# Effect of inertia mass on the stochastic resonance driven by a multiplicative dichotomous noise*

Li Peng(李 鹏), Nie Lin-Ru(聂林如)†, Huang Qi-Rui(黄奇瑞), and Sun Xing-Xiu(孙兴修)

*Faculty of Science, Kunming University of Science and Technology, Kunming* 650093, *China*

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A stochastic system driven by dichotomous noise and periodic signal is investigated in the under-damped case. The exact expressions of output signal amplitude and signal-to-noise ratio (SNR) of the system are derived. Numerical results indicate that the inertial mass greatly affects the output signal amplitude and the SNR. Regardless of whether the noise is symmetric or asymmetric, the inertial mass can influence the phenomenon of stochastic resonance (SR) of the system, leading to two types of resonance phenomenon: one is coherence-resonance-like of the SNR with inertial mass, the other is the SR of the SNR with noise intensity.

**Keywords:** dichotomous noise, inertial mass, output signal amplitude, signal-to-noise ratio

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## 1. Introduction

The term "stochastic resonance (SR)" was first introduced by Benzi *et al.*[1,2] Their goal is to explain the periodic recurrence of ice ages on the earth. Since the introduction of SR, it has been studied extensively. Noise-induced dynamics has become the center of the subject that has been investigated in a variety of nonlinear systems with additive and multiplicative noises.[3−6] The phenomenon of SR has attracted considerable interest due to many applications in biology,[5−7] physics,[8] chemistry,[9] and so on.[10−15] The beneficial role of the background noise in physical systems has been appreciated, particularly since the discovery of SR. There has been a tremendous theoretical development of SR in stochastic systems. In order to describe SR, one believes that the response of the system undergoes a resonance-like behavior, and the typical manifestation is the existence of a maximum or minimum of the output sigal or signal-to-noise ratio (SNR).[5,7−11,14,15] They have studied the SNR on the correlation time of the noise and the frequency of the input signal to testify SR phenomenon.

Recently, it has been shown that SR can occur in linear systems for the dichotomous noise,[16−20] composite noise,[21,22] Gaussian noise,[23−25] and so on.[26−28] It turns out that SR takes place only for a multiplicative colored noise, but disappears for a white noise. In general, one considers an over-damped nonlinear or a linear system. However, only a few previous studies considered the effect of the inertia mass on the system.

In this paper, we investigate the effect of the inertia mass on the phenomenon of stochastic resonance driven by a multiplicative dichotomous noise. This paper is organized as follows. In Section 2, we provide the exact expressions of output signal amplitude and the effect of inertial mass on output signal amplitude. Furthermore, we calculate the SNR and investigate the effect of inertial mass on SNR. In Section 3, the main results and conclusions are summarized.

## 2. Model and its output amplitude and signal-to-noise ratio

A stochastic system subjected to a sinusoidal signal and an inertia mass can be described by the following stochastic differential equation:

$$m\ddot{x} + \dot{x} = -\frac{\mathrm{d}U(x)}{\mathrm{d}x} + \xi(t)x + a\sin(\omega t), \qquad (1)$$

where $\mathrm{d}U(x)/\mathrm{d}x = 0$ (the potential $U(x)$ is a constant), $m$ is the inertial mass, and $\xi(t)$ represents the asymmetric dichotomous noise which takes two asymmetric values of $E$, and $-kE$ ($E > 0$, $k > 0$). The $k$ represents the asymmetric degree of the noise. When

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†Corresponding author. E-mail: lrnie@163.com

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$k = 1$, the noise becomes symmetric. $\gamma$ is the transition rate ranging from $E$ to $-kE$, and $\gamma'$ is the reverse transition rate. We assume that $\xi(t)$ has the statistical properties

$$\begin{aligned}\langle\xi(t)\rangle &= \frac{E\gamma' - kE\gamma}{\gamma+\gamma'} = g_0,\\ \langle\xi(t)\xi(t')\rangle &= D\lambda\exp[-\lambda|t-t'|],\end{aligned} \tag{2}$$

where $g_0$ is a constant that can be zero, negative, or positive, $\lambda = \gamma+\gamma'$ is the autocorrelation rate, $\tau = 1/\lambda$ is the correlation time, and $D$ is the noise intensity. We have[29]

$$D = E^2(1+k)^2\frac{\gamma\gamma'}{(\gamma+\gamma')^3}. \tag{3}$$

We assume

$$\xi(t) = \eta(t) + g, \tag{4}$$

where $g$ is a constant, and $\eta(t)$ is a dichotomous noise which takes two values $c$ and $-c$ ($c > 0$). The mean value and correlation function of $\eta(t)$ are given by

$$\begin{aligned}\langle\eta(t)\rangle &= g_0 - g,\\ \langle\eta(t)\eta(t')\rangle &= (g_0-g)^2 + D\lambda\exp[-\lambda|t-t'|],\end{aligned} \tag{5}$$

and $c$ and $g$ take the forms of

$$c = (1+k)E/2, \quad g = (1-k)E/2. \tag{6}$$

In what follows, the method in Ref. [30] is adopted to deal with Eq. (1). Averaging Eq. (1), we obtain

$$\begin{aligned}&m\frac{\mathrm{d}^2\langle x(t)\rangle}{\mathrm{d}t^2} + \frac{\mathrm{d}\langle x(t)\rangle}{\mathrm{d}t}\\ &= g\langle x(t)\rangle + \langle\eta(t)x(t)\rangle + a\sin(\omega t).\end{aligned} \tag{7}$$

Assuming $\dot{x} = y$, equation (7) can be rewritten as

$$\begin{aligned}&m\frac{d\langle y(t)\rangle}{\mathrm{d}t} + \langle y(t)\rangle\\ &= g\langle x(t)\rangle + \langle\eta(t)x(t)\rangle + a\sin(\omega t).\end{aligned} \tag{8}$$

Applying the formulas of Shapiro and Loginov to the system,[30–32] we have

$$\begin{aligned}\frac{\mathrm{d}\langle\eta(t)x(t)\rangle}{\mathrm{d}t} &= \langle\eta(t)\frac{\mathrm{d}x(t)}{\mathrm{d}t}\rangle - \lambda\langle\eta(t)x(t)\rangle\\ &\quad + \lambda\langle\eta(t)\rangle\langle x(t)\rangle.\end{aligned} \tag{9}$$

Substituting Eq. (8) into Eq. (9), and considering $\dot{x} = y$, we can obtain the differential equation

$$\begin{aligned}&m\frac{\mathrm{d}^2\langle y(t)\rangle}{\mathrm{d}t^2} + \frac{\mathrm{d}\langle y(t)\rangle}{\mathrm{d}t} - g\langle y(t)\rangle - a\omega\cos(\omega t)\\ &= \langle\eta(t)y(t)\rangle - \lambda\Big[m\frac{\mathrm{d}\langle y(t)\rangle}{\mathrm{d}t} + \langle y(t)\rangle\\ &\quad - g\langle x(t)\rangle - a\sin(\omega t)\Big] + \lambda\langle\eta(t)\rangle\langle x(t)\rangle.\end{aligned} \tag{10}$$

Then, Eq. (10) can be rewritten as

$$\begin{aligned}&m\frac{\mathrm{d}^2\langle y(t)\rangle}{\mathrm{d}t^2} + (1+\lambda m)\frac{\mathrm{d}\langle y(t)\rangle}{\mathrm{d}t}\\ &+ (\lambda - g)\langle y(t)\rangle - g_0\lambda\langle x(t)\rangle\\ &= \langle\eta(t)y(t)\rangle + a\omega\cos(\omega t) + a\lambda\sin(\omega t).\end{aligned} \tag{11}$$

Applying the formulas of Shapiro and Loginov once again, we have

$$\begin{aligned}\frac{\mathrm{d}\langle\eta(t)y(t)\rangle}{\mathrm{d}t} &= \langle\eta(t)\frac{\mathrm{d}y(t)}{\mathrm{d}t}\rangle - \lambda\langle\eta(t)y(t)\rangle\\ &\quad + \lambda\langle\eta(t)\rangle\langle y(t)\rangle.\end{aligned} \tag{12}$$

Then, substituting Eqs. (8) and (11) into Eq. (12), and considering $y = \dot{x}$, we obtain a four-order differential equation of $\langle x(t)\rangle$ as

$$\begin{aligned}&\frac{\mathrm{d}^4\langle x(t)\rangle}{\mathrm{d}t^4} + 2\left(\frac{1}{m}+\lambda\right)\frac{\mathrm{d}^3\langle x(t)\rangle}{\mathrm{d}t^3}\\ &+ \left(\lambda^2 + \frac{3\lambda}{m} + \frac{1}{m^2} - \frac{2g}{m}\right)\frac{\mathrm{d}^2\langle x(t)\rangle}{\mathrm{d}t^2}\\ &+ \left(\frac{\lambda^2}{m} + \frac{\lambda}{m^2} - \frac{2g}{m^2} - \frac{2g_0\lambda}{m}\right)\frac{\mathrm{d}\langle x(t)\rangle}{\mathrm{d}t}\\ &+ \left(\frac{-g_0\lambda - mg_0\lambda^2 + g^2 - c^2}{m^2}\right)\langle x(t)\rangle\\ &= \frac{-am\omega^2 + a\lambda + am\lambda^2 - 2ag + g_0a}{m^2}\sin(\omega t)\\ &+ \frac{2am\omega\lambda + a\omega}{m^2}\cos(\omega t),\end{aligned} \tag{13}$$

where $\eta(t)^2 = c^2$ is used since $\eta(t) = \pm c$. From Eq. (13), we can obtain the output signal in the following form:

$$\langle x(t)\rangle = A\sin(\omega t + \varphi), \tag{14}$$

where

$$\begin{aligned}A &= a\left(\frac{f_3^2 + f_4^2}{f_1^2 + f_2^2}\right)^{1/2},\\ \varphi &= \arctan\left(\frac{f_1f_3 - f_2f_4}{f_2f_3 + f_1f_4}\right),\end{aligned} \tag{15}$$

and

$$\begin{aligned}f_1 &= \omega^4 - \left(\lambda^2 + \frac{3\lambda}{m} + \frac{1}{m^2} - \frac{2g}{m}\right)\omega^2\\ &\quad + \frac{-g_0\lambda - mg_0\lambda^2 + g^2 - c^2}{m^2},\\ f_2 &= \left(\frac{\lambda^2}{m} + \frac{\lambda}{m^2} - \frac{2g}{m^2} - \frac{2g_0\lambda}{m}\right)\omega - 2\left(\frac{1}{m}+\lambda\right)\omega^3,\\ f_3 &= \frac{2m\omega\lambda + \omega}{m^2},\end{aligned}$$

$$f_4 = \frac{-m\omega^2 + \lambda + m\lambda^2 - 2g + g_0}{m^2}. \quad (16)$$

The SNR reflects the response of the system to an external signal. Then, the effect of the inertial mass on the SNR of the system will be considered. In order to obtain the analytical expression of SNR, correlation function of the system must be calculated. Multiplying $x(t')$ on both sides of Eq. (1), and then averaging them, we can obtain

$$m\frac{\mathrm{d}^2\langle x(t)x(t')\rangle}{\mathrm{d}t^2} + \frac{\mathrm{d}\langle x(t)x(t')\rangle}{\mathrm{d}t} = \langle \xi(t)x(t)x(t')\rangle + a\sin(\omega t)\langle x(t')\rangle. \quad (17)$$

In the same way as above, we can obtain the four-order equation about $\langle x(t)x(t')\rangle$ as

$$\begin{aligned}
&\frac{\mathrm{d}^4\langle x(t)x(t')\rangle}{\mathrm{d}t^4} + 2\left(\frac{1}{m}+\lambda\right)\frac{\mathrm{d}^3\langle x(t)x(t')\rangle}{\mathrm{d}t^3} \\
&+\left(\lambda^2 + \frac{3\lambda}{m} + \frac{1}{m^2}\right)\frac{\mathrm{d}^2\langle x(t)x(t')\rangle}{\mathrm{d}t^2} \\
&+\left(\frac{\lambda^2}{m} + \frac{\lambda}{m^2} - \frac{2g_0\lambda}{m}\right)\frac{\mathrm{d}\langle x(t)x(t')\rangle}{\mathrm{d}t} \\
&-\left(\frac{g_0\lambda + mg_0\lambda^2 + D\lambda}{m^2}\right)\langle x(t)x(t')\rangle \\
=\ &\frac{1}{m}\left(\frac{a\lambda}{m} + a\lambda^2 - a\omega^2 + \frac{ag_0}{m}\right)\langle x(t')\rangle\sin(\omega t) \\
&+\frac{1}{m}\left(2a\lambda\omega + \frac{a\omega}{m}\right)\langle x(t')\rangle\cos(\omega t) \\
&+\frac{a}{m^2}\Big\{\Big[-mA\omega^2\sin(\varphi) + A\omega\cos(\varphi) \\
&-Ag_0\sin(\varphi)\Big]\cos(\omega t') \\
&+[-mA\omega^2\cos(\varphi) - A\omega\sin(\varphi) \\
&-Ag_0\cos(\varphi) - a]\sin(\omega t')\Big\} \\
&\times \exp(-\lambda|t-t'|)\sin(\omega t). \quad (18)
\end{aligned}$$

The solution of Eq. (18) is expressed in the following form:

$$\langle x(t)x(t')\rangle = M\sin(\omega t + \phi) + N\sin(\omega t + \psi)\exp(-\lambda|t-t'|), \quad (19)$$

where

$$M = \left(\frac{w_3^2 + w_4^2}{w_1^2 + w_2^2}\right)^{1/2},$$

$$\phi = \arctan\left(\frac{w_1w_3 - w_2w_4}{w_2w_3 + w_1w_4}\right), \quad (20)$$

$$N = \left(\frac{z_3^2}{z_1^2 + z_2^2}\right)^{1/2}, \quad \psi = \arctan\left(\frac{z_2}{z_1}\right), \quad (21)$$

with

$$\begin{aligned}
w_1 &= \omega^4 - (\lambda^2 + \frac{3\lambda}{m} + \frac{1}{m^2})\omega^2 - \frac{g_0\lambda + mg_0\lambda^2 + D\lambda}{m^2}, \\
w_2 &= \left(\frac{\lambda^2}{m} + \frac{\lambda}{m^2} - \frac{2g_0\lambda}{m}\right)\omega - 2\left(\frac{1}{m}+\lambda\right)\omega^3, \\
w_3 &= \frac{2am\omega\lambda + a\omega}{m^2}\langle x(t')\rangle, \\
w_4 &= \frac{-am\omega^2 + a\lambda + am\lambda^2 + ag_0}{m^2}\langle x(t')\rangle, \quad (22) \\
z_1 &= \lambda^4 - 6\lambda^2\omega^2 + \omega^4 + 2\left(\frac{1}{m}+\lambda\right)(-\lambda^3 + 3\lambda\omega^2) \\
&\quad + \left(\lambda^2 + \frac{3\lambda}{m} + \frac{1}{m^2}\right)(\lambda^2 - \omega^2) \\
&\quad + \left(\frac{\lambda^2}{m} + \frac{\lambda}{m^2} - \frac{2g_0\lambda}{m}\right)(-\lambda) \\
&\quad - \frac{g_0\lambda + mg_0\lambda^2 + D\lambda}{m^2}, \\
z_2 &= -(-4\lambda^3\omega + 4\lambda\omega^3) - 2\left(\frac{1}{m}+\lambda\right)(3\lambda^2\omega - \omega^3) \\
&\quad - \left(\lambda^2 + \frac{3\lambda}{m} + \frac{1}{m^2}\right)(-2\lambda\omega) \\
&\quad - \left(\frac{\lambda^2}{m} + \frac{\lambda}{m^2} - \frac{2g_0\lambda}{m}\right)\omega, \\
z_3 &= \frac{a}{m^2}\{[-mA\omega^2\sin(\varphi) + A\omega\cos(\varphi) \\
&\quad - Ag_0\sin(\varphi)]\cos(\omega t') \\
&\quad + [-mA\omega^2\cos(\varphi) - A\omega\sin(\varphi) \\
&\quad - Ag_0\cos(\varphi) - a]\sin(\omega t')\}. \quad (23)
\end{aligned}$$

Let $|t-t'| = \tau > 0$, we can calculate the correlation function as $t' = t + \tau$ and $t' = t - \tau$. Next, taking the time average within one period of $2\pi/\omega$ and performing the Fourier transformation regarding $\tau$ to the correlation functions $\langle\langle x(t)x(t+\tau)\rangle\rangle_t$ and $\langle\langle x(t)x(t-\tau)\rangle\rangle_t$, the averaged power spectrum on one side can be obtained

$$\begin{aligned}
&S(\Omega)_{+\Omega} \\
&= \int_{-\infty}^{0}\langle\langle x(t)x(t-\tau)\rangle\rangle_t\exp(-\mathrm{i}\Omega\tau)\mathrm{d}\tau \\
&\quad + \int_{0}^{+\infty}\langle\langle x(t)x(t+\tau)\rangle\rangle_t\exp(-\mathrm{i}\Omega\tau)\mathrm{d}\tau \\
&= \frac{\pi}{2m}\cdot\frac{aA}{w_1^2 + w_2^2}\Big\{\Big[\left(\frac{\lambda}{m} + \lambda^2 - \omega^2 + \frac{g_0}{m}\right)w_1 \\
&\quad + \left(2\lambda\omega + \frac{\omega}{m}\right)w_2\Big]\cos(\varphi) + \Big[\left(2\lambda\omega + \frac{\omega}{m}\right)w_1 \\
&\quad - \left(\frac{\lambda}{m} + \lambda^2 - \omega^2 + \frac{g_0}{m}\right)w_2\Big]\sin(\varphi)\Big\} \\
&\quad \times[\delta(\Omega+\omega) + \delta(\Omega-\omega)]
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\cdot\frac{a}{m^2}\cdot\frac{1}{z_1^2+z_2^2}\{z_2[-mA\omega^2\sin(\varphi)\\
&+A\omega\cos(\varphi)-Ag_0\sin(\varphi)]+z_1[-mA\omega^2\cos(\varphi)\\
&-A\omega\sin(\varphi)-Ag_0\cos(\varphi)-a]\}\\
&\times\left[\frac{\lambda}{(\Omega-\omega)^2+\lambda^2}+\frac{\lambda}{(\Omega+\omega)^2+\lambda^2}\right]\\
&+\frac{1}{2}\cdot\frac{a}{m^2}\cdot\frac{1}{z_1^2+z_2^2}\{z_2[-mA\omega^2\cos(\varphi)\\
&-A\omega\sin(\varphi)-Ag_0\cos(\varphi)-a]\\
&-z_1[-mA\omega^2\sin(\varphi)+A\omega\cos(\varphi)-Ag_0\sin(\varphi)]\}\\
&\times\left[\frac{\Omega+\omega}{(\Omega+\omega)^2+\lambda^2}+\frac{\Omega-\omega}{(\Omega-\omega)^2+\lambda^2}\right]. \qquad (24)
\end{aligned}
$$

The averaged power spectrum on the other side can be obtained in the same way. Thus, the total power spectrum reads

$$S(\Omega)=G_1\delta(\Omega-\omega)+G_2\delta(\Omega)+G_3, \qquad (25)$$

where

$$
\begin{aligned}
G_1 =& -\frac{\pi}{m}\cdot\frac{aA}{w_1^2+w_2^2}\left\{\left[\left(\frac{\lambda}{m}+\lambda^2-\omega^2+\frac{g_0}{m}\right)w_1\right.\right.\\
&\left.+\left(2\lambda\omega+\frac{\omega}{m}\right)w_2\right]\cos(\varphi)\\
&+\left[\left(2\lambda\omega+\frac{\omega}{m}\right)w_1\right.\\
&\left.\left.-\left(\frac{\lambda}{m}+\lambda^2-\omega^2+\frac{g_0}{m}\right)w_2\right]\sin(\varphi)\right\}, \qquad (26)\\
G_2 =& \ 0, \qquad (27)\\
G_3 =& \ \frac{a}{m^2}\cdot\frac{1}{z_1^2+z_2^2}\{z_2[-mA\omega^2\sin(\varphi)\\
&+A\omega\cos(\varphi)-Ag_0\sin(\varphi)]\\
&+z_1[-mA\omega^2\cos(\varphi)-A\omega\sin(\varphi)\\
&-Ag_0\cos(\varphi)-a]\}\\
&\times\left[\frac{\lambda}{(\Omega-\omega)^2+\lambda^2}+\frac{\lambda}{(\Omega+\omega)^2+\lambda^2}\right]\\
&+\frac{a}{m^2}\cdot\frac{1}{z_1^2+z_2^2}\left\{z_2[-mA\omega^2\cos(\varphi)-A\omega\sin(\varphi)\right.\\
&-Ag_0\cos(\varphi)-a]-z_1[-mA\omega^2\sin(\varphi)\\
&\left.+A\omega\cos(\varphi)-Ag_0\sin(\varphi)]\right\}\\
&\times\left[\frac{\Omega+\omega}{(\Omega+\omega)^2+\lambda^2}+\frac{\Omega-\omega}{(\Omega-\omega)^2+\lambda^2}\right]. \qquad (28)
\end{aligned}
$$

The SNR is defined as

$$\mathrm{SNR}=\frac{G_1}{G_3(\Omega=\omega)}. \qquad (29)$$

So far we have obtained the analytical expressions of the amplitude and the signal-to-noise ratio. Now let us discuss the effect of inertial mass on them. By virtue of Eq. (15), the effects of inertia mass $m$ on the amplitude can be discussed through numerical computations (see Figs. 1 and 2). In Figs. 1(a) and 1(b), we plot the dependence of $A$ on the noise intensity $D$ for symmetric noise ($k=1$) and asymmetric noise ($k=3$), respectively. In Fig. 1, we can see that in the case of symmetric dichotomous noise, troughs of the curves of $A$ appear with the increase of inertial mass $m$, while in the case of asymmetric dichotomous noise, the amplitude exhibits that a peak gets flattened and vanishes with the increase of $m$. This means that it is the difference of curves between the symmetric and asymmetric dichotomous noise that makes the underdamped system different. Figure 1(b) shows the resonance, but Fig. 1(a) shows the reverse resonance.

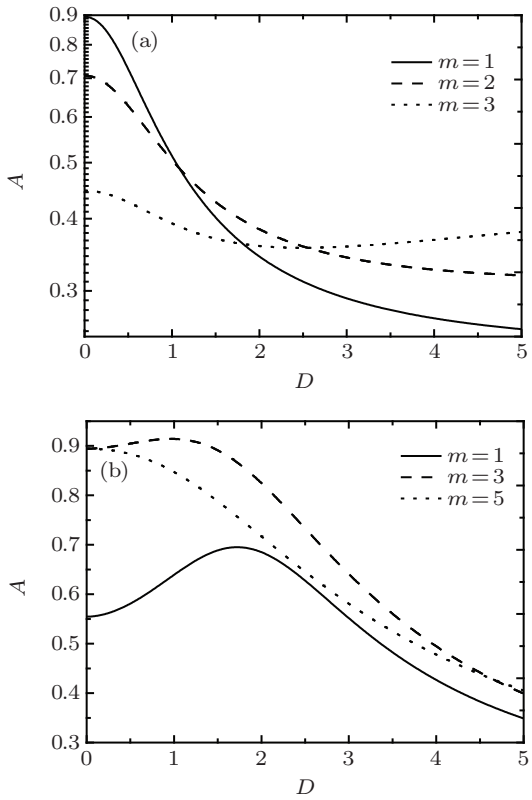



**Fig. 1.** Output signal amplitude $A$ as a function of the noise intensity $D$. (a) $k=1$, (b) $k=3$. The other parameters are $E=1$, $a=0.5$, $\omega=0.5$, and $\gamma=\gamma'$.

In Figs. 2(a) and 2(b), we plot the dependence of $A$ on the inertial mass $m$ for a symmetric noise ($k=1$) and an asymmetric noise ($k=3$), respectively. As shown in Fig. 2(a), a resonance peak occurs. As the value of $D$ decreases, the peak of $A$–$m$ curves becomes higher, and the position of the peak moves toward left. In the case of asymmetric dichotomous noise, the amplitude exhibits that a bimodal structure exists in Fig. 2(b). With $D$ increasing, the highest peak of the curves of $A$ becomes lower and the lowest one vanishes.

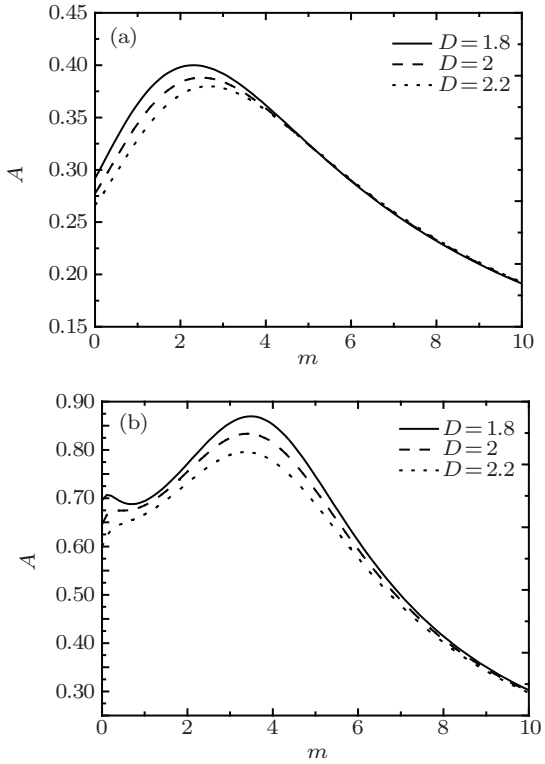



**Fig. 2.** Output signal amplitude $A$ as a function of the inertial mass $m$ for $D$ = 1.8, 2, and 2.2. (a) $k$ = 1, (b) $k$ = 3. The other parameters are the same as that in Fig. 1.

According to Eq. (29), the change of SNR with the noise intensity and inertial mass for symmetric noise and asymmetric noise are shown in Figs. 3 and 4, respectively. In Figs. 3(a) and 3(b), we plot the dependence of SNR on the noise intensity $D$ for symmetric noise ($k$ = 1) and asymmetric noise ($k$ = 3), respectively. From Fig. 3(a), we can see that with the further increment of the inertial mass $m$,the SNR dependence of $D$ becomes non-monotonic. A trough of the curves of SNR appears. The value in the trough is smaller and the position of the trough moves right with the increment of the inertial mass $m$. However, in the case of asymmetric dichotomous noise, the SNR of the system decrease monotonously with increasing noise intensity $D$, as shown in Fig. 3(b) where no peak appears. The curves of SNR illustrate that the SNR decreases with $D$, but increases with $m$.

Figures 4(a) and 4(b) are plots of SNR as a function of the inertial mass $m$ for symmetric noise ($k$ = 1) and asymmetric noise ($k$ = 3), respectively. As shown in Fig. 4(a), a trough appears. The value in the trough is smaller and the position of the trough moves right with the increment of the noise intensity $D$. For a small value of $D$, the SNR shows a small single peak. As the value of $D$ increases, the peak for the smaller value of $D$ decreases and a new peak appears for a larger value of $D$. Figure 4(b) shows that one

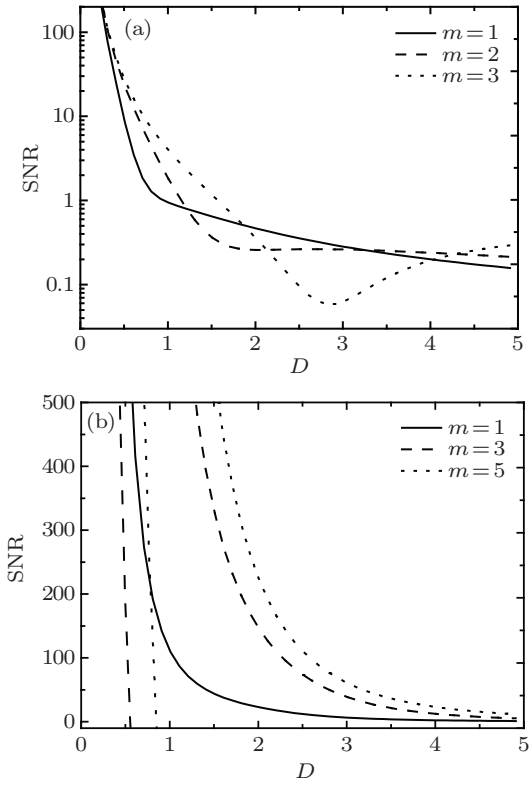



**Fig. 3.** SNR as a function of the noise intensity $D$. (a) $k$ = 1, (b) $k$ = 3. The other parameters are the same as that in Fig. 1.

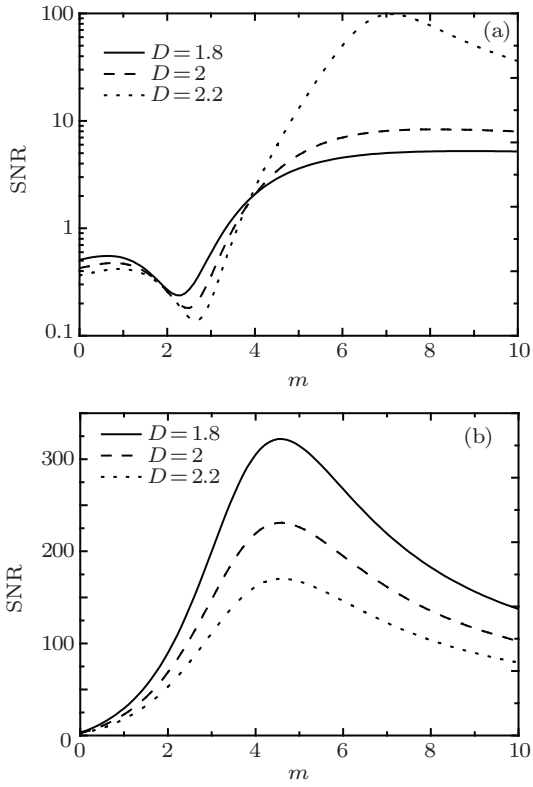



**Fig. 4.** SNR as a function of the inertial mass $m$ for $D$ = 1.8, 2, and 2.2. (a) $k$ = 1, (b) $k$ = 3. The other parameters are the same as that in Fig. 1.

resonance peak appears. The SNR increases for small values of $m$, but decreases for large values of $m$. Namely, the SNR of the system as a function of $m$ is non-monotonic and behaves as a maximum. The SR in the conventional form occurs.

## 3. Numerical simulations

In this section, numerical simulations are presented to verify the effectiveness of the influence of the inertial mass on the SNR, leading to the stochastic resonance of the system. In Fig. 5, we plot the power as a function of frequency at different inertial masses for asymmetric noise. We find that the stochastic resonance appears at $\omega = 0.5$. Figure 6 shows that in the case of asymmetric dichotomous noise, the SNR of the system as a function of inertial mass exhibits a peak.

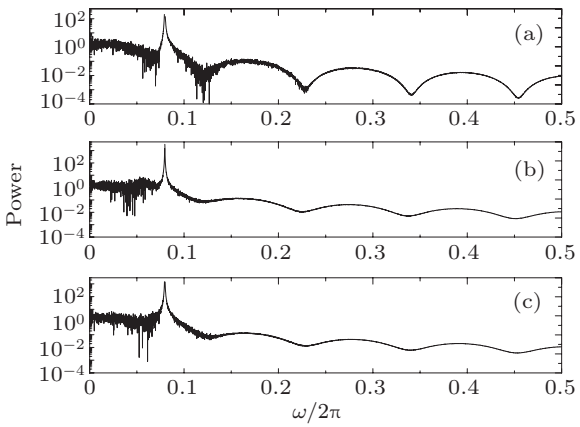



**Fig. 5.** Power as a function of frequency at different inertial masses: $m = 1$, $m = 5$, and $m = 8$ for $D = 2$ and $k = 3$, respectively. The other parameters are the same as in Fig. 1.

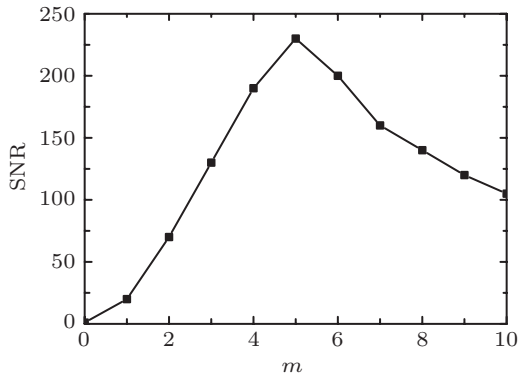



**Fig. 6.** SNR as a function of the inertia mass $m$ for $D = 2$ and $k = 3$. The other parameters are the same as in Fig. 1.

## 4. Conclusions

In summary, we have studied the effect of the inertial mass on SR that is subjected to multiplicative dichotomous noise, and found that the inertial mass greatly affects the output signal amplitude and the SNR. Regardless of whether the noise is symmetric or asymmetric, the inertial mass can influence the stochastic resonance of the system, leading to two types of resonance phenomenon: one is the coherence-resonance-like of the SNR with inertial mass, the other is the SR of the SNR with noise intensity.

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