# Competition between two kinds of information among random-walking individuals*

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A model is proposed to describe the competition between two kinds of information among $N$ random-walking individuals in an $L \times L$ square, starting from a half-and-half mixture of two kinds of information. Individuals remain or change their information according to their neighbors' information. When the moving speed of individuals $v$ is zero, the two kinds of information typically coexist, and the ratio between them increases with $L$ and decreases with $N$. In the dynamic case ($v > 0$), only one information eventually remains, and the time required for one information being left scales as $T_{\rm d} \sim v^{\alpha} L^{\beta} N^{\gamma}$.

**Keywords:** information spread, random walk, complex networks, power law

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## 1. Introduction

A great deal of studies have shown that the underlying issue of many complex phenomena is the spread of information.[1−11] Therefore, information spread is attracting more and more attention. Many researchers have discussed the spread of one information over the system.[8−19] In these studies, systems are usually modeled by complex networks, in which individuals are represented by nodes, and interactions between individuals through which information is transferred are represented by links between nodes.

Two cases are usually considered in modeling the spread of information, that is, the static case and the dynamic case. In the static case, connections between individuals and the topology of the network remain unchanged although information diffuses in the system.[8−15] In the dynamic case, it is usually supposed that individuals random walk in a square, which means that the topology of the network keeps changing with the information spread.[16−19]

Since usually more than one information coexists and spreads among the population, Trpevski *et al.*[20] recently reported their investigation on the competition between two kinds of information in one system. They proposed a spread model, in which the static network and two kinds of unequal information are contained. They analyzed the behavior of the model for different network topologies, and found that the prior information dominates the whole system in most cases and the other information holds only in certain network topologies with the low node degree and the low clustering coefficient.[20] In this study, we will discuss the spread and competition of two kinds of information with the same priority among random-walking individuals.

Firstly, we propose the information spread model, in which $N$ individuals random walk with the same absolute velocity $v$ ($v \geq 0$) in an $L \times L$ square. And one individual only carries one information at a time. If two individuals get close enough to each other, then they are neighbors and can send information from one to another. The moving of individuals and the spreading of two kinds of information are simultaneous. Initially, half of individuals carries one information and another half of them carries the other. At each time step and for each individual, it would switch to dominant information which is carried by more than half of its neighbors, and walk one step. If numbers of its neighbors carrying two kinds of information are the same, then its information would not change.

The spread of information depends on many parameters, and in particular on the moving speed of individuals $v$, the square size $L$, and the total number of individuals $N$. In this paper, we mainly analyze the effect of these three parameters on information spread.

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In the static case ($v = 0$), information spreads between still individuals until the system reaches the stable state, in which two kinds of information typically co-exist. Such a stable state is called the co-existence state, and the ratio between the smaller and larger ones of numbers of individuals carrying two kinds of information is denoted as $r$ ($r \in [0, 1]$). Simulation results show that $r$ increases with $L$ when $N$ is fixed, and decreases with $N$ when $L$ is fixed.

We also introduce a lattice model to analyze the coexistence of two kinds of information. It is found that when individuals are randomly scattered, they naturally form unconnected regions. For regions only containing one information, their information would not change during the system evolution. If a region contains two kinds of information, then whether both of information should remain or not depends on whether stable boundaries between information clusters can exist or not.

In the dynamic case ($v > 0$), positions, neighbors, and information of individuals are simultaneously updated at each time step. In this case, stable boundaries between information clusters cannot maintain. As a result, only one information would remain when the system reaches the stable state. Thus, we mainly analyze the time that is required for the system to enter the stable state. In this work, this time is denoted as $T_{\rm d}$, and the stable state only containing one information is called the consistency state. Numerous simulations are performed to analyze the effect of $v$, $L$, and $N$ on $T_{\rm d}$. Results show that $T_{\rm d}$ depends on $v$, $N$, and $L$ as a power-law, and the exponent corresponding to each parameter changes very little. Denoting exponents of three power-law relations as $\alpha$, $\beta$, and $\gamma$, we have $T_{\rm d} \sim v^{\alpha} L^{\beta} N^{\gamma}$.

## 2. Model

We consider a population of $N$ individuals moving in a square of linear size $L$ with periodic boundary conditions. Let $\boldsymbol{x}_i(t)$ and $\theta_i(t)$ be the position and motion direction of the individual $i$ at time $t$. For each time step $\Delta t = 1$, $\boldsymbol{x}_i(t)$ and $\theta_i(t)$ are updated according to

$$\begin{cases} \boldsymbol{x}_i(t + \Delta t) = \boldsymbol{x}_i(t) + \boldsymbol{v}_i(t)\Delta t, \\ \theta_i(t + \Delta t) = \xi_i(t + \Delta t), \end{cases} \tag{1}$$

where $\boldsymbol{v}_i(t) = (v \cos\theta_i(t), v \sin\theta_i(t))$ is the velocity of the individual $i$ at time $t$, and the absolute velocity $v$ ($v \geq 0$) is the same for all individuals. $\xi_i(t)$ follows the uniform distribution in $[-\pi, \pi]$. For convenience of discussion, we name this model as the square model.

At time $t = 0$, these individuals are randomly distributed in the square with the same probability. For individuals $i$ and $j$, if $|\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t)| \leq r_0, i \neq j$, where $r_0$ is the interaction radius, then they are neighbors. In this study, $r_0$ is set to 1. There are two kinds of information ($A$ and $B$) in the square, and initially each individual carries either information $A$ or information $B$ with the same probability 0.5. For the sake of convenience, individuals carrying information $A$ ($B$) are called $A$ ($B$) individuals. $A$ ($B$) individuals which are connected together constitute $A$ ($B$) clusters.

The spread of information and the movement of individuals are simultaneous. For the individual $i$, numbers of $A$ individuals and $B$ individuals in its neighborhood are denoted as $N_{A_i}$ and $N_{B_i}$, respectively. Information spreads in the following way: if $N_{A_i} > N_{B_i}$ at time $t$, the individual $i$ carries information $A$ at time $t + 1$; if $N_{A_i} < N_{B_i}$ at time $t$, the individual $i$ carries information $B$ at time $t + 1$; if $N_{A_i} = N_{B_i}$ at time $t$, the individual $i$ keeps its information unchanged. For each of individuals, it would not lose its information, unless it switches to dominant information in its neighborhood.

## 3. Results

If one information vanishes in the spread of information, and all the individuals carry the other information, then the system achieves the consistency state, else the co-existence state. Whether the system achieves the consistency state or the co-existence state is mainly determined by the moving speed of individuals $v$. When $v = 0$, the system typically achieves the co-existence state; and when $v$ is significantly larger than zero, the system usually achieves the consistency state. In the following, we discuss the static case and dynamic case, respectively.

### 3.1. The static case

To quantify the relative number of two kinds of information, we denote the ratio between the smaller one and larger one of $A$ and $B$ individual numbers as $r$. Here we mainly discuss the effect of the total number of individuals $N$ and the square size $L$ on $r$. As shown in Fig. 1, for each $N$, when $L$ is very small, $r$ is about 0, and with the increase of $L$, $r$ increases sharply. When $L$ is larger than some threshold, $r$ slowly increases to

1.

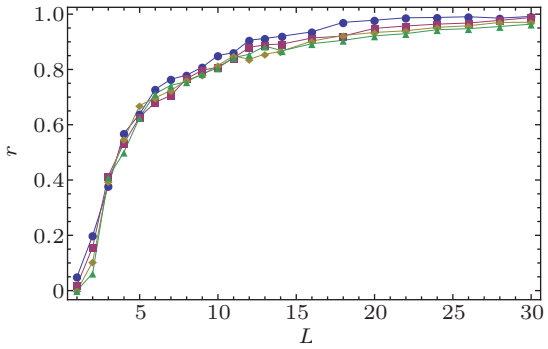



**Fig. 1.** (colour online) The ratio between the smaller one and larger one of $A$ and $B$ individual numbers $r$ as a function of the square size $L$ with $N = 30$ (circle); $N = 50$ (square); $N = 80$ (diamond); $N = 100$ (triangle). Results are averaged over 200 runs.

To analyze the increase of $r$, we introduce a simplified lattice model. In the lattice model, there are $N$ static individuals lying in the $L \times L$ lattice with periodic boundary conditions. Here more than one individual is allowed to lie in one lattice site. Initially each individual is randomly scattered in the lattice and carries information $A$ or $B$ with the same probability. For any given individual, individuals lying in its site and its four nearest-neighbor sites are its neighbors. Information spreads between neighbors in the same way as the square model mentioned in Section 2.

In Fig. 2, we show the information spread in the lattice in the cases that $N = 40$, $L = 9$, and $N = 24$, $L = 6$, respectively. Systems reach the co-existence state with $r = 19/21$ and $r = 1/2$, respectively. As shown in Fig. 2, there are many unconnected information regions in the lattice, whose shapes are determined by positions of individuals and keep unchanged. If a region initially only carries one information, then information of the region keeps unchanged.

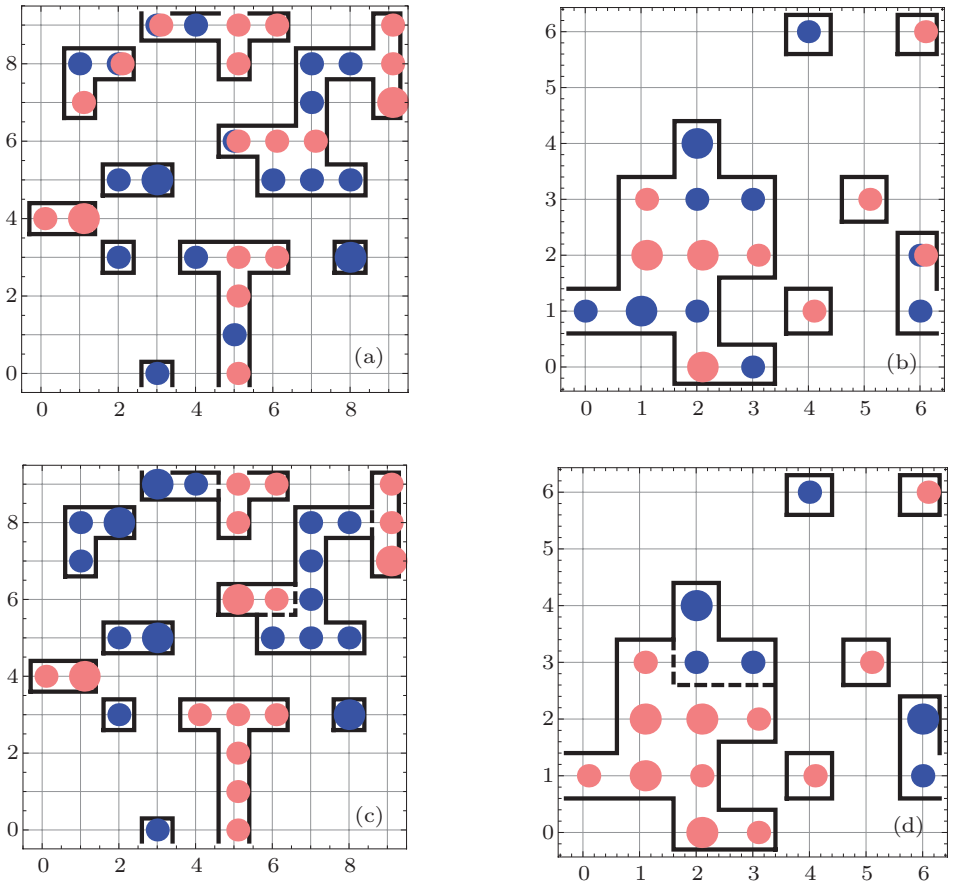



**Fig. 2.** (colour online) Two examples of the lattice model. The initial state for $N = 40$ and $L = 9$ (a), the initial state for $N = 24$ and $L = 6$ (b), the co-existence state for $N = 40$ and $L = 9$ (c), and the co-existence state for $N = 24$ and $L = 6$. At $t = 0$, all individuals are randomly scattered in the lattice when $N = 40$ and $L = 9$ (a), and $N = 24$ and $L = 6$ (b), respectively. After a transient process, the system reaches the stable state for $N = 40$ and $L = 9$, (c) and $N = 24$ and $L = 6$ (d), respectively. The large dot means that several individuals lie in this site, and the more individuals, the larger the dot. Thick lines represent information regions. In panels (c) and (d), dashed lines separate information clusters in one region.

If a region initially has two kinds of information, then the region contains many small cojoint information clusters. Boundaries between clusters are constituted by individuals which have neighbor(s) carrying the other information, that is, $A$ ($B$) individuals that have $B$ ($A$) neighbor(s). With the evolution of the system, shapes and numbers of information clusters would change, until all boundaries reach a stable state. For each $A$ ($B$) individual on stable boundaries, its $A$ ($B$) neighbors are not less than $B$ ($A$) neighbors.

When $L$ is very small, all individuals are connected to a region, and $A$ individuals and $B$ individuals are mixed in this region. Thus, $A$ and $B$ clusters overlap each other, and nearly all individuals are on boundaries. In this case, stable boundaries can hardly form, and individuals would keep changing their information. The random fluctuation increases one information and decreases the other. This eventually causes the system to fall to the stable consistency state, in which only one information exists.

When $L$ is very large, the probability that two individuals are connected is very small and individuals on boundaries are rare. Thus, most individuals would not switch their information during the system evolution, and $r$ remains near the initial value 1. When $L$ is medium, the number of boundary individuals which would be affected by the system evolution is also medium. After the transient process, $r$ would be between 0 and 1.

We can see from Fig. 3 that $r$ decreases from 1 to 0 with increasing $N$. The analysis of the change of $r$ with $N$ is similar to previous discussions. In short, $r$ is determined by the closeness between individuals. The increase of $N$ for fixed $L$ is similar to the decrease of $L$ for fixed $N$, for the closeness between individuals gets tighter in both cases.

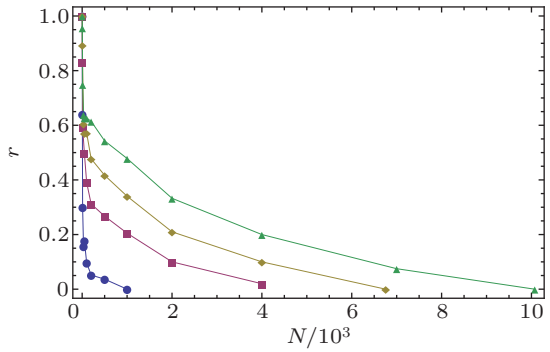



**Fig. 3.** (colour online) The ratio between the smaller and larger of $A$ and $B$ individual numbers $r$ as a function of the individual number $N$ when $L = 2$ (circle); $L = 3$ (square); $L = 4$ (diamond); $L = 5$ (triangle). Results are averaged over 50 runs.

## 3.2. The dynamic case

In the dynamic case, individuals randomly move in the square, and therefore stable boundaries between information clusters cannot form. In this case, the random fluctuation leads to the decrease of one information and the increase of the other information. It is difficult for the significantly lesser information to recover to the original proportion. After the transient process, only one information remains, and the system reaches the consistency state. The time that has passed before the system reaches the consistency state is called the information-consistency time and is denoted as $T_{\rm d}$. In the following, we analyze the effect of the moving speed of individuals $v$, the square size $L$, and the total number of individuals $N$ on $T_{\rm d}$.

Firstly, we measure $T_{\rm d}$ corresponding to each $v$ for various combinations of $N$ and $L$. As shown in Fig. 4, when $v$ is less than the critical value, $T_{\rm d}$ decays as a power law with an exponent $\alpha$

$$T_{\rm d} \sim v^{\alpha}. \tag{2}$$

The mean and variance of $\alpha$ are $-1.59$ and 0.13, respectively. This means the fluctuation of $\alpha$ is very small.

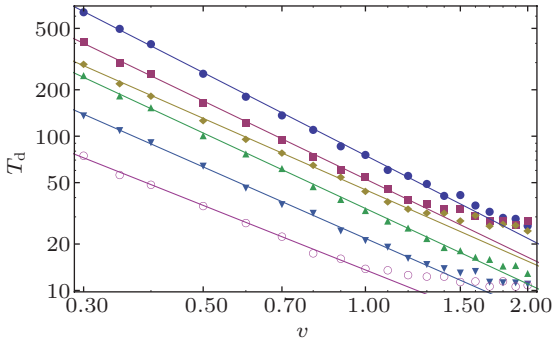



**Fig. 4.** (colour online) The log–log plot of the information consistency time $T_{\rm d}$ versus the moving speed of individuals $v$ with $N = 100$ and $L = 14$ (circle); $N = 50$ and $L = 11$ (square); $N = 40$ and $L = 10$ (diamond); $N = 100$ and $L = 10$ (triangle); $N = 70$ and $L = 8$ (inverted triangle); $N = 30$ and $L = 6$ (ring). Different combinations of $N$ and $L$ correspond to different critical values of $v$. Results are averaged over 200 runs.

Here we analyze the reason for the decrease of $T_{\rm d}$ with $v$. Since each individual moves with the constant speed and random direction, the mean of the displacement of each individual is zero when the evolution time tends to infinity. Moreover, the smaller the moving speed of individual, the smaller the displacement variance. That is, when the individual moving speed is small, individuals tend to walk around their initial

positions. As a result, the variation of shapes of information clusters boundaries is slow and $T_{\rm d}$ is large. When the speed is large, moving ranges of individuals are also large, which enhances the variation of shapes of information clusters boundaries. As a result, $T_{\rm d}$ is small.

Figure 5 shows the dependence of $T_{\rm d}$ on $L$ with fixed $v$ and $N$. Data points in this figure are well fitted by power laws when $L$ is larger than the critical value:

$$T_{\rm d} \sim L^{\beta}. \tag{3}$$

The mean and variance of $\beta$ are 2.44 and 0.29, respectively. This shows the fluctuation of $\beta$ is also small. The reason why $T_{\rm d}$ increases with $L$ is that the larger $L$ leads to the looser connections between individuals. As a result, the variation of shapes of information clusters boundaries is slow, and $T_{\rm d}$ is large.

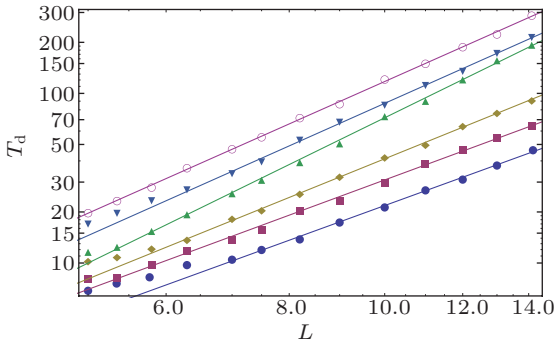



**Fig. 5.** (colour online) The log–log plot of the information consistency time $T_{\rm d}$ versus the square size $L$ with $N = 100$ and $v = 1.3$ (circle); $N = 70$ and $v = 1.1$ (square); $N = 70$ and $v = 0.9$ (diamond); $N = 30$ and $v = 0.8$ (triangle); $N = 50$ and $v = 0.6$ (inverted triangle); $N = 50$ and $v = 0.5$ (ring). Different combinations of $N$ and $v$ correspond to different critical values of $L$. Results are averaged over 200 runs.

The log–log plot of $T_{\rm d}$ versus $N$ for several combinations of $v$ and $L$ is shown in Fig. 6. It can be seen that $T_{\rm d}$ decays as a power law with exponent $\gamma$ in the case that $N$ is larger than the critical value:

$$T_{\rm d} \sim N^{\gamma}. \tag{4}$$

The mean and variance of $\gamma$ are $-0.64$ and 0.13, respectively. It can be seen that the fluctuation of $\gamma$ is also small. The reason why $T_{\rm d}$ decreases with $N$ is that when $v$ and $L$ are fixed, the increase of $N$ tightens the closeness between individuals. Thus, shapes of information clusters boundaries significantly change at each time step, and $T_{\rm d}$ is small.

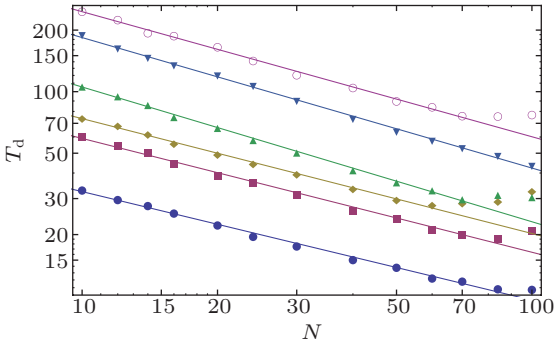



**Fig. 6.** (colour online) The log–log plot of the information consistency time $T_{\rm d}$ versus the individual number $N$ when $L = 7$ and $v = 1.3$ (circle); $L = 8$ and $v = 1$ (square); $L = 8$ and $v = 0.8$ (diamond); $L = 9$ and $v = 0.9$ (triangle); $L = 10$ and $v = 0.8$ (inverted triangle); $L = 11$ and $v = 0.7$ (ring). Different combinations of $L$ and $v$ correspond to different critical values of $N$. Results are averaged over 200 runs.

Exponents $\alpha$, $\beta$, and $\gamma$ change little with varying $v$, $L$, and $N$, respectively. Thus, we have:

$$T_{\rm d} \sim v^{\alpha} L^{\beta} N^{\gamma}. \tag{5}$$

## 4. Conclusion

In this paper, we studied the competition between two kinds of information spreading among a population of individuals. When individuals keep still ($v$=0), two kinds of information typically coexist in the stable state of the system. Simulation results show that the ratio between two kinds of information $r$ increases with $L$ for fixed $N$, and decreases with $N$ for fixed $L$. In the dynamic case ($v > 0$), only one information is left when the system reaches the stable state. Denoting the time required for the system to enter the stable state as $T_{\rm d}$, we found that $T_{\rm d} \sim v^{\alpha} L^{\beta} N^{\gamma}$.

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