Parameter estimation for chaotic systems using the cuckoo search algorithm with an orthogonal learning method*

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We study the parameter estimation of a nonlinear chaotic system, which can be essentially formulated as a multi-dimensional optimization problem. In this paper, an orthogonal learning cuckoo search algorithm is used to estimate the parameters of chaotic systems. This algorithm can combine the stochastic exploration of the cuckoo search and the exploitation capability of the orthogonal learning strategy. Experiments are conducted on the Lorenz system and the Chen system. The proposed algorithm is used to estimate the parameters for these two systems. Simulation results and comparisons demonstrate that the proposed algorithm is better or at least comparable to the particle swarm optimization and the genetic algorithm when considering the quality of the solutions obtained.

Keywords: cuckoo search algorithm, chaotic system, parameter estimation, orthogonal learning

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1. Introduction

As the quintessence of a nonlinear system, chaos is a bounded unstable dynamic behavior that exhibits sensitive dependence on the initial conditions and includes infinite unstable periodic motions.[1] Chaos has been applied in many academic and engineering fields, such as communication, economic systems, and optimization. During the past decade, significant progress on tackling the parameters of chaotic systems has been made. In the real world, the parameters may be difficult to determine due to the complexity of the chaotic systems. Therefore, the parameter estimation for a chaotic system has become an important issue of nonlinear science. So far, different kinds of classical techniques have been developed to handle these problems.[2−7] Among them, the meta-heuristic based method, such as the genetic algorithm (GA), the particle swarm optimization algorithm (PSO), and the differential evolution algorithm (DE), may be one of the most popular methods, which formulates the problem as a multi-dimensional optimization problem.[8−10]

Recently, the cuckoo search (CS) algorithm has been proposed as a novel meta-heuristic based method by Yang and Deb.[11] This algorithm is an evolutionary one,[11] which is inspired by the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host birds. There are three principle rules used during the searching process. The first rule is that each cuckoo can only lay one egg, i.e., one solution for each time, which will be dumped in a randomly chosen nest. The second rule is that the best nests with better eggs will be retained to the next generation. The third rule is that during the whole search process, the number of available host nests is a constant, and the host bird can find the egg laid by a cuckoo with a probability. When it happens, the laid egg will be thrown away, or the host bird will abandon the nest to build a new one. For details of the cuckoo search, readers are referred to Ref. [11]. Although the CS algorithm is good at exploring the search space and locating the region of global minimum, it is slow at exploiting the solutions. It is common to find that the convergence rate of the algorithm is relatively slow for some parameter selections of chaotic systems.

The orthogonal design has been proved to be very useful for many intelligent algorithms.[12−15] The basic idea of the orthogonal design is to utilize the properties of the fractional experiment for the efficient determination of the best combination of levels. In the present paper, by integrating orthogonal learning strategy into the standard CS algorithm, we propose

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an improved version of the CS algorithm, which is called OLCS. The proposed algorithm can inherit both the advantages of the stochastic exploration ability of the cuckoo search and the exploitation ability of the orthogonal learning strategy. Simulation results and comparisons demonstrate that the proposed algorithm is better or at least comparable to state-of-art algorithms when considering the quality of the solutions obtained.

2. Problem formulation

Now, we describe the chaotic system. Let

$$\dot{\boldsymbol{X}} = \boldsymbol{F}(\boldsymbol{X}, \boldsymbol{X}_0, \boldsymbol{\theta}_0) \tag{1}$$

be a continuous nonlinear chaotic system, where $\boldsymbol{X} = (x_1, x_2, \ldots, x_N)' \in \mathrm{R}^N$ denotes the state vector of the chaotic system, $\dot{X}$ is the derivative of $\boldsymbol{X}$, $\boldsymbol{X}_0$ is the initial state, and $\boldsymbol{\theta}_0 = (\theta_{10}, \theta_{20}, \ldots, \theta_{d0})'$ is the set of the original parameters.

Suppose that the structure of system (1) is known, then the estimated system can be written as

$$\dot{\tilde{\boldsymbol{X}}} = \boldsymbol{F}(\boldsymbol{X}, \boldsymbol{X}_0, \tilde{\boldsymbol{\theta}}), \tag{2}$$

where $\tilde{\boldsymbol{X}} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N)' \in \mathrm{R}^N$ denotes the state vector, and $\tilde{\boldsymbol{\theta}} = (\tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_d)'$ is a set of estimated parameters.

Based on the measurable state vector $\boldsymbol{X} = (x_1, x_2, \ldots, x_N)' \in \mathrm{R}^N$, we define the following objective or fitness function:

$$f(\tilde{\theta}_i^n) = \sum_{t=0}^{W} [(x_1(t) - x_{i,1}^n(t))^2 + \cdots + (x_N(t) - x_{i,N}^n(t))^2], \tag{3}$$

where $t = 0, 1, \ldots, W$, and i denotes the i-th state vector. The goal of estimating the parameters of chaotic system (2) is to find out the suitable value of $\tilde{\theta}_i^n$ such that fitness function (3) is globally minimized.

Obviously, the parameter estimation for chaotic system is a multi-dimensional continuous optimization problem, where the decision vector is $\boldsymbol{\theta}$, and the optimization is to minimize $f(\tilde{\theta}_i^n)$. The principle of parameter estimation for the chaotic system in the sense of optimization can be illustrated in Fig. 1.

The chaotic system is not easy to estimate because of its dynamic instability. Moreover, due to the multiple variables in the problem and the multiple local search optima in the objective functions, the traditional optimization cannot be easily achieved in the local optima. Therefore, we try to solve this problem by proposing an orthogonal learning cuckoo search algorithm in this paper.

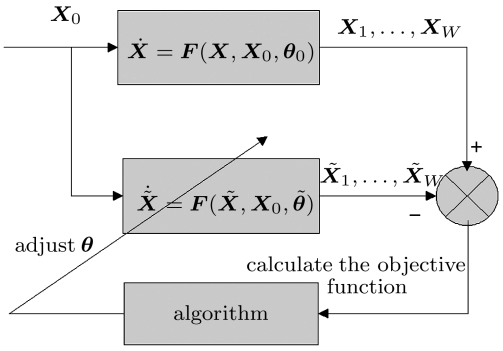


Fig. 1. Principle of parameter estimation for chaotic system.

3. OLCS algorithm

In this section, we discuss how to integrate the orthogonal learning strategy into the standard CS algorithm. The orthogonal design[12] has been used to produce all possible combinations of levels for a complete factorial experiment. The basic idea of the orthogonal design is to utilize the properties of the fractional experiment to efficiently determine the best combination of levels. Consider a system whose cost depends on K factors and each factor can take one of Q levels. To find the best level for each factor, experiments for all combinations of factor levels should be done to choose the best one if K and Q are small. Obviously, the number of all combinations is Q^K. However, it is not efficient to test all combinations of factor levels if K and Q are large. The orthogonal design is one of the most popular experimental design tools, and provides a series of orthogonal arrays for accommodating different numbers of factors and different levels. An orthogonal array of K factors with Q levels and M combinations is denoted as $L_M(Q^K)$, where Q is a prime number, $M = Q^J$, J is a positive integer satisfying $K = (Q^J - 1)/(Q - 1)$. Constructing the orthogonal array is shown below. The procedure is to first construct the basic column and then the non-basic columns.

Procedure constructing the orthogonal array

Step 1: construct the basic columns

For $k = 1$ to j do

$$j = \frac{Q^{k-1} - 1}{Q - 1} + 1$$

For $i = 1$ to Q^J do

$$a_{i,j} = \left\lfloor \frac{i-1}{Q^{J-K}} \right\rfloor \bmod Q;$$

End for

End for.

Steps 2: construct the non-basic columns

For $k = 2$ to J do

$$j = \frac{Q^{k-1}-1}{Q-1} + 1$$

For $s = 1$ to $j-1$ do

For $t = 1$ to $Q-1$ do

$$\boldsymbol{a}_{j+(s-1)(Q-1)+1} = (\boldsymbol{a}_s \times t + \boldsymbol{a}_j) \bmod Q;$$

End for

End for

End for.

Step 3: increment $a_{i,j}$ by 1 for $1 \leq i \leq M, 1 \leq j \leq N$.

Using the orthogonal array design, the orthogonal learning strategy can be integrated into the original algorithm to improve the search mechanism. In order to balance the exploration and the exploitation abilities of the algorithm, we produce a rand individual based on the orthogonal design[12] as the local search algorithm to enhance the exploitation of the basic CS. In this way, we can build an orthogonal learning strategy procedure. Generally speaking, the orthogonal learning strategy acts on two individuals, one is a rand individual of CS, and the other is produced according to

$$v_i = x_a + \text{rand} \times (x_b - x_c), \tag{4}$$

where $i \in \{1, 2, \ldots, N_{\text{P}}\}$, and $a, b, c, d \in \{1, 2, \cdots, N_{\text{P}}\}$ are randomly chosen indexes, N_{P} is the population size. In this way, we can add more variations to the search process and reduce the number of controlling parameters. This procedure quantizes the solution space defined by these parents into a finite number of points, and then the best orthogonal learning strategy is used to select the smallest individual as the new solution in the current form. The procedure of the orthogonal learning strategy is as follows.

Step 1 Construct the orthogonal array.

Step 2 Select two solutions from the population, one is the rand solution in the current population x_d, the other is produced as follows:

$$v_i = x_a + \text{rand} \times (x_b - x_c).$$

Step 3 Quantize the domain formed by the two solutions.

Step 4 Randomly generate $k-1$ integers $p_1, \ldots, p_{c1-1}$ such that $1 < p_1 < \cdots < p_{k-1} < n$.

Step 5 Use $L_M(Q^k)$ to generate M potential offspring.

Step 6 Randomly select a solution from the population.

Step 7 Compare the solution with the best solution from the orthogonal M offsprings, and obtain the better solution.

Based on the orthogonal learning strategy, an orthogonal learning cuckoo search algorithm can be described as follows.

Procedure orthogonal learning Cuckoo Search via lévy flights

Generation $t = 1$;
initialized with random vector values, and initialize parameters N_{P}, D;
evaluate fitness for each individual and determine the best individual with the best objective value;
While (stopping criterion is not met)
get a cuckoo randomly by lévy flights;
evaluate fitness for cuckoo F;
choose a nest from n (say j) randomly;
If $(F_i > F_j)$
replace j by the new solution;
End if;
a fraction (p_a) of worse nests are abandoned and new ones are built;
keep the best solution;
rank the solutions and find the current best,
% orthogonal learning strategy
construct the orthogonal array;
select two solutions from the population, one is the rand solution in the current population x_d, the other is produced as follows:

$$v_i = x_a + \text{rand} * (x_b - x_c);$$

quantize the domain formed by the two solutions;
randomly generate $k-1$ integers p_1, …, p_{k-1}, such that $1 < p_1 < \cdots < p_{k-1} < n$;
use $L_M(Q^k)$ to generate M potential offsprings;
randomly select a solution from the population;
compare the solution with the best solution from the orthogonal M offsprings and obtain the better solution;
update the generation number $t = t + 1$;
End while.

4. Simulation results

The Lorenz and the Chen chaotic systems are chosen to test the performance of the algorithm. So

far, these two systems have been widely used in many studies. In our experiments, each algorithm runs 30 times for each system. The parameters of the algorithm are configured as follows. After periods of the transient process, a state vector $\boldsymbol{X}_0$ is chosen as the initial state for the parameter estimation system, as shown in Fig. 1. The successive W states ($W = 30$) of both the estimated and the original systems are used to calculate the fitness. For the cuckoo search algorithm, the probability rate p is 0.25. For the PSO algorithm, the inertia weight is 0.729, and the acceleration coefficient is set to be 1.49. For the GA, the crossover rate is 0.8, and the mutation rate increases linearly from 0.001 to 0.6 during the evolution. In order to perform a fair comparison, the maximum generation number is 100 for the Lorenz system and 200 for the Chen system. The population size of each algorithm is 30. All the algorithms are coded in MATLAB 7.0, and the experiments are performed on a Pentium 3.0 GHz processor with 1.0 GB of memory. The source code can be obtained from the authors upon request.

4.1. Lorenz system

In the first example, the well-known Lorenz system[16] is employed. The chaotic system can be described as

$$\begin{cases} \dot{x}_1 = \theta_1(x_2 - x_1), \\ \dot{x}_2 = (\theta_2 - x_3)x_1 - x_2, \\ \dot{x}_3 = x_1 x_2 - \theta_3 x_3, \end{cases}$$

where x_1, x_2, and x_3 are the state variables; θ_1, θ_2, and θ_3 are unknown positive constant parameters. The original parameters are $\theta_1 = 10$, $\theta_2 = 28$, and $\theta_3 = 8/3$. For the simulation, we set the parameters of the Lorenz system to be $\theta_1 = 10$, $\theta_2 = 28$, and $\theta_3 = 8/3$. Table 1 lists the statistical results of the best fitness values, the mean values, the standard deviations, and the identified parameters of the Lorenz system. From Table 1, it can be seen that the best fitness values obtained by the OLCS are better than those obtained by the CS, the PSO, and the GA, and the mean values of the identified parameters by the OLCS and the CS are more accurate than those identified by the PSO and the GA. Figure 2 shows the convergence rates of the fitness values and the values of the three parameters of all the algorithms. From the figure, it can be seen that the OLCS converges to the optimal solution more rapidly than the other algorithms. Moreover, the convergence processes of the parameters in a single run of OLCS are much better than those of the other algorithms.

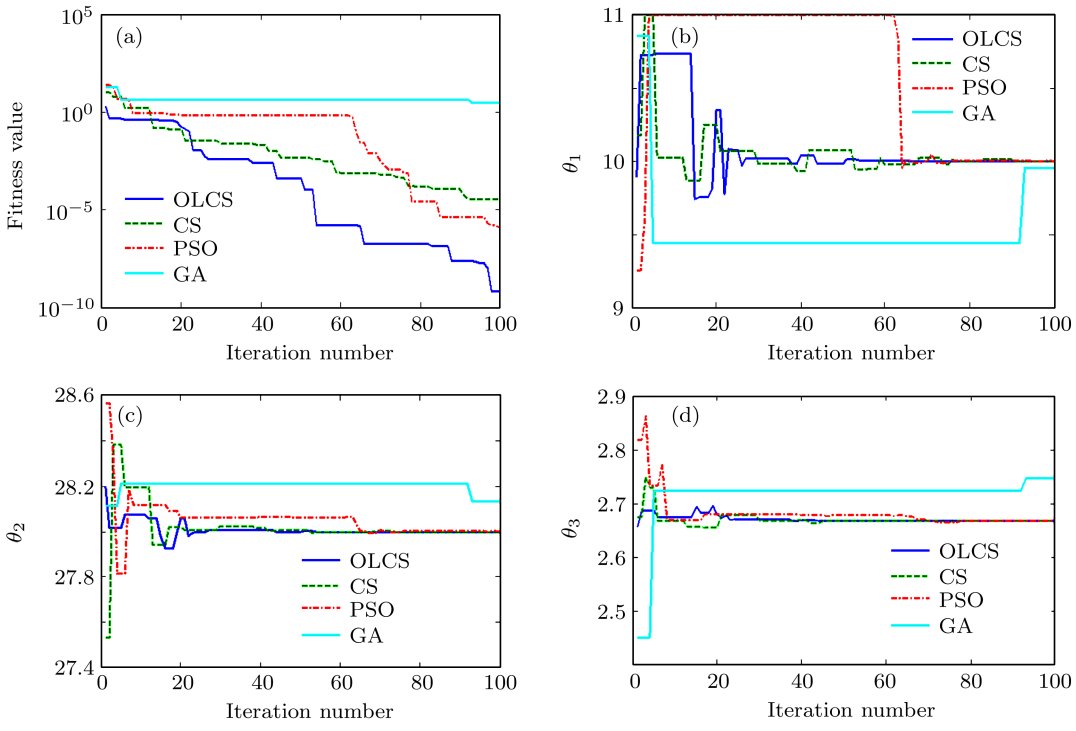


Fig. 2. (colour online) (a) The averaged convergence processes of fitness value over 30 runs for the Lorenz system; the convergence processes of (b) θ_1, (c) θ_2, and (d) θ_3 in a single run generating the best solution.

Table 1. Statistical results for the Lorenz system.

Algorithm	Mean of best fitness	Std. dev. of best fitness	Mean value and best value obtained (in brackets) of identified parameters		
			θ_1	θ_2	θ_3
OLCS	1.35×10^{-7}	1.81×10^{-7}	10.0000 (9.9984)	28.0000 (27.9997)	2.6667 (2.6666)
CS	1.81×10^{-4}	1.66×10^{-4}	9.9984 (10.0000)	27.9997 (28.0000)	2.6666 (2.6665)
PSO	0.11788	0.268094	10.1667 (9.9999)	28.0105 (27.9999)	2.6684 (2.6666)
GA	1.331401	2.783125	9.9824 (10.0274)	28.0175 (28.0058)	2.6824 (2.6692)

In general, the performance of the OLCS is highly competitive with those of the CS, the PSO, and the GA for the well-known Lorenz system.

4.2. Chen system

The second example is the Chen system.[17,18] The model of this system can be described as

$$\begin{cases} \dot{x}_1 = \theta_1(x_2 - x_1), \\ \dot{x}_2 = (\theta_3 - \theta_1)x_1 - \theta_3 x_2 - x_1 x_3, \\ \dot{x}_3 = x_1 x_2 - \theta_2 x_3, \end{cases}$$

where x_1, x_2, and x_3 are the state variables; θ_1, θ_2, and θ_3 are unknown positive constant parameters. The original parameters are $\theta_1 = 35$, $\theta_2 = 3$, and $\theta_3 = 28$. In order to perform the simulation of the Chen system, we set the parameters to be $\theta_1 = 35$, $\theta_2 = 3$, and $\theta_3 = 28$. The statistical results of the best fitness values, the mean fitness values, the standard deviations, and identified parameters of the Chen system are listed in Table 2. From Table 2, it can be seen that the best fitness values obtained by the OLCS are better than those obtained by the CS, the PSO, and the GA. Figure 3 shows the convergence rates of the fitness values and the values of the three parameters of all the algorithms. From the figure, we can see that

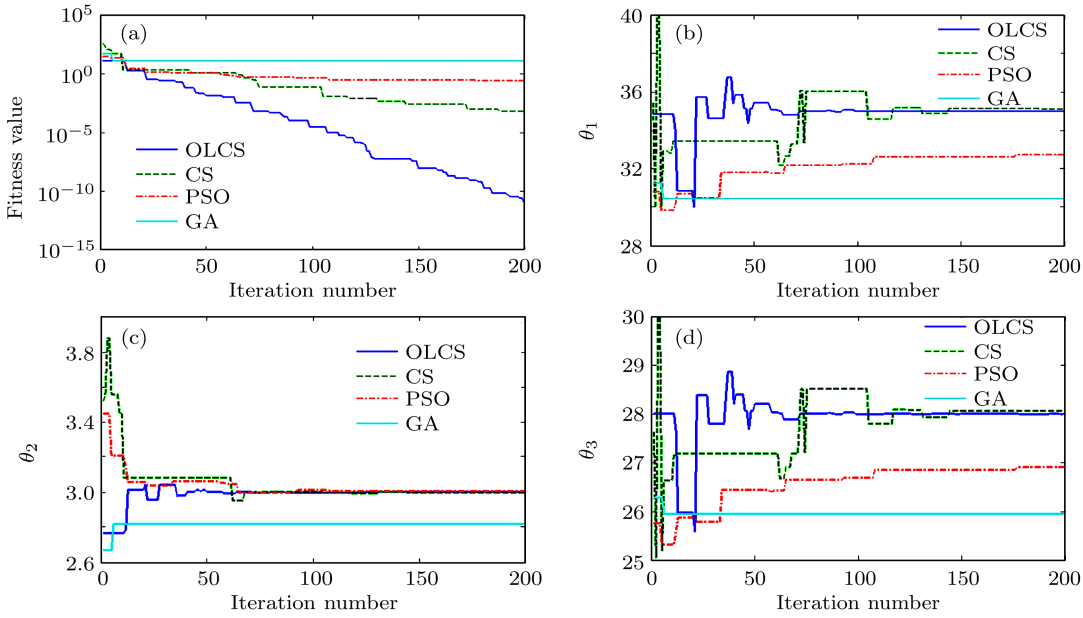


Fig. 3. (colour online) (a) The averaged convergence processes of fitness value over 30 runs for Chen system; the convergence processes of (b) θ_1, (c) θ_2, and (d) θ_3 in a single run generating the best solution.

Table 2. Statistical results for the Chen system.

Algorithm	Means of best fitness	Std. dev. of best fitness	Mean value and best value obtained (in brackets) of identified parameters					
			θ_1		θ_2		θ_3	
OLCS	4.71×10^{-11}	1.07×10^{-10}	35.0000	(34.99999)	2.99999	(2.99999)	28.000	(27.9999)
CS	9.66×10^{-7}	0.003717	35.0084	(35.0007)	2.9994	(2.9999)	28.0045	(28.0003)
PSO	8.461767	24.18407	38.1409	(34.4876)	3.0556	(2.9968)	29.4815	(27.7544)
GA	4.769291	21.53209	35.2407	(35.2199)	2.9884	(3.0196)	28.1173	(28.0935)

compared with the other algorithms, the OLCS can converge to the optimal solution more rapidly, and the convergence processes of the parameters in a single run of the OLCS are also much better than those of the other algorithms.

5. Conclusion

In this paper, a novel evolutionary algorithm, the orthogonal learning cuckoo search algorithm, is used for the parameter estimation of the chaotic system. A new search strategy based on the orthogonal learning strategy is used to enhance the exploitation ability of the basic cuckoo search algorithm. This new algorithm is very simple in structure and easy in application. To verify the performance of the OLCS, the algorithm is employed to estimate the parameters of two chaotic dynamical systems. The experimental results show that the OLCS can identify the parameters of the Lorenz and the Chen systems more accurately, more rapidly, and more stably than the CS, the PSO, and the GA. To the best of our knowledge, this is the first report using the improved cuckoo search algorithm to estimate the parameters of a chaotic system. The orthogonal learning cuckoo search algorithm can be applied to other chaotic systems, the OLCS can be a promising tool for various numerical optimization problems in physics.

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