Generalized synchronization between different chaotic maps via dead-beat control

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This paper presents a new scheme to achieve generalized synchronization (GS) between different discrete-time chaotic (hyperchaotic) systems. The approach is based on a theorem, which assures that GS is achieved when a structural condition on the considered class of response systems is satisfied. The method presents some useful features: it enables exact GS to be achieved in finite time (i.e., dead-beat synchronization); it is rigorous, systematic, and straightforward in checking GS; it can be applied to a wide class of chaotic maps. Some examples of GS, including the Grassi-Miller map and a recently introduced minimal 2-D quadratic map, are illustrated.

Keywords: generalized synchronization, chaos synchronization, discrete-time chaotic systems, dead-beat control, chaotic maps

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1. Introduction

It is well-known that chaotic systems are characterized by a strong sensitivity to the initial conditions.[1−2] Namely, the dynamics of two chaotic systems starting from slightly different initial conditions diverge exponentially with the passing of time. Thus, the topic of controlling two chaotic systems to get synchronized dynamics has attracted great interest in nonlinear science.[3−30] In the pioneering paper, Carroll and Pecora[3] showed that a drive system and a driven (response) system achieve synchronized dynamics when the Lyapunov exponents of the driven system are negative. Most of the methods that have been inspired by Ref. [3] focus on complete (identical) synchronization, where two identical chaotic systems asymptotically approach the same trajectory.[3−13] However, different types of synchronization have been proposed in the literatures.[14−30] For example, in projective synchronization all the states of the response system are scaled replicas of the states of the drive system,[14,15] whereas in full-state hybrid projective synchronization the scaling factor is different for each state variable.[16,17]

Another interesting dynamic behaviour is the so-called generalized synchronization (GS),[18−30] where two chaotic systems are said to be synchronized if a functional relationship exists between the states of the two systems. Note that in GS the two chaotic systems can be different. For example, in Refs. [19]–[21] GS is achieved between the Lorenz system and the Rössler system, whereas in Ref. [22] GS is obtained between the Chen system and the Lorenz system. In Ref. [23] GS is achieved between the Lorenz system and Chua's circuit.

It is worth noting that most methods to achieve GS are related to continuous-time systems.[18−25] Few methods are related to discrete-time systems.[26−30] For example, in Ref. [26] the GS of two unidirectionally coupled discrete-time systems is analysed, showing that there exists unsmoothed Hölder continuous GS when the coupling parameter is sufficiently small. In Ref. [27] GS for discrete-time systems in lack of inherent symmetries is analysed. In Ref. [28] the occurrence of GS is explained by means of the modified system approach, and an example involving two logistic maps is illustrated. In Ref. [30] a theoretical analysis on the existence of GS in bidirectionally coupled discrete-time systems is made.

Even though the methods illustrated in Refs. [26]–[30] are interesting, the conditions to be checked (in order to verify GS) are not straightforward. Moreover, the methods in Refs. [26]–[30] enable asymptotic GS to be achieved (i.e., the error asymptotically tends toward zero when the number of steps k goes to infinity), whereas a desirable feature in discrete-time systems is achieving exact synchronization in fi-

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nite time (i.e., dead-beat synchronization).[12] Based on these considerations, this paper presents a further contribution to the topic of GS by introducing a novel scheme, where two different chaotic (hyperchaotic) discrete-time systems synchronize via dead-beat control. Specifically, the proposed approach enables synchronization between two different chaotic maps to be exactly achieved in finite time, i.e., for an n-th order map the error is exactly zero after n steps. To the best of our knowledge, this represents a new result in the field of GS. Moreover, the proposed approach is straightforward in checking GS and can be applied to a wide class of discrete-time chaotic (hyperchaotic) systems. The paper is organized as follows. Section 2 defines the proposed GS and presents a theorem, which assures that GS is achieved when a structural condition on the considered class of response systems is satisfied, provided that a suitable dead-beat controller is adopted. Finally, in order to show the effectiveness of the approach, Section 3 illustrates some examples of GS, which involve the so-called Grassi-Miller map[16,17] and the minimal two-dimensional quadratic map illustrated in Ref. [31].

2. Generalized synchronization via dead-beat control

Definition I Given the discrete-time drive and response systems described, respectively, by the state vectors $\boldsymbol{x}(k)$ and $\boldsymbol{z}(k)$, the two systems are in generalized synchronization with respect to an arbitrary continuous differentiable function $\boldsymbol{\varphi}(\boldsymbol{x}(k))$, where $\boldsymbol{\varphi}: \Re^n \to \Re^m$. The synchronization error is given by[22]

$$\|\boldsymbol{e}(k)\| = \|(\boldsymbol{z}(k) - \boldsymbol{\varphi}(\boldsymbol{x}(k)))\| \to 0, \quad (k \to \infty). \quad (1)$$

Referring to the systems to be synchronized, this paper focuses on the class of drive systems defined by the following state equations:

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}f(\boldsymbol{x}(k)), \quad (2)$$

where $\boldsymbol{x}(k) \in \Re^{n\times 1}$, $\boldsymbol{A} \in \Re^{n\times n}$, $\boldsymbol{b} \in \Re^{n\times 1}$, and $f: \Re^n \to \Re$ is a nonlinear vector field.

The response systems considered herein are different from that defined by Eq. (2). Namely, they are described by the following state equations:

$$\boldsymbol{z}(k+1) = \boldsymbol{C}\boldsymbol{z}(k) + \boldsymbol{d}\boldsymbol{g}(\boldsymbol{z}(k)) + \boldsymbol{u}(\boldsymbol{z}(k), \boldsymbol{x}(k)), \quad (3)$$

where $\boldsymbol{z}(k) \in \Re^{m\times 1}$, $\boldsymbol{C} \in \Re^{m\times m}$, $\boldsymbol{d} \in \Re^{m\times 1}$, $\boldsymbol{g}: \Re^m \to \Re$, and $\boldsymbol{u}(z, x) \in \Re^{m\times 1}$ is a control vector to be determined.

Remark 1 As pointed out in Ref. [12], the class described by Eq. (2) (or, equivalently, by Eq. (3) with $\boldsymbol{u} = 0$) includes several chaotic and hyperchaotic discrete-time systems, that is, the logistic map, the cubic map, the Duffing map, the Gauss map, the Lozi map, and the generalized Henon map. The class (2) also includes the so-called Grassi–Miller map,[16,17] the gingerbreadman map,[32] the double scroll map,[33] and the minimal 2-D quadratic map illustrated in Ref. [31].

Given the drive system (2) and the response system (3), a theorem that GS is achievable using a suitable dead-beat control input $\boldsymbol{u}(\boldsymbol{z}, \boldsymbol{x})$ can be developed.

Theorem 1 Given the drive system (2) and the response system (3), if the matrix

$$\left[\boldsymbol{d}\ \boldsymbol{C}\boldsymbol{d}\ \boldsymbol{C}^2\boldsymbol{d}\ \cdots\ \boldsymbol{C}^{m-1}\boldsymbol{d}\right] \quad (4)$$

is full rank and the control $\boldsymbol{u}(\boldsymbol{z}, \boldsymbol{x})$ is chosen as

$$\begin{aligned}\boldsymbol{u}(\boldsymbol{z}(k), \boldsymbol{x}(k)) = &-\boldsymbol{d}\boldsymbol{k}\boldsymbol{z}(k) + \boldsymbol{d}\boldsymbol{k}\boldsymbol{\varphi}(\boldsymbol{x}(k)) - \boldsymbol{d}\boldsymbol{g}(\boldsymbol{z}(k)) \\ &+ \boldsymbol{D}\boldsymbol{\varphi}(\boldsymbol{x}(k))\boldsymbol{x}(k+1) - \boldsymbol{C}\boldsymbol{\varphi}(\boldsymbol{x}(k)), \end{aligned} \quad (5)$$

where $\boldsymbol{D}\boldsymbol{\varphi}(\boldsymbol{x}(k))$ is the Jacobian matrix of the function $\boldsymbol{\varphi}(\boldsymbol{x}(k))$ and $\boldsymbol{k} \in \Re^{1\times m}$ is a gain vector, then the dead-beat GS is achieved between systems (2) and (3) for arbitrary continuous differentiable function $\boldsymbol{\varphi}(\boldsymbol{x}(k))$, provided that the eigenvalues of $[\boldsymbol{C} - \boldsymbol{d}\boldsymbol{k}]$ are placed at zero by choosing suitable $\boldsymbol{k}$.

Proof Taking Eq. (1) into account, the error dynamics are described as

$$\begin{aligned}\boldsymbol{e}(k+1) &= (\boldsymbol{z}(k+1) - \boldsymbol{\varphi}(\boldsymbol{x}(k+1))) \\ &= (\boldsymbol{z}(k+1) - (\boldsymbol{D}\boldsymbol{\varphi}(\boldsymbol{x}(k)))\boldsymbol{x}(k+1)) \\ &= C\boldsymbol{z}(k) + \boldsymbol{d}\boldsymbol{g}(\boldsymbol{z}(k)) - \boldsymbol{d}\boldsymbol{k}\boldsymbol{z}(k) + \boldsymbol{d}\boldsymbol{k}\boldsymbol{\varphi}(\boldsymbol{x}(k)) \\ &\quad - \boldsymbol{d}\boldsymbol{g}(\boldsymbol{z}(k)) + (\boldsymbol{D}\boldsymbol{\varphi}(\boldsymbol{x}(k)))\boldsymbol{x}(k+1) \\ &\quad - \boldsymbol{C}\boldsymbol{\varphi}(\boldsymbol{x}(k)) - (\boldsymbol{D}\boldsymbol{\varphi}(\boldsymbol{x}(k)))\boldsymbol{x}(k+1) \\ &= (\boldsymbol{C} - \boldsymbol{d}\boldsymbol{k})\boldsymbol{z}(k) - (\boldsymbol{C} - \boldsymbol{d}\boldsymbol{k})\boldsymbol{\varphi}(\boldsymbol{x}(k)) \\ &= (\boldsymbol{C} - \boldsymbol{d}\boldsymbol{k})\boldsymbol{e}(k). \end{aligned} \quad (6)$$

Note that the matrix (4), derived from the response system (3), proves to be the controllability matrix of the linear time-invariant error system $\boldsymbol{e}(k+1) = \boldsymbol{C}\boldsymbol{e}(k) + \boldsymbol{d}\boldsymbol{u}(k)$, where $\boldsymbol{u}(k) = -\boldsymbol{k}\boldsymbol{e}(k)$. By virtue of the assumption of full rank, all the error system eigenvalues can be placed at the origin by choosing suitable $\boldsymbol{k}$, indicating that the dead-beat control is achieved.[34,35] Thus, the error dynamics will reach exactly zero in m steps. This means that the dead-beat GS is achieved, i.e., the drive and response systems

will exactly synchronize in m steps for arbitrary continuous differentiable function $\boldsymbol{\varphi}(\boldsymbol{x}(k))$.

Remark 2 It can be readily verified that matrix (4) is full rank for several well-known discrete-time chaotic (hyperchaotic) systems. Specifically, all the maps mentioned in Remark I satisfy this assumption. As a consequence, according to Theorem I, all these maps achieve dead-beat GS for arbitrary continuous differentiable function $\boldsymbol{\varphi}(\boldsymbol{x}(k))$.

Remark 3 The proposed GS method has the following advantages: 1) it is rigorous and systematic; 2) it enables exact synchronization to be achieved in finite time (while in Refs. [26]–[30] and [36] the error asymptotically tends toward zero when k goes to infinity); 3) it is straightforward in checking GS (while other approaches require complex conditions to be verified[26−28]); 4) it does not require computation of Lyapunov exponents (different from Ref. [36]); 5) it can be applied to a wide class of discrete-time systems; 6) any continuous differentiable function $\boldsymbol{\varphi}(\cdot)$ can be selected (while other approaches require $\boldsymbol{\varphi}(\cdot)$ to be an invertible transformation[29] to achieve GS).

3. Examples

The drive system considered here is the hyperchaotic Grassi–Miller map,[16,17] which is the $n = 3$ case of the generalized Henon map.[37] The hardware implementation of this map proposed by us was illustrated in Ref. [12]. The system dynamics can be written in form (2) as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -0.1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \left(1.76 - (x_2(k))^2\right). \quad (7)$$

The system exhibits a hyperchaotic attractor shown in Fig. 1. The response system is represented by the minimal 2-D quadratic map, with the quasi-periodic route to chaos, recently introduced in Ref. [31]. This map, which presents a very rich dynamic behaviour characterized by several different chaotic attractors, can be written in form (3) as

$$\begin{aligned} &\begin{bmatrix} z_1(k+1) \\ z_2(k+1) \end{bmatrix} \\ &= \begin{bmatrix} b & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(1 - a\,(z_2(k))^2\right) \\ &\quad + \begin{bmatrix} u_1\,(z_1(k), z_2(k), x_1(k), x_2(k), x_3(k)) \\ u_2\,(z_1(k), z_2(k), x_1(k), x_2(k), x_3(k)) \end{bmatrix}, \end{aligned} \quad (8)$$

where a and b are bifurcation parameters. When $a = 0.7$ and $b = 0.9$, the uncontrolled map (8) (i.e., the map with $u_1 = u_2 = 0$) exhibits the chaotic attractor presented in Fig. 1.

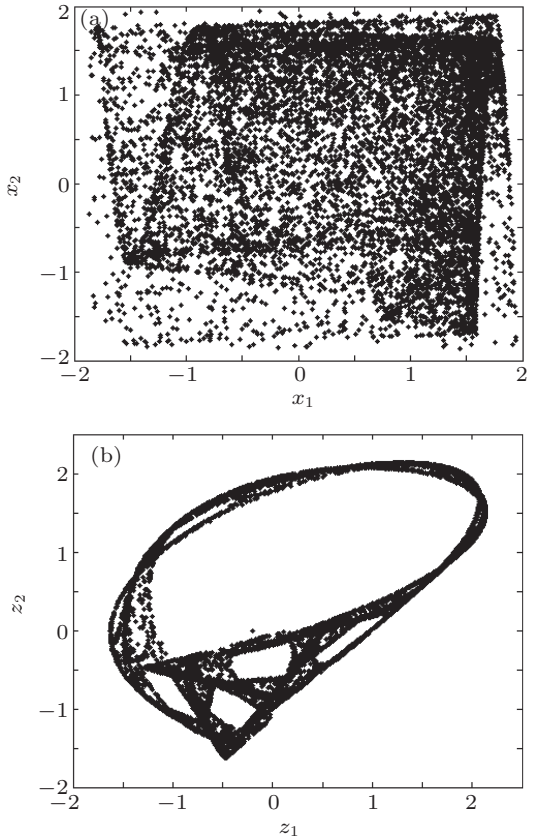


Fig. 1. (a) Attractor of the drive system (7). (b) Attractor of the uncontrolled system (8) when $a = 0.7$, $b = 0.9$, and $u_1 = u_2 = 0$.

By choosing the function $\boldsymbol{\varphi}(\boldsymbol{x}(k))$ as

$$\begin{bmatrix} \varphi_1\,(x_1(k), x_2(k), x_3(k)) \\ \varphi_2\,(x_1(k), x_2(k), x_3(k)) \end{bmatrix} = \begin{bmatrix} x_1(k) \\ x_2(k) + x_3(k) \end{bmatrix}, \quad (9)$$

the control (5) can be written as

$$\begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k)+x_3(k) \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(1 - 0.7\,(z_2(k))^2\right) + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} - \begin{bmatrix} 0.9 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k)+x_3(k) \end{bmatrix}, \tag{10}$$

where $[\,k_1 \;\; k_2\,]$ is the gain vector to be determined. By considering the drive system (7), response system (8), function (9), and control (10), the error dynamics can be described as

$$\begin{bmatrix} e_1(k+1) \\ e_2(k+1) \end{bmatrix} = \left(\begin{bmatrix} 0.9 & 0 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} \right) \times \begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix}. \tag{11}$$

Since the matrix (4) is full rank, the eigenvalues of the error system (11) can be placed anywhere. By placing them at zero, it results in $k = [\,0.9 \;\; 0\,]$. By virtue of Theorem I, the dead-beat GS is achieved between systems (7) and (8), that is, the error dynamics will reach exactly zero after two steps. In order to show the effectiveness of the approach, Fig. 2 shows the attractor of the response system (8) in the (z_1, z_2)-plane when the control is given by Eq. (10) with $\boldsymbol{k} = [\,0.9 \;\; 0\,]$.

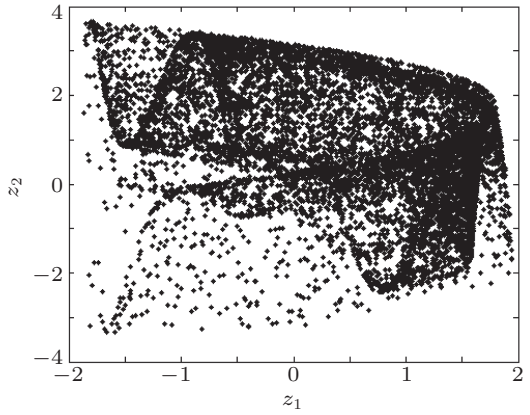


Fig. 2. Attractor of the response system (8) when $a = 0.7$, $b = 0.9$, and the control is given by Eq. (10) with $\boldsymbol{k} = [\,0.9 \;\; 0\,]$.

Note that the response system attractor presented in Fig. 2 is completely different from the uncontrolled response system attractor reported in Fig. 1(a). Namely, the role of the GS is to produce, in the response system, an attractor that represents the nonlinear image of the drive system attractor. In other words, the GS behaves like a nonlinear filter, which turns the original response system attractor (see Fig. 1(a)) into a nonlinear image of the drive system attractor (see Fig. 2). The errors (1), which are given by

$$\begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix} = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} - \begin{bmatrix} \varphi_1\,(x_1(k), x_2(k), x_3(k)) \\ \varphi_2\,(x_1(k), x_2(k), x_3(k)) \end{bmatrix} = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} - \begin{bmatrix} x_1(k) \\ x_2(k)+x_3(k) \end{bmatrix}, \tag{12}$$

are plotted in Fig. 3 as functions of k. This figure clearly shows that the dead-beat GS is achieved between systems (7) and (8), since the errors are exactly zero after two steps, i.e., the errors are zero from the third step on.

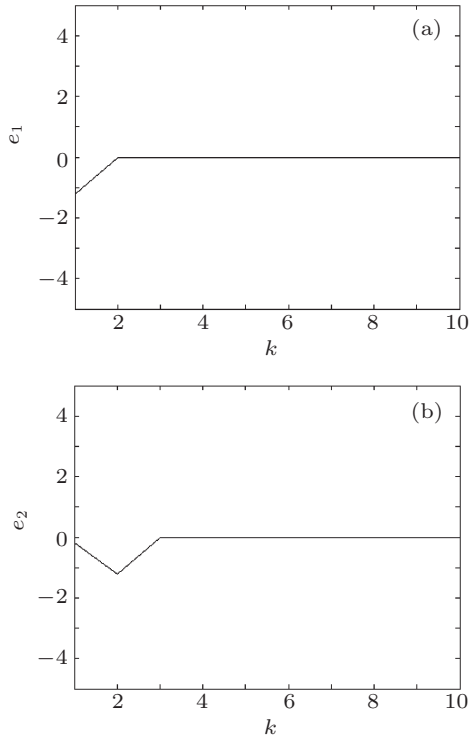


Fig. 3. The errors $e_1(k)$ (a) and $e_2(k)$ (b) as functions of k, for the function φ given by Eq. (9).

Now the second example is presented to show how the response system attractor changes by varying function $\boldsymbol{\varphi}\,(\boldsymbol{x}(k))$. For example, by taking the following function:

$$\begin{bmatrix} \varphi_1\,(x_1(k), x_2(k), x_3(k)) \\ \varphi_2\,(x_1(k), x_2(k), x_3(k)) \end{bmatrix} = \begin{bmatrix} x_1(k)+x_2(k) \\ x_2(k)+x_3(k) \end{bmatrix}, \tag{13}$$

the control (5) can be written as

$$\begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0 \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0.9 & 0 \end{bmatrix} \begin{bmatrix} x_1(k)+x_2(k) \\ x_2(k)+x_3(k) \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(1-0.7\left(z_2(k)\right)^2\right) + \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} - \begin{bmatrix} 0.9 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k)+x_2(k) \\ x_2(k)+x_3(k) \end{bmatrix}. \quad (14)$$

Figure 4 shows the attractor of response system (8) with the control (14), in the (z_1, z_2)-plane.

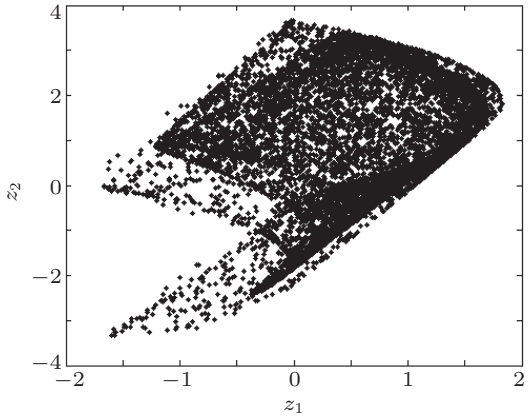


Fig. 4. Attractor of the response system (8) when $a = 0.7$, $b = 0.9$, and the control is given by Eq. (14).

Note that the attractor presented in Fig. 4 is completely different from the one reported in Fig. 2 since function (13) is different from function (9). The errors (1) given by

$$\begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix} = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} - \begin{bmatrix} \varphi_1\left(x_1(k), x_2(k), x_3(k)\right) \\ \varphi_2\left(x_1(k), x_2(k), x_3(k)\right) \end{bmatrix} = \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} - \begin{bmatrix} x_1(k)+x_2(k) \\ x_2(k)+x_3(k) \end{bmatrix} \quad (15)$$

are plotted in Fig. 5 as functions of k.

Figure 5 clearly shows that the dead-beat GS is achieved between systems (7) and (8), since the error is exactly zero after two steps, i.e., the error is zero from the third step on. Note that all the discrete-time GS methods developed in Refs. [26]–[30] and [38]–[40] do not offer this capability, since GS is achieved only for $k \to \infty$ and the error in finite time is different from zero. These considerations clearly highlight the advantages of the conceived scheme over existing discrete-time GS techniques.

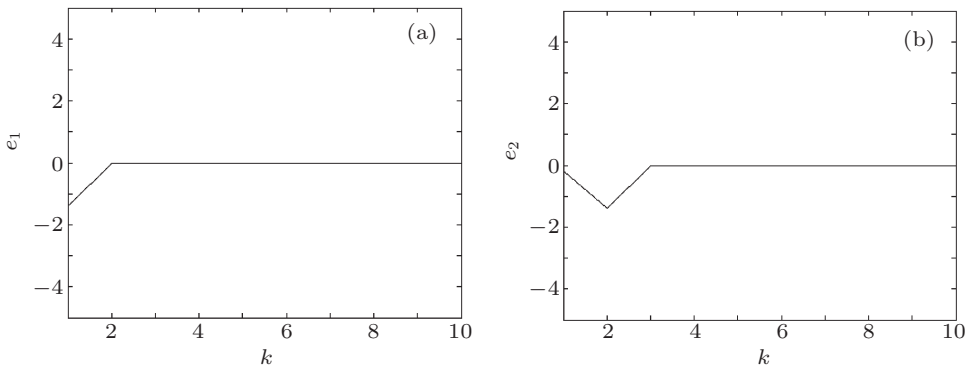


Fig. 5. The errors $e_1(k)$ (a) and $e_2(k)$ (b) as functions of k, for the function φ given by Eq. (13).

Finally, a third synchronization example is given, which exploits a different chaotic attractor generated by system (8). Namely, when $a = 1$ and $b = 0.3$, the map (8) with $u_1 = u_2 = 0$ exhibits the chaotic attractor reported in Fig. 6. Since the matrix (4) obtained from map (8) when $a = 1$ and $b = 0.3$ is full rank, the eigenvalues of the error system can be placed at zero by $\boldsymbol{k} = [\,0.3\ 0\,]$. By taking the following nonlinear function $\boldsymbol{\varphi}\left(\boldsymbol{x}(k)\right)$:

$$\begin{bmatrix} \varphi_1\left(x_1(k), x_2(k), x_3(k)\right) \\ \varphi_2\left(x_1(k), x_2(k), x_3(k)\right) \end{bmatrix} = \begin{bmatrix} x_1(k)-x_2(k) \\ \left(x_3(k)\right)^2 \end{bmatrix}, \quad (16)$$

the control (5) can be written as

$$\begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0.3 & 0 \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0.3 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) - x_2(k) \\ (x_3(k))^2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(1 - (z_2(k))^2\right) + \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 2x_3(k) \end{bmatrix} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} - \begin{bmatrix} 0.3 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) - x_2(k) \\ (x_3(k))^2 \end{bmatrix}. \quad (17)$$

Figure 6 shows the attractor of response system (8) with control (17), in the $(z_1,\ z_2)$-plane. By virtue of the selected function (16), the proposed GS scheme enables the dynamics of response system (8) to be switched from the attractor reported in Fig. 6(a) to the attractor reported in Fig. 6(b), which is synchronized with the drive system attractor reported in Fig. 1(a). By virtue of Theorem 1, the synchronization error is exactly zero after two steps. Finally, the condition is checked in order to verify that GS is straightforward (i.e., the eigenvalues of $[\boldsymbol{C} - \boldsymbol{dk}]$ must be placed at zero by choosing suitable $\boldsymbol{k}$), while approaches reported in other literature require complex conditions to be verified[26−28] or the Lyapunov exponents to be computed.[36]

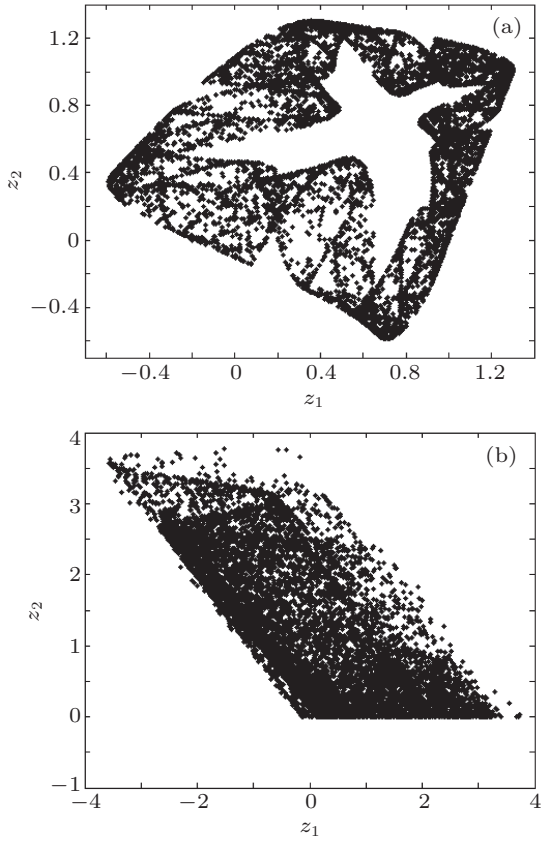


Fig. 6. (a) Attractor of response system (8) when $a = 1$, $b = 0.3$, and $u_1 = u_2 = 0$. (b) Attractor of system (8) when $a = 1$, $b = 0.3$, and the control is given by Eq. (17).

4. Conclusions

This paper has introduced a new approach, based on the concept of dead-beat GS. The proposed method enables synchronization between different chaotic maps to be exactly achieved in finite time and presents some useful features: it is rigorous, systematic, and straightforward in checking GS; it can be applied to a wide class of chaotic maps; GS is achievable regardless of the choice of function $\boldsymbol{\varphi}(\boldsymbol{x}(k))$. Some GS examples involving the Grassi–Miller map and a recently introduced minimal 2-D quadratic map have been illustrated.

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