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Cosmological applications in Kaluza–Klein theory

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The field equations of Kaluza–Klein (KK) theory have been applied in the domain of cosmology. These equations are solved for a flat universe by taking the gravitational and the cosmological constants as a function of time t. We use Taylor's expansion of cosmological function, $\Lambda(t)$, up to the first order of the time t. The cosmological parameters are calculated and some cosmological problems are discussed.

Keywords: Kaluza–Klein theory, cosmology, Taylor's expansion of cosmological function

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1. Introduction

Recent observation of distant supernovae type Ia indicated that the universe is presently accelerating.[1−4] Cosmic acceleration is attributed to the presence of an unknown form of energy violating the strong energy conditions $\rho + 3p > 0$ where ρ and p are energy density and pressure of dark energy, respectively. Different candidates for dark energy,[5,6] have attempted to yield accelerating cosmologies at a late times. The cosmological constant Λ and phantom fields[7−19] violating weak energy conditions $\rho + p > 0$ are the most popular ones.

Kaluza's[20] has shown that the five-dimensional general relativity contains both Einstein's 4-dimensional theory of gravity and Maxwell's theory of electromagnetism. He however imposed a somewhat artificial restriction (the cylindrical condition) on the coordinates, essentially barring the fifth one a priori from making a direct appearance in the laws of physics. Klein's[21] contribution was to make this restriction less artificial by suggesting a plausible physical basis for it in the compactification of the fifth dimension. This idea was enthusiastically received by unified-field theorists, and when the time came to include the strong and weak forces by extending Kaluza's mechanism to higher dimensions, it was assumed that these too would be compact. This line of thinking has led through eleven-dimensional supergravity theories in 1980 to the current favorite contenders for a possible theory of everything, ten-dimensional superstrings.

Most existing astrophysical work on the cosmological term, operate on the assumption that Λ is a constant. However, quantum field theorists and others treat the cosmological term as a dynamical quantity (cf. Refs. [22]–[29]). Anything which contributes to the energy density ρ of the vacuum behaves like a cosmological term via $\Lambda = 8\Pi G\rho$. Many potential sources of fluctuating vacuum energy have now been identified, including scalar fields,[31−39] tensor fields,[40−44] nonlocal effects,[45,46] wormholes,[47,48] inflationary mechanisms,[49] and even cosmological perturbations.[50] Each of these has been claimed to give rise to a negative energy density which grows with time, tending to cancel out any pre-existing positive cosmological term and drive the net value of Λ toward zero. Processes of this kind are among the most promising ways to resolve the longstanding cosmological constant problem[31] (see Ref. [50] for review).

It is the aim of the present work to solve the gravitational field equations of Kaluza–Klein (KK) theory in the domain of cosmology considering the gravitational and cosmological constants as a function of time t. Then, we assume that $\Lambda(t) = \epsilon R(t)^{-2}$, where $R(t)$ is the scale factor used to calculate the cosmological

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parameters. In Section 2, the field equations of KK are applied to the FRW metric. In Section 3, we give the solution of the field equation and calculate the corresponding cosmological parameters. In Section 4, we give the solution of the field equation for a closed universe and also calculate the cosmological parameters associated with this model. The final section is devoted a conclusion.

2. The FRW model and KK theory

The FRW line element in five-dimensional spacetime has the form[52]

$$\begin{aligned} \mathrm{d}s^2 \;=\; & \mathrm{d}t^2 - R^2(t)\bigg[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) \\ & + (1-kr^2)\,\mathrm{d}\psi^2\bigg], \end{aligned} \tag{1}$$

where $R(t)$ is the scale factor, k is the curvature parameter ($k = +1,\ 0,\ -1$) corresponding to closed flat and open universe respectively. The Einstein field equations are given by

$$R_{\mu\nu} - g_{\mu\nu}\frac{R}{2} = g_{\mu\nu}\Lambda - 8\pi G T_{\mu\nu} = 8\pi G T'_{\mu\nu}, \tag{2}$$

where $R_{\mu\nu}$, R, $g_{\mu\nu}$, $\Lambda = \Lambda(t)$, $G = G(t)$ are Ricci tensor, Ricci scalar, metric tensor, cosmological function, and gravitational function, respectively.

Assuming that matter filling the universe is in the form of a perfect fluid, then

$$\begin{aligned} & T'^{0}_{0} = \rho + \frac{\Lambda}{8\pi G}, \\ & T'^{1}_{1} = T'^{2}_{2} = T'^{3}_{3} = T'^{4}_{4} = -P + \frac{\Lambda}{8\pi G}, \end{aligned} \tag{3}$$

where $\rho = \rho(t)$ is the density and $P = P(t)$ is the pressure. Applying Eq. (2) to metric given by Eq. (1) and using Eq. (3) we obtain the following set of differential equations

$$8\pi G\rho + \Lambda = 6\bigg[\bigg(\frac{\dot{R}}{R}\bigg)^2 + \frac{k}{R^2}\bigg], \tag{4}$$

$$8\pi G P - \Lambda = -3\bigg[\frac{\ddot{R}}{R} + \bigg(\frac{\dot{R}}{R}\bigg)^2 + \frac{k}{R^2}\bigg]. \tag{5}$$

Now we have two differential equations in five unknown functions $R(t)$, $\rho(t)$, $P(t)$, $\Lambda(t)$, and $G(t)$. Therefore we need some extra conditions to be able to solve Eqs. (4) and (5), for each value of k. One of these conditions is usually provided by an equation of state characterizing the fluid filling the model. Here we take the equation of state in the form

$$P = (\omega - 1)\rho, \tag{6}$$

where $\omega \in [0, 2]$. When $\omega = 0$, then $P = -\rho$ (dark energy case), when $\omega = 1$, then $P = 0$ (dust case), when $\omega = 4/3$ then $P = \rho/3$ (radiation case), and when $\omega = 2$ then $P = \rho$ (stiff matter case). The conservation equation has the form $T'^{\mu\nu}{}_{;\,\nu} = 0$. This will gives rise to the relation,

$$\dot{\rho} + 4\frac{\dot{R}(t)}{R(t)}(\rho + P) + \frac{\Lambda}{8\pi G}\bigg(\frac{\dot{\Lambda}}{\Lambda} - \frac{\dot{G}}{G}\bigg) = 0. \tag{7}$$

Equations (4), (5), and (6) are three nonlinear differential equations in the five unknown functions. Therefore, we spilt Eq. (7), for simplicity, into the two equations

$$\dot{\rho} + 4\frac{\dot{R}(t)}{R(t)}(\rho + P) = 0, \tag{8}$$

and

$$\frac{\dot{\Lambda}}{\Lambda} - \frac{\dot{G}}{G} = 0, \tag{9}$$

in order to impose two further additional conditions on the unknown function. Now, we have five differential equations (4), (5), (6), (8), and (9) in the five unknown functions $R(t)$, $\rho(t)$, $P(t)$, $\Lambda(t)$, and $G(t)$.

Equation (9) shows that Λ is constant whenever G is constant and *vice versa*. Solving Eq. (8), we obtain

$$\rho(t) = \frac{c_1}{R(t)^{4\omega}}, \tag{10}$$

where equation (6) has been used and c_1 is a constant of integration. The constant c_1 can be determined by using $\omega = \omega_0$ and $\rho = \rho_{\mathrm{c}}$ (critical density) at time $t = t_0$, where the subscript 0 refers to the present value. We are going to use the following definitions of some cosmological parameters:

$$\begin{aligned} & \text{the Hubble's parameter} \quad H_0 = \frac{\dot{R}(t_0)}{R(t_0)} = \frac{\dot{R}_0}{R_0}, \\ & \text{the Hubble's time} \quad \tau_0 \overset{\text{def.}}{=\!=} {H^{-1}}_0, \\ & \text{the deceleration parameter} \\ & q_0 \overset{\text{def.}}{=\!=} -\frac{\ddot{R}_0}{R_0}{H^{-2}}_0 = -1 - \frac{\dot{H}_0}{{H_0}^2}, \\ & \text{the density parameter} \quad \sigma_0 \overset{\text{def.}}{=\!=} \frac{\rho_0}{\rho_c}, \end{aligned}$$

where ρ_{c} is the critical density defined by

$$\rho_c \overset{\text{def.}}{=\!=} \frac{3{H^2}_0}{8\pi G}. \tag{11}$$

3. Solution for $k = 0$

We are going to solve the field equations[1] (4) and (5), and then we evaluate the cosmological parameters. Here we assume[2]

$$\Lambda(t) \simeq \epsilon_0 + \epsilon_1 t + \cdots , \tag{12}$$

where ϵ_0 and $\epsilon_1 \cdots$ are arbitrary constants. From Eq. (9) we obtain

$$G(t) = c_2(\epsilon_0 + \epsilon_1 t), \tag{13}$$

where c_2 is another constant of integration.

Equations (4) and (5), in the case $k = 0$, take the forms

$$\begin{aligned} 8\pi G\rho + \Lambda &= 6\left[\left(\frac{\dot{R}(t)}{R(t)}\right)^2\right], \\ 8\pi GP - \Lambda &= -3\left[\frac{\ddot{R}(t)}{R(t)} + \left(\frac{\dot{R(t)}}{R(t)}\right)^2\right]. \end{aligned} \tag{14}$$

From Eq. (14) we obtain

$$\begin{aligned} &3\frac{\ddot{R}(t)}{R(t)} - 3\left(\frac{\dot{R}(t)}{R(t)}\right)^2 + \omega\left[6\frac{\dot{R}^2(t)}{R^2(t)} - \Lambda\right] \\ &\equiv \dot{H(t)} + \omega\left[2H^2(t) - \frac{\epsilon_0}{3} - \frac{\epsilon_1 t}{3}\right] = 0, \end{aligned} \tag{15}$$

where equation (6) has been used again. Equation (15) is a differential equation with two variables so we are going to study the following cases, dust, radiation, and stiff matter.

We will discard the case $\omega = 0$ since this will make the density parameter, given by Eq. (10), always constant, in spite of the expansion of the universe. This needs an extensive discussion which will be done in the future work.

(i) ***Dust case ($\omega = 1$)***

The integration of Eq. (15) has the form

$$\begin{aligned} H(t) &= A_0 \frac{c_3\,\mathrm{A}(1, \Lambda(t)/A_{00}) - c_4\,\mathrm{B}(1, \Lambda(t)/A_{00})}{c_3\,\mathrm{A}(\Lambda(t)/A_{00}) - c_4\,\mathrm{B}(\Lambda(t)/A_{00})}, \\ A_0 &= \left(\frac{\epsilon_1}{12}\right)^{1/3}, \quad A_{00} = \left(\frac{3{\epsilon_1}^2}{2}\right)^{1/3}, \end{aligned} \tag{16}$$

where c_3 and c_4 are constants of integration, $\mathrm{A}(1, x)$ and $\mathrm{B}(1, x)$ are the $\mathrm{AiryAi}(1, x)$ and $\mathrm{AiryBi}(1, x)$ wave functions.[3]

From Eq. (16) we obtain the scale factor in the form

$$R(t) = \sqrt{\frac{c_3\,\mathrm{A}(\Lambda(t)/A_{00}) - c_4\,\mathrm{B}(\Lambda(t)/A_{00})}{A_0\,[\,\mathrm{A}(1, \Lambda(t)/A_{00})\,\mathrm{B}(\Lambda(t)/A_{00}) - \mathrm{B}(1, \Lambda(t)/A_{00})\,\mathrm{A}(\Lambda(t)A_{00})]}}. \tag{17}$$

(ii) ***Radiation case ($\omega = 4/3$)***

The integration of Eq. (15) has the form

$$H(t) = B_0 \frac{c_5\,\mathrm{A}(1, \Lambda(t)/B_{00}) c_6\,\mathrm{B}(1, \Lambda(t)/B_{00})}{c_5\,\mathrm{A}(\Lambda(t)/B_{00}) - c_6\,\mathrm{B}(\Lambda(t)/B_{00})}, \quad B_0 = \left(\frac{{\epsilon_1}^3}{16}\right)^{1/3}, \quad B_{00} = \left(\frac{27{\epsilon_1}^2}{32}\right)^{1/3}, \tag{18}$$

where c_5 and c_6 are constants of integration. From Eq. (18) we obtain the scale factor in the form

$$R(t) = \left\{\frac{c_5\,\mathrm{A}(\Lambda(t)/B_{00}) - c_6\,\mathrm{B}(\Lambda(t)/B_{00})}{B_0\,[\mathrm{A}(1, \Lambda(t)/B_{00})\,\mathrm{B}(\Lambda(t)/B_{00}) - \mathrm{B}(1, \Lambda(t)/B_{00})\,\mathrm{A}(\Lambda(t)/B_{00})]}\right\}^{3/8}. \tag{19}$$

(iii) ***Stiff matter case ($\omega = 2$)***

The integration of Eq. (15) has the form

$$H(t) = C_0 \frac{c_7\,\mathrm{A}(1, \Lambda(t)/C_{00}) - c_8\,\mathrm{B}(1, \Lambda(t)/C_{00})}{c_5\,\mathrm{A}(\Lambda(t)/C_{00}) + \mathrm{B}(\Lambda(t)/C_{00})}, \quad C_0 = \left(\frac{\epsilon_1}{24}\right)^{1/3}, \quad C_{00} = \left(\frac{3{\epsilon_1}^2}{8}\right)^{1/3}, \tag{20}$$

where c_7 and c_8 are constants of integration. From Eq. (20) we obtain the scale factor in the form

$$R(t) = \left\{\frac{c_7\,\mathrm{A}(\Lambda(t)/C_{00}) - c_8\,\mathrm{B}(\Lambda(t)/C_{00})}{C_{00}[\,\mathrm{A}(1, \Lambda(t)/C_{00})\,\mathrm{B}(\Lambda(t)/C_{00}) - \mathrm{B}(1, \Lambda(t)/C_{00})\,\mathrm{A}(\Lambda(t)/C_{00})]}\right\}^{1/4}. \tag{21}$$

[1] In Ref. [52] a solution of the field equations (4) and (5) has been obtained for the case $k = 0$, $\Lambda(t) \simeq \epsilon H^2(t)$ and the relevant cosmological parameters are also calculated. The continuity equation given in this case, is the same as Eq. (8), and not like Eq. (7), which does not include the variation of $\Lambda(t)$ and $G(t)$.

[2] In these calculations we will take Taylor's expansion, of $\Lambda(t)$, up to the first order in (t) and we will neglect the higher orders.

[3] AiryAi and AiryBi(z) are linearly independent solutions for w in the equation $w'' - zw = 0$. Specifically $\mathrm{AiryAi}(z) = C_1\,\mathrm{F}_1(2/3; z^3/9) - C_2\,\mathrm{F}_1(4/3; z^3/9)$ and $\mathrm{AiryBi}(z) = \sqrt{3}\,[C^*{}_1\,\mathrm{F}_1(2/3; z^3/9) + C^*{}_2\,\mathrm{F}_1(4/3; z^3/9)]$ where $C^*{}_1 = \mathrm{AiryAi}(0)$ and $C^*{}_2 = \mathrm{AiryAi}'(0)$ and F_1 is the generalized hypergeometric function. Also $\mathrm{AiryAi}(1, x)$ is the first derivative of $\mathrm{AiryAi}(x)$.

4. Physical properties of the models obtained

4.1. Red shift

Consider an observer O, located at the origin of space coordinate and a galaxy S at an arbitrary point with spatial coordinates (r, θ, ϕ, ψ). According to Eq. (1), the velocity of a radial ray of light ($\mathrm{d}s = 0, \mathrm{d}\theta = 0, \mathrm{d}\phi = 0, \mathrm{d}\psi = 0$) is given by

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \pm\frac{\sqrt{1-kr^2}}{R(t)}. \tag{22}$$

Hence if a light pulse is emitted by the galaxy S at the instant t_1, is received by O at the instant t_0, ($t_0 > t_1$) we obtain from Eq. (22)

$$f(r_1) = -\int_{t_1}^{t_0}\frac{\mathrm{d}t}{R(t)} = \begin{cases} \sin^{-1} r_1, & k = +1, \\ r_1, & k = 0, \\ \sinh^{-1} r_1, & k = -1. \end{cases} \tag{23}$$

The value of the red-shift Z in all obtained models can be proved to have the form

$$Z = \frac{\delta\lambda}{\lambda} = \frac{R_0}{R_1} - 1. \tag{24}$$

4.2. Particle horizons

One of the important questions in cosmological theory is to find out, according to a certain model, the answer of the following questions: How much of our universe can be observed at a given instant? To answer this question we consider, as mentioned above, a galaxy S at the point $(r, \theta, \phi,\ \psi)$ and an observer O, located at the origin space coordinates. Then consider a light signal emitted by S at the instant t_1 and received by O at the instant t_0. Light signal moves along null geodesic whose equation gives rise to

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\sqrt{1-kr^2}}{R(t)} \Rightarrow \frac{\mathrm{d}r}{\sqrt{1-kr^2}} = \frac{\mathrm{d}t}{R(t)} = f(r_1). \tag{25}$$

We have to distinguish between two different distances, the radial coordinate distance defined by

$$r \overset{\text{def.}}{=\!=} \int_{t_1}^{t_0}\frac{\mathrm{d}t}{R(t)}, \tag{26}$$

and the proper distance s defined by

$$s \overset{\text{def.}}{=\!=} R(t_0)r. \tag{27}$$

Recalling the red-shift as given by Eq. (24), one can always express the proper distance s in terms of the red-shift Z. Then if $s \to \infty$ as $Z \to \infty$, we can say that the model is free from particle horizons, i.e., isotropic observers can communicate by sending and receiving signals. But if s attains a finite value as $Z \to \infty$ we say that the model contains particle horizons.

For example we are going to explore the existence of particle horizons in one of the models discussed above. For the dust model ($\omega = 1$, $k = 0$, $\Lambda(t) = \epsilon_0 + \epsilon_1 t$), given by Eq. (17), we have

$$f(r_1) = r_1 = \int_{t_1}^{t_0}\frac{\mathrm{d}t}{R(t)} = \int_{t_1}^{t_0}\frac{\mathrm{d}t}{\sqrt{\dfrac{c_3\,\mathrm{A}(\Lambda(t)/A_{00}) - c_4\,\mathrm{B}(\Lambda(t)/A_{00})}{A_0\,[\,\mathrm{A}(1,\Lambda(t)/A_{00})\mathrm{B}(\Lambda(t)/A_{00}) - \mathrm{B}(1,\Lambda(t)/A_{00})\,\mathrm{A}(\Lambda(t)/A_{00})]}}}.$$

Taking Taylor's expansion of the above equation, up to the first order in t, we obtain

$$r_1 = \int_{t_1}^{t_0}\frac{\mathrm{d}t}{\sqrt{-\dfrac{c_3\Gamma[\frac{1}{3}]}{2\sqrt[6]{3}} + \dfrac{c_4\sqrt[3]{3}\Gamma[\frac{1}{3}]}{2}} + \dfrac{t\left(\sqrt{2}\sqrt[3]{9}c_3\Gamma[\frac{2}{3}] + 3\sqrt{2}\sqrt[6]{3}c_4\Gamma[\frac{2}{3}]\right)}{4\sqrt[6]{3}\sqrt{-\sqrt{3}c_3 + 3c_4}\sqrt{\Gamma[\frac{1}{3}]}}}},$$

which can be integrated to give,

$$r_1 = \frac{2\sqrt{-\sqrt{3}c_3 + 3c_4}\sqrt{2\Gamma[\frac{1}{3}]}}{\sqrt{\sqrt{3}c_3 + 3c_4}\Gamma[\frac{2}{3}]}\ln\left\{\frac{\left[-2\sqrt[6]{3}c_3\Gamma\left[\frac{1}{3}\right] + 2c_4\sqrt[3]{9}\Gamma\left[\frac{1}{3}\right] + t_0\left(\left[\sqrt{3}c_3 + 3c_4\right]\Gamma\left[\frac{2}{3}\right]\right)\right]}{\left[-2\sqrt[6]{3}c_3\Gamma\left[\frac{1}{3}\right] + 2c_4\sqrt[3]{9}\Gamma\left[\frac{1}{3}\right] + t_1\left(\left[\sqrt{3}c_3 + 3c_4\right]\Gamma\left[\frac{2}{3}\right]\right)\right]}\right\},$$

$$\Rightarrow r_1 = \frac{2\sqrt{-\sqrt{3}c_3 + 3c_4}\sqrt{2\Gamma[\frac{1}{3}]}}{\sqrt{\sqrt{3}c_3 + 3c_4}\Gamma[\frac{2}{3}]}\ln\{1+Z\},$$

where Γ is the gamma function. Then from Eq. (26) we have,

$$s = R_0 r_1 = \frac{\ln\left[-2\sqrt[6]{3}c_3\Gamma\left[\frac{1}{3}\right] + 2c_4\sqrt[3]{9}\Gamma\left[\frac{1}{3}\right] + t_0\left(\left[\sqrt{3}c_3 + 3c_4\right]\Gamma\left[\frac{2}{3}\right]\right)\right]}{\sqrt{\sqrt{3}c_3 + 3c_4}\Gamma[\frac{2}{3}]} \ln\{1+Z\}\,. \tag{28}$$

From Eq. (27) we see that there is no particle horizon since when $Z \to \infty$ the proper distance s tends to infinity as well.

Using the same method we can show that radiation and stiff matter models ($\omega = 4/3, k = 0, \Lambda(t) = \epsilon_0 + \epsilon_1 t$) and ($\omega = 2, k = 0, \Lambda(t) = \epsilon_0 + \epsilon_1 t$), respectively, are free from particle horizons.

5. Main results and discussion

In the present work, we consider the cosmological application in the domain of KK theory, taking the Newtonian and the cosmological constants, G and Λ, to be functions of time t. Assuming homogeneity and isotropy, the field equations of KK theory give two differential equations. We have added to them an equation of sate and two further conditions related to conservation. This is done in order to gain the five unknown functions $R(t)$, $P(t)$, $\rho(t)$, $\Lambda(t)$, and $G(t)$. One of these equations shows, in a clear way, that the relation between $\Lambda(t)$ and $G(t)$ indicates that $\Lambda(t)$ is constant whenever $G(t)$ is constant and *vice versa*. We study the flat space, i.e., $k = 0$, and assume a Taylor expansion of $\Lambda(t)$ keeping up to the first order of t, i.e., $O(t)$. We can see from Eqs. (12), (17), (19), and (21) that the values of $\Lambda(t)$ and the scale factor $R(t)$ are finite while, when $t \to 0$ the values of $\Lambda(t)$ and the scale factor $R(t)$ are unbounded when $t \to \infty$.

A summary of the results obtained are compared below in the following table.

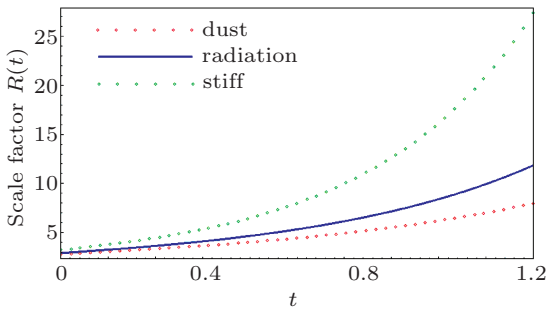


Fig. 1. The scale factor $R(t)$ of different cases for $\epsilon_0 = 1$, $\epsilon_1 = 1$, $c_3 = 1$, $c_4 = 1$, $c_5 = 1$, $c_6 = 1$, $c_7 = 1$, and $c_8 = 1$.

Different figures (Figs. 1–5) related to this summary are given. From these figures we can conclude as follows.

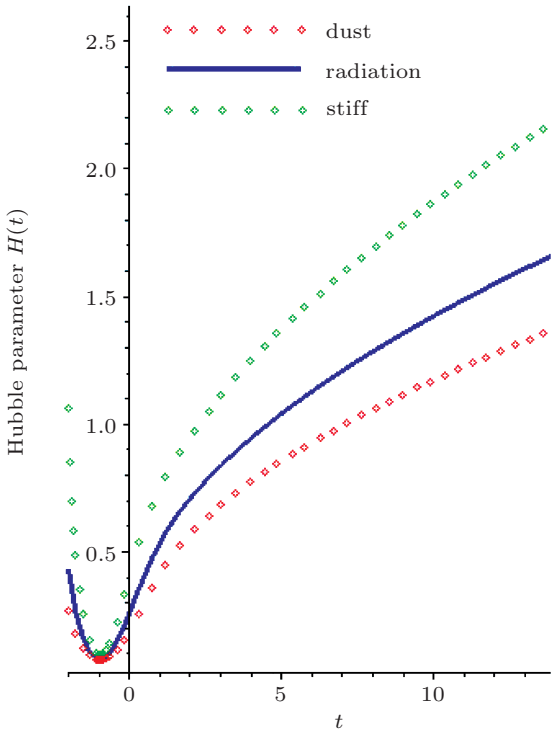


Fig. 2. The Hubble parameter $H(t)$ of different cases for $\epsilon_0 = 1$, $\epsilon_1 = 1$, $c_3 = 1$, $c_4 = 1$, $c_5 = 1$, $c_6 = 1$, $c_7 = 1$, $c_8 = 1$.

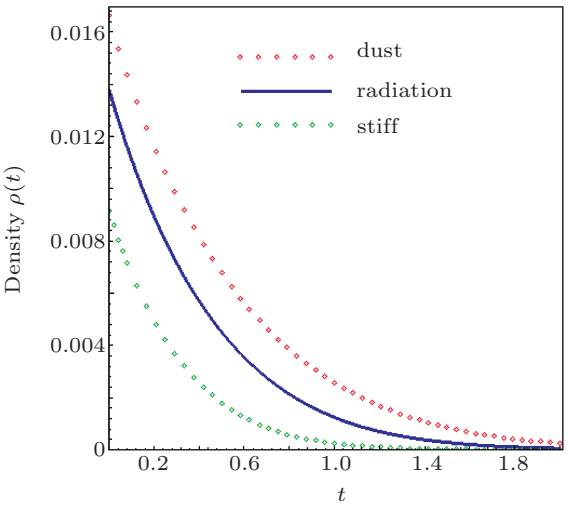


Fig. 3. The density $\rho(t)$ of different cases for $\epsilon_0 = 1$, $\epsilon_1 = 1$, $c_3 = 1$, $c_4 = 1$, $c_5 = 1$, $c_6 = 1$, $c_7 = 1$, $c_8 = 1$.

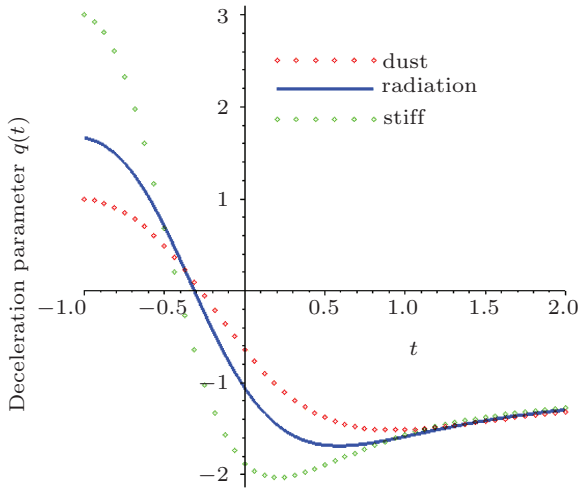


Fig. 4. The deceleration parameter $q(t)$ of different cases for $\epsilon_0 = 1$, $\epsilon_1 = 1$, $c_3 = 1$, $c_4 = 1$, $c_5 = 1$, $c_6 = 1$, $c_7 = 1$, $c_8 = 1$.

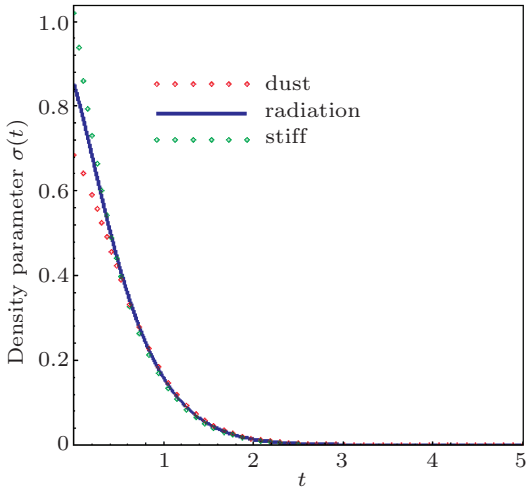


Fig. 5. The density parameter $\sigma(t)$ of different cases for $\epsilon_0 = 1$, $\epsilon_1 = 1$, $c_3 = 1$, $c_4 = 1$, $c_5 = 1$, $c_6 = 1$, $c_7 = 1$, $c_8 = 1$.

(I) The models resulting from the three studied cases, i.e., dust, radiation, and stiff matter have no singularity unless when $A_0 = (\epsilon_1/12)^{1/3}$, $B_0 = ({\epsilon_1}^3/16)^{1/3}$, and ${C^*}_0 = (\epsilon_1/24)^{1/3}$ vanish, which contradicts our assumption that $\Lambda(t) = \epsilon_0 + \epsilon_1 t$ is a function of t. In other words the models are singular when $\Lambda(t)$ is constant.

(II) The resulting models are accelerating which is consistent with most recent observation.[1−3]

(III) Figure 3 shows that density in the three models are decreasing with time which ensures that our universe is expanding.

(IV) The three models are free from particle horizons, which is consistent with the previous result.[53]

(V) We have considered the cosmological term to contribute to the material-energy contents of the universe.

(VI) Equation (7) satisfies the conservation law when $G(t)$ and $\Lambda(t)$ are functions of time in contradiction with the result obtained before (Eq. (8)).[52]

Table 1. Comparison between the results of scale factors $R(t)$, Hubble parameters $H(t)$, density $\rho(t)$, and deceleration parameters $q(t)$ for $\Lambda(t) = \epsilon_0 + \epsilon_1 t$.

Equation of state	$\omega = 1$	$\omega = 4/3$	$\omega = 2$
Scale factor	$\sqrt{\frac{c_3\mathrm{A}(f(t))-c_4\mathrm{B}(f(t))}{A_0[\mathrm{A}(f'(t))\mathrm{B}(f(t))-\mathrm{B}(f'(t))\mathrm{A}(f(t))]}}$	$\left\{\frac{c_5\mathrm{A}(f_1(t))-c_6\mathrm{B}(f_1(t))}{B_0[\mathrm{A}(f_1'(t))\mathrm{B}(f_1(t))-\mathrm{B}(f_1'(t))\mathrm{A}(f_1(t))]}\right\}^{3/8}$	$\left\{\frac{c_7\mathrm{A}(f_2(t))-c_8\mathrm{B}(f_2(t))}{C_0[\mathrm{A}(f_2'(t))\mathrm{B}(f_2(t))-\mathrm{B}(f_2'(t))\mathrm{A}(f_2(t))]}\right\}^{1/4}$
Hubble parameter	$A_0\,\frac{c_3\mathrm{A}(f'(t))-c_4\mathrm{B}(f'(t))}{c_3\mathrm{A}(f(t))-c_4\mathrm{B}(f(t))}$	$B_0\,\frac{c_5\mathrm{A}(f_1'(t))c_6\mathrm{B}(f_1'(t))}{c_5\mathrm{A}(f_1(t))-c_6\mathrm{B}(f_1(t))}$	$C_0\,\frac{c_7\mathrm{A}(f_2'(t))-c_8\mathrm{B}(f_2'(t))}{c_5\mathrm{A}(f_2(t))+\mathrm{B}B(f_2(t))}$
Density	$\frac{c_1(A_0[\mathrm{A}(f'(t))\mathrm{B}(f(t))-\mathrm{B}(f'(t))\mathrm{A}(f(t))])^2}{(c_3\mathrm{A}(f(t))-c_4\mathrm{B}(f(t)))^2}$	$c_1\left\{\frac{B_0[\mathrm{A}(f_1'(t))\mathrm{B}(f_1(t))-\mathrm{B}B(f_1'(t))\mathrm{A}(f_1(t))]}{c_5\mathrm{A}(f_1(t))-c_6\mathrm{B}(f_1(t))}\right\}^{3/2}$	$\frac{c_1C_0[\mathrm{A}(f_2'(t))\mathrm{B}(f_2(t))-\mathrm{B}(f_2'(t))\mathrm{A}(f_2(t))]}{c_7\mathrm{A}(f_2(t))-c_8\mathrm{B}(f_2(t))}$
Deceleration parameter	$\frac{1}{3\sqrt[3]{{\epsilon_1}^2}[c_3\mathrm{A}(f'(t))-c_4\mathrm{B}(f'(t))]^2}$ $\left\{3\sqrt[3]{{\epsilon_1}^2}[c_3\mathrm{A}(f'(t))-c_4\mathrm{B}(f'(t))]^2\right.$ $\left.-\sqrt[3]{144}\Lambda(t)[c_3\mathrm{A}(f(t))-c_4\mathrm{B}(f(t))]^2\right\}$	$\frac{1}{18\sqrt[3]{4{\epsilon_1}^2}[c_5\mathrm{A}(f_1'(t))-c_6\mathrm{B}(f_1'(t))]^2}$ $\left\{15\sqrt[3]{2{\epsilon_1}^2}[c_5\mathrm{A}(f_1'(t))-c_6\mathrm{B}(f_1'(t))]^2\right.$ $\left.-32\Lambda(t)[c_5\mathrm{A}(f_1(t))-c_6\mathrm{B}(f_1(t))]^2\right\}$	$\frac{1}{3\sqrt[3]{{\epsilon_1}^2}[c_7\mathrm{A}(f_2'(t))-c_8\mathrm{B}(f_2'(t))]^2}$ $\left\{3\sqrt[3]{3{\epsilon_1}^2}[c_7\mathrm{A}(f_2'(t))-c_8\mathrm{B}(f_2'(t))]^2\right.$ $\left.-8\Lambda(t)[c_7\mathrm{A}(f_2(t))-c_8\mathrm{B}(f_2(t))]^2\right\}$
Particle horizons	$\frac{\ln\{1+Z\}}{\sqrt{\sqrt{3}c_3+3c_4}\Gamma[\frac{2}{3}]}$ $\left\{\ln[-2\sqrt[6]{3}c_3\Gamma[\frac{1}{3}]+2c_4\sqrt[3]{9}\Gamma[\frac{1}{3}]\right.$ $\left.+t_0([\sqrt{3}c_3+3c_4]\Gamma[\frac{2}{3}])]\right\}$	$\frac{\ln\{1+Z\}}{\sqrt{\sqrt{3}c_5+3c_6}\Gamma[\frac{2}{3}]}$ $\left\{\ln[-2\sqrt[6]{3}c_5\Gamma[\frac{1}{3}]+2c_6\sqrt[3]{9}\Gamma[\frac{1}{3}]\right.$ $\left.+t_0([\sqrt{3}c_5+3c_6]\Gamma[\frac{2}{3}])]\right\}$	$\frac{\ln\{1+Z\}}{\sqrt{\sqrt{3}c_7+3c_8}\Gamma[\frac{2}{3}]}$ $\left\{\ln[-2\sqrt[6]{3}c_7\Gamma[\frac{1}{3}]+2c_8\sqrt[3]{9}\Gamma[\frac{1}{3}]\right.$ $\left.+t_0([\sqrt{3}c_7+3c_8]\Gamma[\frac{2}{3}])]\right\}$

$f(t) = \Lambda(t)/A_{00}$, $A_{00} = (3{\epsilon_1}^2/2)^{1/3}$, $f'(t) = (1, \Lambda(t)/A_{00}$, $f_1(t) = \Lambda(t)/B_{00}$, $B_{00} = (27{\epsilon_1}^2/32)^{1/3}$, $f_1'(t) = (1, \Lambda(t)/B_{00})$, $f_2(t) = \Lambda(t)/C_{00}$, $C_{00} = (3{\epsilon_1}^2/8)^{1/3}$, and $f_2'(t) = (1, \Lambda(t)/C_{00})$.

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