Nonlocal quantum cloning via quantum dots trapped in distant cavities*

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A scheme for implementing nonlocal quantum cloning via quantum dots trapped in cavities is proposed. By modulating the parameters of the system, the optimal $1 \to 2$ universal quantum cloning machine, $1 \to 2$ phase-covariant cloning machine, and $1 \to 3$ economical phase-covariant cloning machine are constructed. The present scheme, which is attainable with current technology, saves two qubits compared with previous cloning machines.

Keywords: quantum cloning, quantum dots, cavity quantum electrodynamics

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1. Introduction

Perfect quantum cloning of an unknown state is impossible because of the no-cloning theorem,[1] which describes a fundamental difference between classical and quantum information. Therefore, much attention has been paid to approximate cloning. Bužek and Hillery first proposed a universal quantum cloning machine (UQCM) for an arbitrary unknown state,[2,3] which plays an important role in quantum key distribution and cryptography. The phase-covariant cloning machine (PCCM) proposed by Bruß *et al.* is better than UQCM when the input state to be cloned is an equatorial state.[4,5] Later, the economical phase-covariant cloning machine (EPCCM) without any ancilla was proposed by Niu and Griffiths.[6] More recently, Fiurášek proposed a generalized version of this cloning machine.[7]

As one of the most promising candidates for realizing quantum information processing,[8−15] a solid-state system serves as a very nice candidate for accomplishing quantum cloning. Recently, many schemes have been proposed to prepare a multi-qubit entanglement state by using remote quantum dots (QDs) and nonlocal quantum gates have been realized,[16,17] such as the controlled-phase gate, controlled-NOT gate, and the swap gate. All these schemes exploit the advantages of QDs and photons, i.e., the photons act as the flying qubits to transfer quantum information and the QDs trapped in the optical cavity as the storing qubits, which are actually suitable for long-distance communication and quantum information processing. In this paper we present a scheme for nonlocally implementing quantum cloning machine (QCM) using the QDs trapped in distant cavities. The optimal symmetric $1 \to 2$ UQCM, $1 \to 2$ PCCM, and optimal symmetric $1 \to 3$ EPCCM can be realized by choosing the strength of magnetic field B_x. It saves two qubits compared with a previous cloning scheme.[18]

The paper is organized as follows. In Section 2, the interaction between QDs and polarized photons assisted by a magnetic field is described and the Hamiltonian of the system is deduced. In Section 3, based on the Hamiltonian, we propose the scheme for implementing QCM, and the experimental feasibility is discussed. The paper ends with a conclusion.

2. Interaction between singly-charged QDs and linearly-polarized light assisted by a magnetic field

We consider the model that one excess electron spins in a self-assembled InAs QD embedded in the Schottky diode structure, which enables the control over the charging state of the QD.[12] The modelled energy-level structure of the QD at zero magnetic field is shown in Fig. 1(a). The dipole-allowed transition is

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from the electron spin ground state $|\uparrow\rangle$ ($|\downarrow\rangle$) to the trion state $|\Uparrow\rangle$ ($|\Downarrow\rangle$) with σ^+ (σ^-) polarized light excitation, where $|\uparrow\rangle$ ($|\Uparrow\rangle$) and $|\downarrow\rangle$ ($|\Downarrow\rangle$) represent $\pm 1/2$ ($\pm 3/2$) electron (hole) spins. In this condition, the total transitions can be considered as a double two-level structure. To create a three-level lambda system, a magnetic field in Voigt geometry (x-axis), perpendicular to the growth direction (z-axis) of QDs should be used.[12] This magnetic field switches on the forbidden Raman transitions at zero magnetic field, and thus the coherent population trapping is realized. The energy-level structure of a negatively-charged QD can be illustrated, as shown in Fig. 1(b), where new spin states $|\mathrm{X}^{\pm}\rangle$ are formed by the mixture of electron spin ground states $|\uparrow\rangle(|\downarrow\rangle)$. According to the associated optical selection rule, the new electron spin eigenstates $|\mathrm{X}^{\pm}\rangle$ can be excited to trion states $|\mathrm{T}^{\pm}\rangle$ with a linearly-polarized light. The electron spin transitions $|\mathrm{X}^{+}\rangle \leftrightarrow |\mathrm{T}^{-}\rangle$ and $|\mathrm{X}^{-}\rangle \leftrightarrow |\mathrm{T}^{-}\rangle$ are due to the coupling between the electron spin and two degenerate cavity modes (horizontally polarized H and vertically polarized V). Each transition channel is governed by the Hamiltonian of the Jaynes–Cummings model. Supposing two such quantum dots are trapped in spatially-separated cavities, the interaction Hamiltonian in the interaction picture can be written as

$$\begin{aligned}\widehat{H}_i =\ & g_{\mathrm{H}}\left(a_{\mathrm{H}}|\mathrm{T}^{-}\rangle\langle\mathrm{X}^{+}| + a_{\mathrm{H}}^{\dagger}|\mathrm{X}^{+}\rangle\langle\mathrm{T}^{-}|\right) \\ & + g_{\mathrm{V}}\left(a_{\mathrm{V}}|\mathrm{T}^{-}\rangle\langle\mathrm{X}^{-}| + a_{\mathrm{V}}^{\dagger}|\mathrm{X}^{-}\rangle\langle\mathrm{T}^{-}|\right) \\ & + \frac{1}{2}B_x\sigma_x, \end{aligned} \tag{1}$$

where $a_{\mathrm{H}}^{\dagger}$ and a_{H} ($a_{\mathrm{V}}^{\dagger}$ and a_{V}) are the creation and annihilation operators for the cavity mode H (V), respectively. B_x is the strength of the homogeneous magnetic field along the x-axis. $\sigma_x = |\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|$ is the Pauli operator for the excess electron spin. The new electron spin eigenstates $|\mathrm{X}^{\pm}\rangle$ are defined as[13]

$$|\mathrm{X}^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle). \tag{2}$$

We assume that the cavity modes are detuned from their respective transitions by Δ, and the coupling strength of two cavity modes are equal ($g_{\mathrm{H}} = g_{\mathrm{V}} = g$). In the case that $\Delta \gg g_{\mathrm{H}}, g_{\mathrm{V}}$, the trion state $|\mathrm{T}^{-}\rangle$ can be adiabatically eliminated. Thus, the effective Hamiltonian is written as (assuming $\hbar = 1$)

$$\begin{aligned}\widehat{H}_e =\ & -\left(\frac{g^2}{\Delta}a_{\mathrm{H}}^{\dagger}a_{\mathrm{H}}|\mathrm{X}^{+}\rangle\langle\mathrm{X}^{+}| + \frac{g^2}{\Delta}a_{\mathrm{V}}^{\dagger}a_{\mathrm{V}}|\mathrm{X}^{-}\rangle\langle\mathrm{X}^{-}|\right) \\ & - \left(\frac{g^2}{\Delta}a_{\mathrm{V}}^{\dagger}a_{\mathrm{H}}|\mathrm{X}^{-}\rangle\langle\mathrm{X}^{+}| + \frac{g^2}{\Delta}a_{\mathrm{H}}^{\dagger}a_{\mathrm{V}}|\mathrm{X}^{+}\rangle\langle\mathrm{X}^{-}|\right) \\ & + B_x S_z, \end{aligned} \tag{3}$$

where $S_z = (|\mathrm{X}^{+}\rangle\langle\mathrm{X}^{+}| - |\mathrm{X}^{-}\rangle\langle\mathrm{X}^{-}|)/2$.

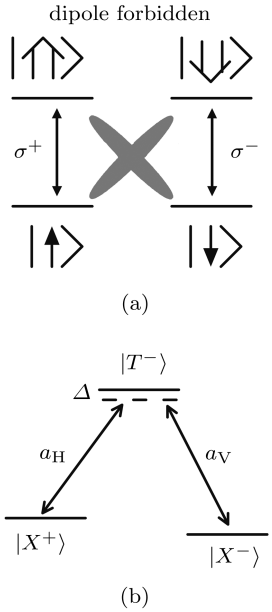


Fig. 1. (a) Trion energy-level diagram in the absence of a magnetic field in the Voigt geometry. (b) Structure of a Λ-type three-level quantum dot,[19] where new electron spin states $|\mathrm{X}^{\pm}\rangle$ are formed by the mixture of electron spin ground states $|\uparrow\rangle(|\downarrow\rangle)$.

If the singly-charged QD is in a state of $|X^{+}\rangle$ and the photon in a state of $|\mathrm{H}\rangle$ initially, then according to the cavity electrodynamics, the evolution of the system stays at a subspace spanned by $\{|\mathrm{X}^{+}\rangle|\mathrm{H}\rangle, |\mathrm{X}^{-}\rangle|\mathrm{V}\rangle\}$. Then the final state can be written as

$$|\varphi(t)\rangle = a(t)|\mathrm{X}^{+}\rangle|\mathrm{H}\rangle + b(t)|\mathrm{X}^{-}\rangle|\mathrm{V}\rangle. \tag{4}$$

Based on the Schrödinger equation, the following equation set can be obtained:

$$\begin{aligned}\mathrm{i}\dot{a}(t) &= -\frac{g^2}{\Delta}a(t) - \frac{g^2}{\Delta}b(t) + \frac{1}{2}B_x a(t), \\ \mathrm{i}\dot{b}(t) &= -\frac{g^2}{\Delta}a(t) - \frac{g^2}{\Delta}b(t) - \frac{1}{2}B_x b(t), \end{aligned} \tag{5}$$

with the initial conditions

$$a(0) = 1, \quad b(0) = 0. \tag{6}$$

Let the strength of magnetic field $B_x = 2\sqrt{5}g^2/\Delta$, we obtain the solution of Eq. (5)

$$\begin{aligned} a(t) &= \frac{1}{12}\,\mathrm{e}^{\mathrm{i}\frac{g^2 t}{\Delta}}\left(12\cos\frac{\sqrt{6}}{\Delta}g^2 t - 2\sqrt{30}\mathrm{i}\sin\frac{\sqrt{6}}{\Delta}g^2 t\right), \\ b(t) &= \frac{1}{12}\,\mathrm{e}^{\mathrm{i}\frac{g^2 t}{\Delta}}2\sqrt{6}\mathrm{i}\sin\frac{\sqrt{6}}{\Delta}g^2 t. \end{aligned} \tag{7}$$

Through similar calculations, the evolution of the system can be obtained

$$\begin{aligned}
&|\mathrm{X}^+\rangle|\mathrm{H}\rangle \rightarrow a(t)|\mathrm{X}^+\rangle|\mathrm{H}\rangle + b(t)|\mathrm{X}^-\rangle|\mathrm{V}\rangle,\\
&|\mathrm{X}^+\rangle|\mathrm{V}\rangle \rightarrow \mathrm{e}^{-\mathrm{i}B_x t}|\mathrm{X}^+\rangle|\mathrm{V}\rangle,\\
&|\mathrm{X}^-\rangle|\mathrm{H}\rangle \rightarrow \mathrm{e}^{-\mathrm{i}B_x t}|\mathrm{X}^-\rangle|\mathrm{H}\rangle,\\
&|\mathrm{X}^-\rangle|\mathrm{V}\rangle \rightarrow a(t)|\mathrm{X}^-\rangle|\mathrm{V}\rangle + b(t)|\mathrm{X}^+\rangle|\mathrm{H}\rangle. \qquad (8)
\end{aligned}$$

3. Implementation of quantum cloning machines

To implement quantum cloning transformation, a controlled-phase gate should be used, as shown in Fig. 2.[19] The polarization beam splitter (PBS) transmits the H-polarized photon and reflects the V-polarized photon, the optical path from the PBS to the cavity is equal to that from the PBS to the mirror.

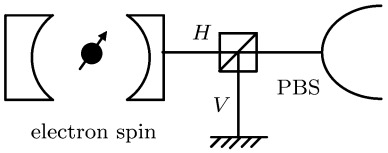


Fig. 2. Schematic setup of the controlled-phase gate between a photon and an electron spin inside a cavity.

When the incoming photon is V-polarized, it will be reflected by the mirror without any phase change, while an H-polarized photon will resonate with the cavity mode and obtain a phase of π after its reflection when the electron spin is in the $|\mathrm{X}^-\rangle$ state. If the electron spin is in the $|\mathrm{X}^+\rangle$ state, it is far-off resonant with the cavity mode. Hence, the controlled-phase gate can be described by a unitary operator

$$U = \mathrm{e}^{\mathrm{i}\pi|\mathrm{X}^-\rangle\langle\mathrm{X}^-|\otimes|\mathrm{H}\rangle\langle\mathrm{H}|}. \qquad (9)$$

The setup for implementing QCMs is shown in Fig. 3. TR1, TR2, and TR3 are optical devices that can be exactly controlled to transmit or reflect photons with a very fast switch. The quarter wave plate (QWP) implements the transformations $|\mathrm{H}\rangle \rightarrow (-|\mathrm{H}\rangle + |\mathrm{V}\rangle)/\sqrt{2}$ and $|\mathrm{V}\rangle \rightarrow (|\mathrm{H}\rangle + |\mathrm{V}\rangle)/\sqrt{2}$, and the half wave plate (HWP) implements the transformations $|\mathrm{H}\rangle \rightarrow |\mathrm{V}\rangle$ and $|\mathrm{V}\rangle \rightarrow |\mathrm{H}\rangle$. The QDs 1 and 2 are trapped in the cavities 1 and 2, respectively. The electron spin of QD 1 is in an arbitrary state (to be cloned) $\alpha|\mathrm{X}_1^+\rangle + \beta\,\mathrm{e}^{\mathrm{i}\varphi}|\mathrm{X}_1^-\rangle$ (α and β are real), and the electron spin of QD 2 is in the $|\mathrm{X}_2^+\rangle$ state initially. A single-photon pulse in the $|\mathrm{H}\rangle$ state passes through TR1 and the QWP moves along the path 2. Then the PBS1 transmits the H-polarized light and reflects the V-polarized light. After the H-polarized light is reflected by the cavity 1 and the V-polarized light is reflected by the mirror, the system evolves into the following state

$$\begin{aligned}
&\frac{1}{\sqrt{2}}\alpha|\mathrm{X}_1^+\rangle|\mathrm{X}_2^+\rangle(-|\mathrm{H}\rangle + |\mathrm{V}\rangle)\\
&\quad + \frac{1}{\sqrt{2}}\beta\,\mathrm{e}^{\mathrm{i}\varphi}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^+\rangle(|\mathrm{H}\rangle + |\mathrm{V}\rangle). \qquad (10)
\end{aligned}$$

Then, the photon moves to PBS1 and then passes through the QWP, TR1, TR2, TR3, and PBS2 to reach the cavity 2 along the path 4. At this time, we need to perform a Hadamard gate operation ($|\mathrm{X}^\pm\rangle \rightarrow (\pm|\mathrm{X}^+\rangle + |\mathrm{X}^-\rangle)/\sqrt{2}$ on the spin of QD 2 by using a $\pi/2$ microwave pulse before the photon enters into

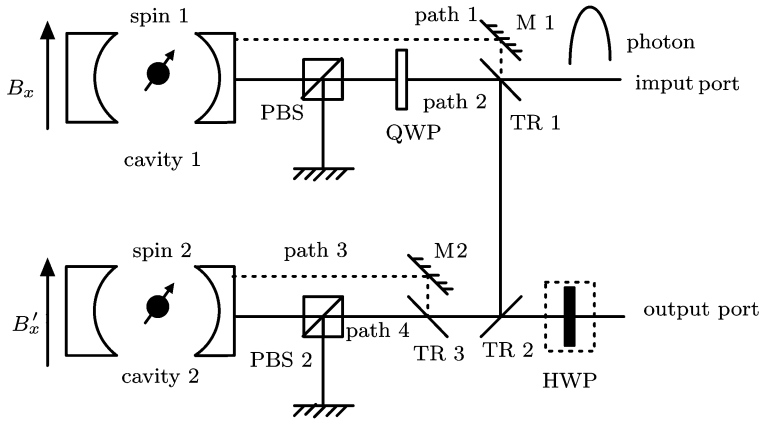


Fig. 3. Schematic setup of the quantum cloning machine. B_x (B_x') is the magnetic field along the x-axis in the cavity 1 (2).

the cavity 2. The state of the system is

$$\frac{1}{\sqrt{2}}(\alpha|\mathrm{X}_1^+\rangle|\mathrm{H}\rangle+\beta\,\mathrm{e}^{\mathrm{i}\varphi}|\mathrm{X}_1^-\rangle|\mathrm{V}\rangle)(|\mathrm{X}_2^+\rangle+|\mathrm{X}_2^-\rangle). \quad (11)$$

After being reflected out of the cavity 2, the photon passes through the PBS2, TR3, TR2, and TR1. Then a Hadadmard gate operation is applied to the spin of QD 2 again. The state of the system is

$$\alpha|\mathrm{X}_1^+\rangle|\mathrm{X}_2^+\rangle|\mathrm{H}\rangle+\beta\,\mathrm{e}^{\mathrm{i}\varphi}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^-\rangle|\mathrm{V}\rangle. \quad (12)$$

After a photon interacts with the spins 1 and 2 in sequence, a hybrid spin-spin-photon system is prepared in a GHZ state.

Then the photon is transmitted by TR1 to reach the mirror 1 (M1) and cavity 1. By setting the interaction time $t_1^{\mathrm{UQCM}}=\pi\Delta/2\sqrt{6}g^2$, from Eqs. (7) and (8), the system state can be given as

$$\begin{aligned}&\alpha\left(-\sqrt{\frac{5}{6}}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^+\rangle|\mathrm{H}\rangle+\sqrt{\frac{1}{6}}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^+\rangle|\mathrm{V}\rangle\right)\\&+\beta\,\mathrm{e}^{\mathrm{i}\varphi}\Big(-\sqrt{\frac{5}{6}}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^-\rangle|\mathrm{V}\rangle\\&+\sqrt{\frac{1}{6}}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^-\rangle|\mathrm{H}\rangle\Big). \quad (13)\end{aligned}$$

After the photon is reflected out of the cavity 1, it moves to M1 along the path 1, and then passes through the TR1, TR2, TR3, and M2 to reach the cavity 2 along the path 3. From Eqs. (7) and (8), we obtain the final state of the system as follows:

$$\begin{aligned}|\varphi(t)\rangle=\;&m(t)|X_1^+\rangle|X_2^+\rangle|\mathrm{H}\rangle+n(t)|\mathrm{X}_1^+\rangle|\mathrm{X}_2^-\rangle|\mathrm{V}\rangle\\&+q(t)|\mathrm{X}_1^-\rangle|\mathrm{X}_2^+\rangle|\mathrm{V}\rangle, \quad (14)\end{aligned}$$

where

$$\begin{aligned}m(t)=\;&-\frac{1}{2\sqrt{30}}\,\mathrm{e}^{-\frac{\mathrm{i}g^2t}{\Delta}}\\&\times\left(10\cos\frac{\sqrt{5}g^2t}{\Delta}-4\sqrt{5}\mathrm{i}\sin\frac{\sqrt{5}g^2t}{\Delta}\right),\\n(t)=\;&-\frac{1}{2\sqrt{30}}\,\mathrm{e}^{-\frac{2\mathrm{i}g^2t}{\Delta}}\left(2\sqrt{5}\mathrm{i}\sin\frac{\sqrt{5}g^2t}{\Delta}\right),\\q(t)=\;&\frac{1}{\sqrt{6}}\mathrm{i}\,\mathrm{e}^{-\frac{2\mathrm{i}g^2t}{\Delta}}. \quad (15)\end{aligned}$$

We choose the strength of magnetic field $B_x^{'}=4g^2/\Delta$. By choosing the interaction time $t_2^{\mathrm{UQCM}}=\pi\Delta/2\sqrt{5}g^2$, and applying $\hat{\sigma}_z$ to the electron spin 2, the state of the entire system can be written as

$$\begin{aligned}&\alpha\Big(\sqrt{\frac{2}{3}}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^+\rangle|\mathrm{H}\rangle+\sqrt{\frac{1}{6}}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^-\rangle|\mathrm{V}\rangle\\&+\sqrt{\frac{1}{6}}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^+\rangle|\mathrm{V}\rangle\Big)\\&+\beta\,\mathrm{e}^{\mathrm{i}\varphi}\Big(\sqrt{\frac{2}{3}}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^-\rangle|\mathrm{V}\rangle\\&+\sqrt{\frac{1}{6}}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^+\rangle|\mathrm{H}\rangle\\&+\sqrt{\frac{1}{6}}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^-\rangle|\mathrm{H}\rangle\Big). \quad (16)\end{aligned}$$

After the photon is reflected out of the cavity 2, it passes through the mirror 2 (M2), TR3, and TR2 to reach the output port. Thus, we obtain the transformation of the optimal symmetric $1\to2$ UQCM with two spatially-separated QDs by using the photon as an ancilla.

If the input state of the electron spin 1 to be cloned is an equatorial state, i.e., $(|\mathrm{X}_1^+\rangle+\mathrm{e}^{\mathrm{i}\varphi}|\mathrm{X}_1^-\rangle)/\sqrt{2}$, we can choose $B_x=2\sqrt{3}g^2/\Delta$ and $B_x^{'}=2\sqrt{2}g^2/\Delta$. The optimal symmetric $1\to2$ PCCM can be constructed by choosing the interaction times $t_1^{\mathrm{PCCM}}=\pi\Delta/4g^2$ and $t_2^{\mathrm{PCCM}}=\pi\Delta/2\sqrt{3}g^2$, i.e.,

$$\begin{aligned}&\sqrt{\frac{1}{2}}\Big(\sqrt{\frac{1}{2}}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^+\rangle|\mathrm{H}\rangle+\frac{1}{2}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^-\rangle|\mathrm{V}\rangle\\&+\frac{1}{2}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^+\rangle|\mathrm{V}\rangle\Big)+\sqrt{\frac{1}{2}}\,\mathrm{e}^{\mathrm{i}\varphi}\Big(\sqrt{\frac{1}{2}}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^-\rangle|\mathrm{V}\rangle\\&+\frac{1}{2}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^+\rangle|\mathrm{H}\rangle+\frac{1}{2}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^-\rangle|\mathrm{H}\rangle\Big). \quad (17)\end{aligned}$$

When $B_x=-2\sqrt{2}g^2/\Delta$ and $B_x^{'}=2g^2/\Delta$, the photon pulse passes through a HWP, the optimal symmetric $1\to3$ EPCCM can be constructed by choosing the interaction times $t_1^{\mathrm{EPCCM}}=\pi\Delta/2\sqrt{3}g^2$ and $t_2^{\mathrm{EPCCM}}=3\pi\Delta/2\sqrt{2}g^2$, i.e.,

$$\begin{aligned}&\sqrt{\frac{1}{2}}\Big(\sqrt{\frac{1}{3}}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^+\rangle|\mathrm{V}\rangle+\sqrt{\frac{1}{3}}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^-\rangle|\mathrm{H}\rangle\\&+\sqrt{\frac{1}{3}}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^+\rangle|\mathrm{H}\rangle\Big)\\&+\sqrt{\frac{1}{2}}\,\mathrm{e}^{\mathrm{i}\varphi}\Big(\sqrt{\frac{1}{3}}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^-\rangle|\mathrm{H}\rangle\\&+\sqrt{\frac{1}{3}}|\mathrm{X}_1^-\rangle|\mathrm{X}_2^+\rangle|\mathrm{V}\rangle\\&+\sqrt{\frac{1}{3}}|\mathrm{X}_1^+\rangle|\mathrm{X}_2^-\rangle|\mathrm{V}\rangle\Big), \quad (18)\end{aligned}$$

where no ancilla is included, i.e., the two QDs and one photon are all clones.

Compared with a previous proposal for quantum cloning in separate cavities,[18] our scheme is advantageous to save two qubits. In Ref. [18], three flying qubits are used to realize the QCM and the information is encoded in the first flying qubit. The other two

qubits act as ancilla. In our scheme, only one photon acts as the ancilla, and the quantum information can be copied from the electron spin to another distant spin.

Recently, the experimental conditions in cavity $g \sim 11.2$ MHz and $\Delta \sim 70\pi$ MHz can be achieved easily.[20] Under this condition, the total time of manipulating UQCM ($t_1^{\mathrm{UQCM}}+t_2^{\mathrm{UQCM}}$) is about 0.279 μs. The construction of PCCM and EPCCM takes about 0.3 μs and 0.753 μs, respectively, which are less than the time of spin decoherence.[21]

4. Conclusion

In summary, we have proposed a scheme to implement the optimal $1 \rightarrow 2$ UQCM, $1 \rightarrow 2$ PCCM, and $1 \rightarrow 3$ EPCCM by choosing suitable parameters. It saves two photons compared with the previous scheme. The proposed scheme can be achieved with current experimental technology.

References

[1] Wootters W K and Zurek W H 1982 *Nature* **299** 802

[2] Bužek V and Hillery M 1996 *Phys. Rev.* A **54** 1844

[3] Werner R F 1998 *Phys. Rev.* A **58** 1827

[4] BrußD, Cinchetti M, D'Ariano G M and Macchiavello C 2000 *Phys. Rev.* A **62** 012302

[5] Fan H, Imai H, Matsumoto K and Wang X B 2003 *Phys. Rev.* A **67** 022317

[6] Niu C S and Griffiths R B 1999 *Phys. Rev.* A **60** 2764

[7] Fiurášek J 2003 *Phys. Rev.* A **67** 052314

[8] Loss D and DiVincenzo D P 1998 *Phys. Rev.* A **57** 120

[9] Imamoglu A, Awschalom D D, Burkard G, Loss D, DiVincenzo D P, Sherwin M and Small A 1999 *Phys. Rev. Lett.* **83** 4204

[10] Lin Z R, Guo G P, Tu T, Zhu F Y and Guo G C 2008 *Phys. Rev. Lett.* **101** 230501

[11] Hao X and Zhu S Q 2007 *Phys. Rev.* A **76** 044306

[12] Xu X D, Bo S, Berman P R, Steel D G, Bracker A S, Gammon D and Sham L J 2008 *Nature* **4** 629

[13] Kosaka H, Shigyou H, Mitsumori Y, Rikitake Y, Imamura H, Kutsuwa T, Arai K and Edamatsu K 2008 *Phys. Rev. Lett.* **100** 096602

[14] Zhou X Q, Wu Y W and Zhao H 2011 *Acta Phys. Sin.* **4** 60 (in Chinese)

[15] Yin J Q, Li Y, Bao X Q, Peng X H, Cheng Z, Yang T and Pan G S 2011 *Acta Phys. Sin.* **6** 60 (in Chinese)

[16] Lin X M, Zhou Z W, Ye M Y, Xiao Y F and Guo G C 2006 *Phys. Rev.* A **73** 012323

[17] Xiao Y F, Lin X M, Gao J, Yang Y, Han Z F and Guo G C 2004 *Phys. Rev.* A **70** 042314

[18] Fang B L, Song Q M and Ye L 2011 *Phys. Rev.* A **83** 042309

[19] Duan L M and Kimble H J 2004 *Phys. Rev. Lett.* **92** 127902

[20] Maunz P, Puppe T, Schuster I, Syassen N, Pinkse P W H and Rempe G 2005 *Phys. Rev. Lett.* **94** 033002

[21] Press D, Greve K D, McMahon P L, Ladd T D, Friess B, Schneider C, Kamp M, Höfling S, Forchel A and Yamamoto Y 2010 *Nature* **4** 367