlinear media if a is a constant. Soliton solutions in this case are standard types, which can be obtained from Eqs. (3) and (4).

Case 2 Linear potential. Next, we study the dynamics of Eq. (1) with a space linear potential. For simplicity, we set $\alpha = a = b = 1$, the coefficients f and g are constants, and the phase ϕ is the linear nature with respect to the x coordinate. Thus, $\beta = \beta_0\sin(t)$ corresponds to the space linear potential

$$V(x,t) = \beta_0\sin(t)x + g_0\beta_0^2\cos(t)^2/2. \tag{10}$$

The corresponding bright one- and two-soliton solutions are displayed in Figs. 1(a) and 1(b), respectively.

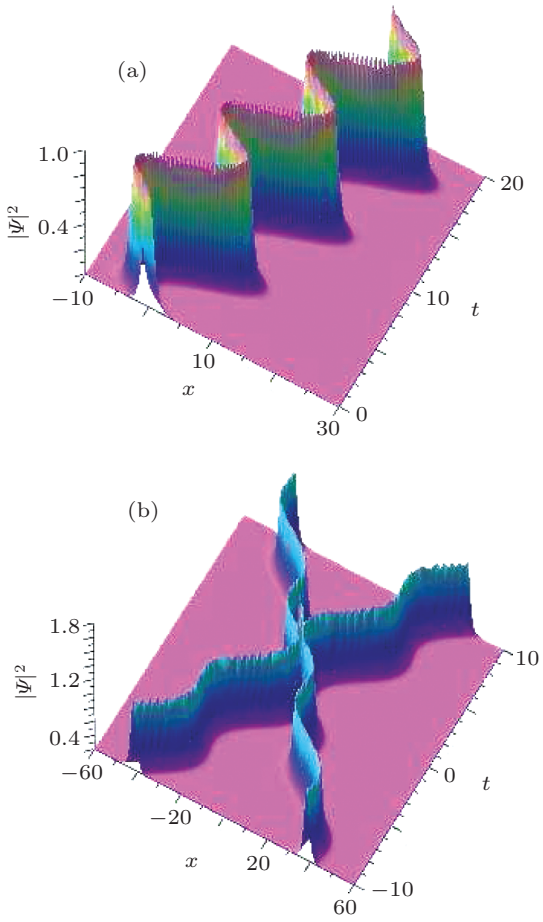


Fig. 1. (colour online) Intensity profiles of (a) the bright one-soliton solution and (b) two-soliton solution of Eq. (1) for $\beta_0 = 5$, in the case of a linear potential. The parameters are: $g_0 = 1$, $T_0 = 0$, $\omega_0 = k = v = 1$, $\epsilon = 0.5$, $\xi_{10,20} = 0$, $r_1 = 1$, $r_2 = 1$, $s_1 = 4, and\ s_2 = -4$.

Case 3 Harmonic potential. We suppose that β and b are arbitrary constants, which leads to $V_2(t) = V_3(t) = 0$. The harmonic potential that is important in realizing BEC in experiments has been studied in Refs. [22]–[27]. It reads

$$V(x,t) = V_1(t)x^2. \tag{11}$$

Next, we study the dynamics of the nonautonomous bright solitons and soliton collisions for three types of harmonic potentials.

1) Harmonic potential without time modulation. We first consider the harmonic potential without time modulation by setting $a = \alpha^2$, and $V_1(t) = \lambda$, where λ is a constant. This choice leads to $\alpha = 2C_0\,\mathrm{e}^{C_0 t}/(C_1\,\mathrm{e}^{2C_0 t} - C_2)$, where $C_0 = \sqrt{2\lambda/g_0}$, C_1 and C_2 are arbitrary real constants. Thus, the potential has the following form:

$$V(x,t) = \lambda x^2, \tag{12}$$

An interesting phenomenon in this case is that one can obtain broadened waves by selecting special values of C_1 and C_2. Examples of the broadened waves are displayed in Fig. 2(a) by choosing the parameters $C_1 = 1$ and $C_2 = -1$.

2) Harmonic trapping potential. Still letting $a = \alpha^2$ and defining a function $\chi = 1/\alpha$, we have the Mathieu equation $\mathrm{d}^2\chi/\mathrm{d}t^2 - (2/g_0)\theta^2(t)\chi = 0$ with $\theta^2(t) = 1 + \varepsilon\cos(\theta_0 t)$. Then, the trapping potential is given by

$$V(x,t) = [1 + \varepsilon\cos(\theta_0 t)]x^2, \tag{13}$$

where $\varepsilon \in (-1,1)$ and $\theta_0 \in \mathbb{R}$. For simplicity, we choose the parameters $g_0 = \theta_0 = 1$, and $\varepsilon = 0.5$. The solution of the above Mathieu equation is $\chi = AC_{\mathrm{M}}(-8,2,t/2) + BS_{\mathrm{M}}(-8,2,t/2)$, where A and B are constants, C_{M} and S_{M} refer to the cosine and sine Mathieu functions, respectively. An example of a soliton solution is shown in Fig. 2(b).

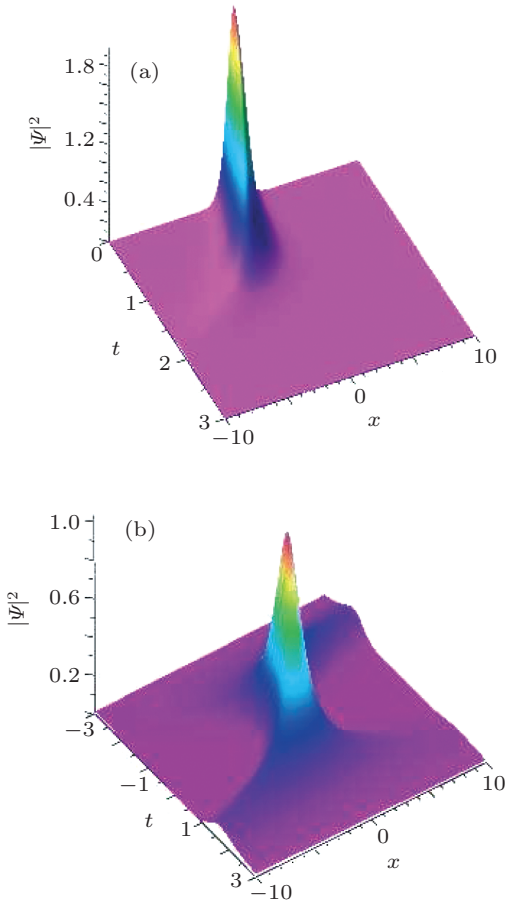


Fig. 2. (colour online) Intensity profiles of (a) the bright one-soliton solution in the case of a harmonic potential without time modulation, and (b) the bright one-soliton solution in the case of a harmonic trapping potential. Parameters are the same as those used in Fig. 1.

3) Harmonic potential with a periodic optical superlattice modulation. For instance, we also investigate the behaviour of the solution in the case of a harmonic potential with a time-periodic optical superlattice modulation. To this end, we choose $a = 1$, which leads to $\rho = 1/\sqrt{\alpha}$. Now, the potential takes the following form:

$$V(x,t) = \frac{1}{4}\epsilon g_0\omega_0^2[\epsilon + 2\cos(\omega_0 t) + \epsilon\cos(2\omega_0 t)]x^2. \tag{14}$$

This potential is modulated by a periodic optical superlattice, which includes two different frequencies (ω_0 and $2\omega_0$).[45] In this case, the soliton presents an interesting periodical moving pattern, as shown in Figs. 3(a) and 3(b).

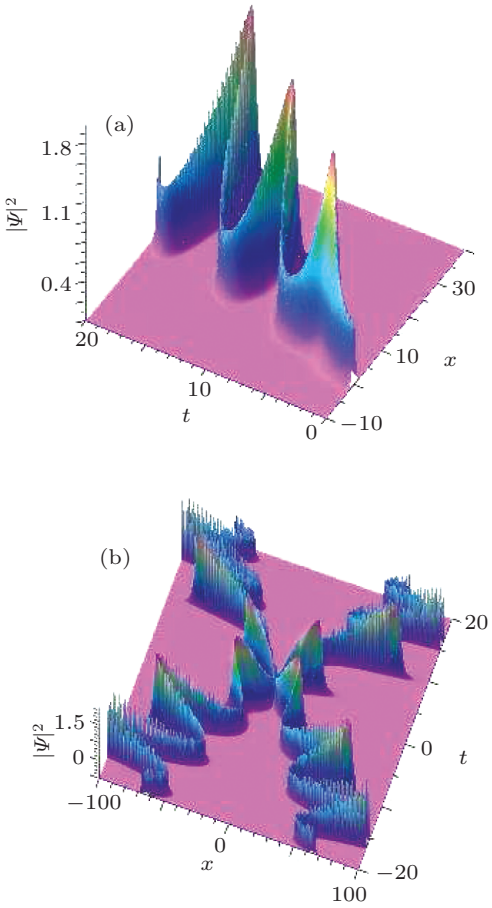


Fig. 3. (colour online) Intensity profiles of (a) the bright one-soliton solution and (b) two-soliton solution of Eq. (1), in the case of a harmonic potential with a periodic optical superlattice modulation. Parameters are the same as those used in Fig. 1.

Case 4 Another potential. Equation (8) can also stimulate other potentials. For instance, by setting $a = \alpha^2$, $\beta = \alpha$, and $b = 1$, we obtain the following potential:

$$V(x,t) = \frac{1}{2(\cos(t)^2 + 4\cos(t) + 4)} \times [V_1(t)x^2 + V_2(t)x + \sin(t)^2]. \tag{15}$$

where $V_1(t) = -\cos(t)^2 + 2\cos(t) + 2, V_2(t) = \cos(t)^3/2 + 2\cos(t) + 2$. For simplicity, we set $g_0 = 1$, $\epsilon = 0.5$, and $\omega_0 = 1$. This potential is plotted in Fig. 4(a). In this case, the soliton presents a periodical oscillation with time, as shown in Fig. 4(b). It is observed that the amplitude and width of the soliton vary with time periodically. Note that the two-soliton solution, which is not shown here due to its similarity with the profile in Fig. 3(b), can also be obtained from Eq. (4).

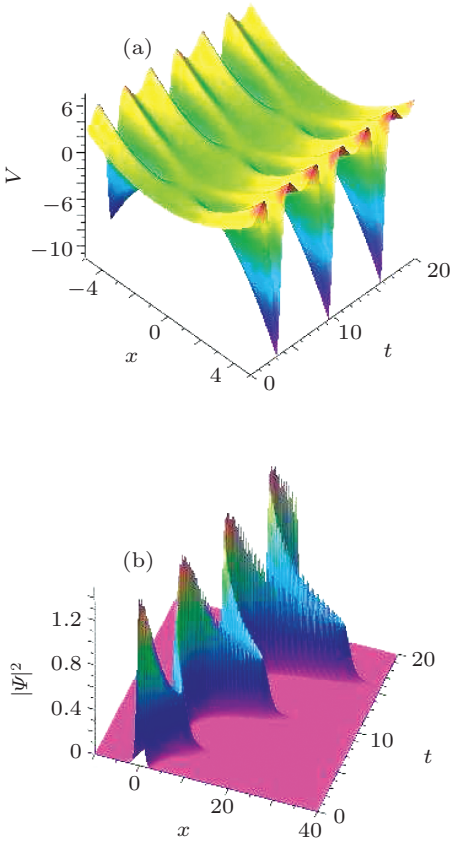


Fig. 4. (colour online) (a) Plot of the potential given by Eq. (15). (b) Corresponding intensity profile of the bright one-soliton solution. Parameters are the same as those used in Fig. 1.

We note that the above solutions may exist under the dispersion management in BECs only if a controlled variation of the effective mass is available.[43]

2.2. Dynamics of the nonautonomous bright solitons and soliton collisions in Fourier-synthesized optical lattice potential

We show that, in another form of similarity transformation, the matter-wave solitons trapped in a rapidly driven asymmetric optical lattice where their mass-dependent transport is available can be formed in the following Fourier-synthesized optical lattice potential:

$$V(x,t) = V_0 h(t)[V_1 + V_2\cos(x) + V_3\cos(2x)], \quad (16)$$

where $h(t) = \sin(\omega t) + \sin(2\omega t)$, $V_1 = 24V_0$, $V_2 = 4\iota$, and $V_3 = 8V_0$. V_0 denotes the intensity of the laser beams forming the lattice, and ι is a positive constant. This potential can be obtained by using the transformation

$$\psi(x,t) = \rho(x,t)\Phi(X(x),T(t))\,\mathrm{e}^{\mathrm{i}\phi(x,t)}, \quad (17)$$

and defining $X(x) = \int_0^x 1/[\iota + 8V_0\cos(s)]\mathrm{d}s$, $T(t) = 2\int_0^t h(t)\mathrm{d}t + T_0$, where $V_0 \in (-\iota/8, \iota/8)$ and T_0 is an arbitrary constant. Substituting Eq. (17) into Eq. (1) with $\Phi(X,T)$ satisfying Eq. (2) leads to the following solutions:

$$f(x,t) = \frac{h(t)}{X_x^2},\ g(x,t) = \frac{2g_0 h(t) X_x}{a},$$
$$\gamma(x,t) = -\frac{a_t}{2a}, \quad (18a)$$
$$\rho(x,t) = \sqrt{\frac{a}{X_x}}, \quad \phi(x,t) = \phi_0, \quad (18b)$$

where $a = a(t)$ is an arbitrary function of time, and ϕ_0 is an arbitrary constant.

From Eq. (18a) one can see that the gain (loss) coefficient $\gamma(t)$ depends on the parameter a. Thus, one can obtain various types of solutions by choosing the form of a. When a is a constant and $\gamma(t)$ equals zero, the parameter $\rho(x,t) = \rho(x)$ is space-dependent. If a is a function of time, we especially choose

$$\gamma(t) = \gamma_0\,\mathrm{e}^{-\sigma t}, \quad \gamma_0, \sigma \in \mathbb{R}, \quad (19)$$

which is attenuated (grew) exponentially with time. The control of $\gamma(t)$ is dependent on the optical lattice and FR techniques in experiments. In this case, we obtain $a = C_0\exp[2\gamma_0\exp(-\sigma t)/\sigma]$.

We show the dynamics of the solutions in Fig. 5, from which one can see that after an initial rapid decrease, the pulse turns into a slow M-shape trace. Similarly, after an initial rapid decrease, the two pulses interact with each other and propagate in the previous shape before and after collision, as shown in Fig. 5(b). Note that we have obtained similar results (without an initial decrease of pulses) with Ref. [31] by choosing $\gamma(t)$ as a periodic function.

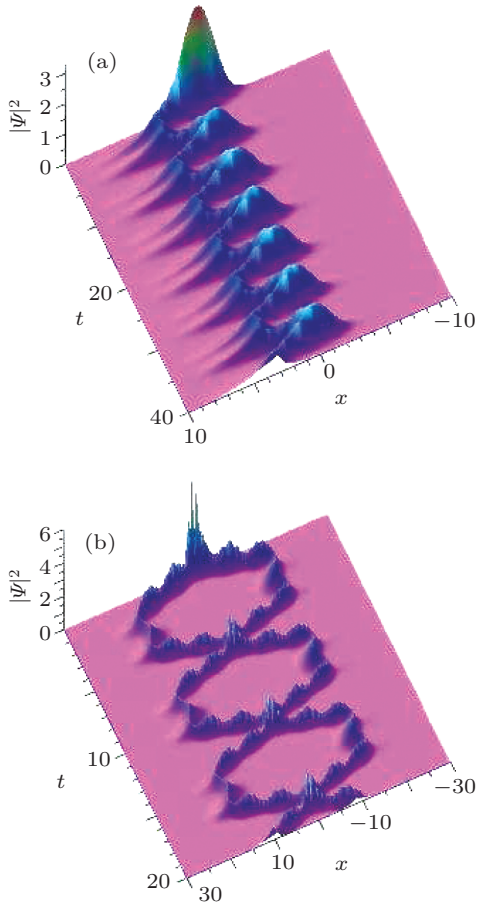


Fig. 5. (colour online) Intensity profiles of (a) the bright one-soliton solution and (b) two-soliton solution of Eq. (1) with $C_0 = \gamma_0 = \sigma = 1$, in the case of an optical lattice potential. Other parameters are the same as those used in Fig. 1.

2.3. Dynamics of the nonautonomous bright solitons and soliton collisions in flying bird potential

Now we use the general similarity transformation form

$$\psi(x,t) = \rho(x,t)\Phi(X(x,t),T(t))\,\mathrm{e}^{\mathrm{i}\phi(x,t)}, \qquad (20)$$

by defining $X(x,t) = \int_0^{\xi} F[\xi'(x,t)]\mathrm{d}\xi'$, where $\xi(x,t) = \tau(t)x$ and $\tau(t)$ is a positive definite function of time. Particularly, we choose the nonlinearity $g(x,t) = \tau\,\mathrm{e}^{\xi^2/b^2}$, where b is a real constant. This nonlinearity may simulate an interesting external potential and solutions of Eq. (1) in BECs with controlled optical interactions.[21] Given that $\Phi(X,T)$ satisfies Eq. (2), after substituting Eq. (20) into Eq. (1), we find the following solutions:

$$\rho(x,t) = \mathrm{e}^{-\xi^2/(2b^2)}/\sqrt{\tau},\ T(t) = t/g_0 + T_0, \qquad (21a)$$

$$\phi(x,t) = -g_0\tau\tau_t \int_0^x \mathrm{e}^{2\xi^2/b^2} x'\,\mathrm{d}x', \qquad (21b)$$

$$f(x,t) = \mathrm{e}^{-2\xi^2/b^2}/(2g_0\tau^2),$$

$$\gamma(x,t) = \frac{\tau_t}{\tau} + \frac{2\tau_t\xi}{b^2}x. \qquad (21c)$$

The potential is given as follows:

$$V(x,t) = l(t) + h_1(\xi,t)\,\mathrm{e}^{2\xi^2/b^2} + h_2(\xi,t)\,\mathrm{e}^{-2\xi^2/b^2}, \qquad (22)$$

where

$$l(t) = \frac{g_0 b^2}{4\tau^2}(\tau\tau_{tt} - \tau_t^2),$$

$$h_1(\xi,t) = \frac{g_0 b^2[\tau_t^2(1 - 2\xi^2/b^2) - \tau\tau_{tt}]}{4\tau^2},$$

$$h_2(\xi,t) = \frac{1 + \xi^2/b^2}{2g_0 b^2}.$$

One can see that the potential includes an arbitrary function $\tau(t)$ which influences the profile of the potential. Without the loss of generality, we define $\tau(t) = 1 + \epsilon\cos(\omega_0 t)$ so that it takes the same form as $\alpha(t)$. In this case, the potential exhibits a flying bird feature when $\epsilon \neq 0$, as displayed in Fig. 6(a). Similar to the solutions reported in Ref. [30], the pulses obtained here, both in the potential barrier (well) and in

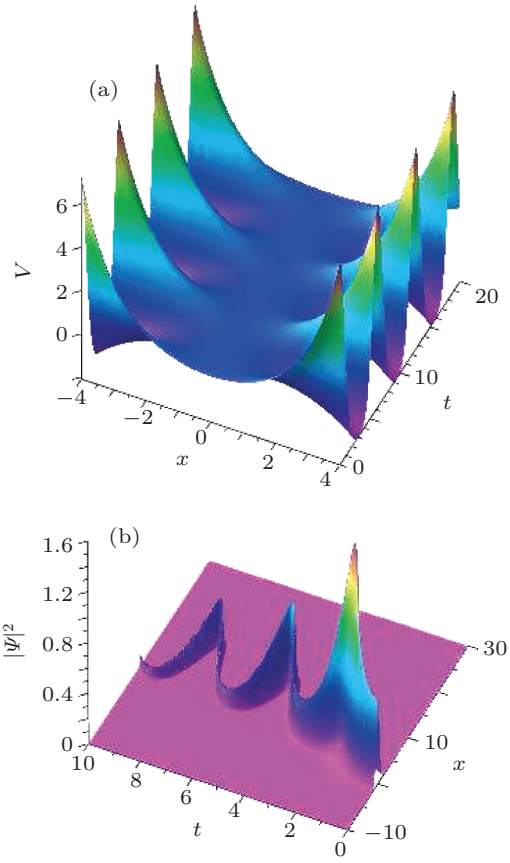


Fig. 6. (colour online) (a) Plot of the potential given by Eq. (22). (b) Intensity profile of the bright one-soliton solution, corresponding to (a). $\omega_0 = v = 2$, $b = 6$, and $\epsilon = 0.5$. Other parameters are the same as those used in Fig. 1.

the flying bird potential, decay with time. We show the propagation of the pulse in the flying bird potential in Fig. 6(b) for $\omega_0 = v = 2$, $b = 6$, and $\epsilon = 0.5$.

3. Stability analysis

Stability of the soliton solutions is another important issue in the present work. We start by employing the direct numerical simulations of Eq. (1). Using the exact solution as an initial pulse, one can find the numerical solutions. We take the solution in Case 2 as an example to demonstrate the numerical calculation. The simulation is performed by means of the split-step Fourier-transform method. The result of the numerically found soliton is displayed in Fig. 7(a). It is observed that the soliton enters a stable state with time.

The stability of the solitons can be further checked by adding a perturbation to the exact solutions. We still take the solution in Case 2 as an example. By adding white noise to the pulse $\psi(x,0)$, then the perturbed pulse reads $\psi_{\rm pert} = \psi(x,0)[1 + 0.1{\rm random}(x)]$. Figure 7(b) shows the numerical result of the evolution of the soliton under the initial perturbation of withe noise. The result reveals that the soliton can propagate in a stable way under the initial perturbation of white noise.

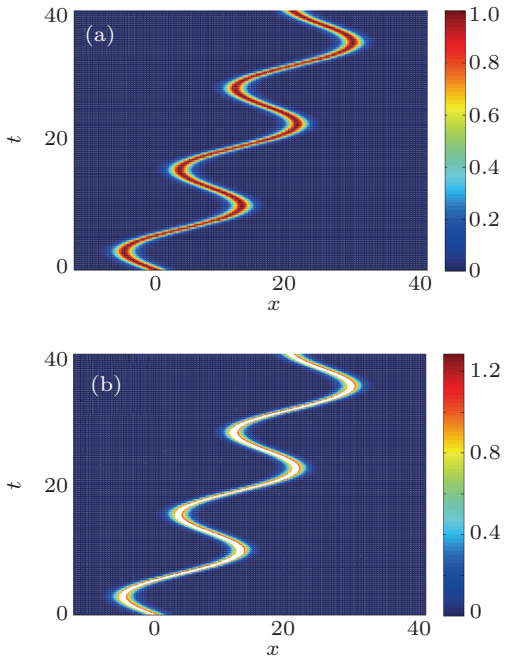


Fig. 7. (colour online) (a) Evolution of the bright soliton from Fig. 1(a). (b) Evolution of the bright soliton from Fig. 1(a) under the initial perturbation of white noise. Other parameters are the same as those used in Fig. 1.

Although we have displayed here only one example regarding the stability study of Eq. (1), a similar conclusion also applies to some other solutions.

4. Conclusion

In conclusion, we have studied the generalized nonautonomous NLS equation with time- and space-dependent distributed coefficients and an external potential. With the aid of the similarity transformation technique, the analytical nonautonomous bright multi-soliton solutions of this model are presented. Some interesting properties of the bright solitons and soliton collisions for different external potentials have been analyzed and discussed in detail for different choices of parameters. Physical applications of interest to the field of BEC and stability of the solitons are discussed. We believe the results will be useful for the further study in nonlinear optics, BECs, and other relevant subjects, and stimulate novel experiments in relevant research fields.

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