Linear matrix inequality approach for synchronization control of fuzzy cellular neural networks with mixed time delays*

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Fuzzy cellular neural networks (FCNNs) are special kinds of cellular neural networks (CNNs). Each cell in an FCNN contains fuzzy operating abilities. The entire network is governed by cellular computing laws. The design of FCNNs is based on fuzzy local rules. In this paper, a linear matrix inequality (LMI) approach for synchronization control of FCNNs with mixed delays is investigated. Mixed delays include discrete time-varying delays and unbounded distributed delays. A dynamic control scheme is proposed to achieve the synchronization between a drive network and a response network. By constructing the Lyapunov–Krasovskii functional which contains a triple-integral term and the free-weighting matrices method an improved delay-dependent stability criterion is derived in terms of LMIs. The controller can be easily obtained by solving the derived LMIs. A numerical example and its simulations are presented to illustrate the effectiveness of the proposed method.

Keywords: asymptotic stability, chaos, fuzzy cellular neural networks, linear matrix inequalities, synchronization

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1. Introduction

Cellular neural networks (CNNs) are locally connected nonlinear networks. They originally stemmed from cellular automata and artificial neural networks. Local connectedness is the most significant property of CNN. The local connectedness restricts the ability of a CNN to solve many global problems that cannot be decomposed into local components. However, the local property has its advantages, such as easy implementation using VLSI technology and efficiency for solving local problems, as proposed by Chua and Yang in Refs. [1] and [2]. Fuzzy set theory provides mathematical support to the capture of uncertainties associated with human cognitive processes. A fuzzy cellular neural network (FCNN) is a generalized case of the CNN structure proposed by Yang in Refs. [3] and [4].

In the past decade, synchronization of chaotic systems has been extensively studied due to its potential applications in many different areas including secure communication, chemical and biological systems, information science, optics, and so on. Carroll and Pecora[5] have introduced the drive-response concept, and used the output of the drive system to control the response system so that state synchronization was achieved.

The study of the dynamical properties of FCNNs has mainly concentrated on stability analysis.[6,7] Time delays in the FCNNs make the dynamic behaviors become more complicated, and may destabilize the stable equilibria and admit periodic oscillation, bifurcation, and chaos. Therefore, considerable attention has been paid to the study of delay systems in control theory.[8] In fact, as special complex networks, delayed neural networks (DNNs) have been found to exhibit complicated dynamics and even chaotic behavior if the parameters and time delays are appropriately chosen.[9−18] However, CNNs and FCNNs have also been found to exhibit complicated dynamics and even chaotic behavior if the parameters and time delays are appropriately chosen.[19−24] For instance, Xia *et al.*[22] investigated the synchronization schemes for coupled identical Yang–Yang type FCNNs.

To the best of the authors' knowledge, there are

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no results on the linear matrix inequality (LMI) approach for synchronization control of FCNNs with discrete and unbounded distributed delays. Based on the Lyapunov–Krasovskii functional, which contains a triple-integral term and free-weighting matrices method, an improved delay-dependent stability criterion is derived in terms of linear matrix inequalities (LMIs). For the synchronization conditions given in Refs. [21] and [24], the time delays are constant delays or time-varying delays that are differentiable such that their derivatives are not greater than one or finite. A new result is presented by relaxing these restrictive conditions on time-varying delays, which means that our presented results have a wider adaptive range. Finally, a numerical example and numerical simulations are given to show the effectiveness of our result.

This paper is organized as follows. In Section 2, we introduce some model formulation and preliminaries. In Section 3, the main results for asymptotical synchronization of chaotic FCNNs are presented. In Section 4, a numerical example is given in order to demonstrate the effectiveness of our results. Finally, in Section 5, some conclusions are summarized.

Notations $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. For any matrix $A = [a_{ij}]_{n\times n}$, let A^{T} and A^{-1} denote the transpose and the inverse of A, respectively. $|A| = [|a_{ij}|]_{n\times n}$. Let $A > 0$ ($A < 0$) denote the positive-definite (negative-definite) symmetric matrix, respectively. I denotes the identity matrix of appropriate dimension and $\Lambda = \{1, 2, \ldots, n\}$. Symbol $*$ denotes the symmetric terms in a symmetric matrix.

2. Model formulation and preliminaries

Most of the synchronization methods belong to the master-slave (drive-response) type, which means that the two systems are coupled so that the behavior of the second one is influenced by the behavior of the first one, but the behavior of the first one is independent of the second one. The first system will be called the master system or drive system, and the second system will be the slave system or response system.

The main objective of this paper is to design a controller to let the slave system synchronize with the master system. Now let us consider the following chaotic FCNN with discrete and unbounded distributed delays

$$\begin{cases} \dot{x}_i(t) = -d_i x_i(t) + \sum\limits_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum\limits_{j=1}^{n} b_{ij} f_j(x_j(t-\tau(t))) + I_i \\ \qquad\quad + \bigwedge\limits_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))\,\mathrm{d}s + \bigvee\limits_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))\,\mathrm{d}s, \quad i \in \Lambda, \\ x_i(s) = \phi_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{1}$$

where $\phi_i(\cdot) \in C((-\infty, 0], \mathbb{R})$; α_{ij} and β_{ij} are the elements of fuzzy feedback MIN template, fuzzy feedback MAX template, respectively; a_{ij} and b_{ij} are the elements of feedback template; $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations, respectively; x_i and I_i denote the state and external input of the i-th neuron, respectively; d_i is a diagonal matrix, d_i represents the rates with which the i-th neuron will reset their potential to the resting state in isolation when disconnected from the networks and external inputs; f_j represents the neuron activation function; $k_j(\cdot)$ is the delay kernel function.

In this section, the following assumptions are needed:

A1 The neuron activation functions $f_j(\cdot)$ are continuously bounded and satisfy

$$l_j^- \le \frac{f_j(u) - f_j(v)}{u - v} \le l_j^+,$$

for any $u, v \in \mathbb{R}, u \ne v, j \in \Lambda$,

where l_j^- and l_j^+ are some real constants and they may be positive, zero or negative.

A2 The transmission delay $\tau(t)$ is a time varying delay, and it satisfies $0 \le \tau(t) \le \tau$, where τ is a positive constant.

A3 The delay kernels k_j are some real-valued non-negative continuous functions defined in $[0, \infty)$ satisfying

$$\tilde{k}_j := \int_0^\infty k_j(s)\,\mathrm{d}s > 0 \ \text{ and}$$
$$\hat{k}_j := \int_0^\infty k_j(s)\,\mathrm{e}^{\eta s}\,\mathrm{d}s < \infty, \ j \in \Lambda,$$

for some $\eta > 0$.

Moreover, considering that a chaotic FCNN system depends extremely on initial values, the initial condition of the response system is defined to be different from that of the drive system. Therefore, a response system with the same form and parameters of drive system (1) is introduced as follows:

$$\begin{cases} \dot{y}_i(t) = -d_i y_i(t) + \sum\limits_{j=1}^{n} a_{ij} f_j(y_j(t)) + \sum\limits_{j=1}^{n} b_{ij} f_j(y_j(t-\tau(t))) + I_i + \bigwedge\limits_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(y_j(s)) \mathrm{d}s \\ \qquad + \bigvee\limits_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(y_j(s)) \mathrm{d}s + u_i(t), \quad i \in \Lambda, \\ y_i(s) = \varphi_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{2}$$

where $\varphi_i(\cdot) \in C((-\infty, 0], \mathbb{R})$; d_i, a_{ij}, b_{ij}, α_{ij}, β_{ij}, I_i, and f_j are the same as system (1) and $u_i(t)$ is the appropriate control input that will be designed in order to obtain a certain control objective.

Let $e(t) = \big(e_1(t), e_2(t), \ldots, e_n(t)\big) := x(t) - y(t)$ be the error state. Then, the error dynamical system between system (1) and system (2) is given by

$$\begin{cases} \dot{e}_i(t) = -d_i e_i(t) + \sum\limits_{j=1}^{n} a_{ij} g_j(e_j(t)) + \sum\limits_{j=1}^{n} b_{ij} g_j(e_j(t-\tau(t))) \\ \qquad + \bigwedge\limits_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s)) \mathrm{d}s - \bigwedge\limits_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(y_j(s)) \mathrm{d}s \\ \qquad + \bigvee\limits_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s)) \mathrm{d}s - \bigvee\limits_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(y_j(s)) \mathrm{d}s - u_i(t), \quad i \in \Lambda, \\ e_i(s) = \phi_i(s) - \varphi_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{3}$$

where $g_j(e_j(\cdot)) = f_j(x_j(\cdot)) - f_j(y_j(\cdot))$, $j \in \Lambda$.

The appropriate control input is

$$u_i(t) := h_{ij} e_i(t), \quad i,\ j \in \Lambda. \tag{4}$$

Substituting Eq. (4) into system (3), it is easy to have

$$\begin{cases} \dot{e}_i(t) = (-d_i - h_{ij}) e_i(t) + \sum\limits_{j=1}^{n} a_{ij} g_j(e_j(t)) + \sum\limits_{j=1}^{n} b_{ij} g_j(e_j(t-\tau(t))) \\ \qquad + \bigwedge\limits_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s)) \mathrm{d}s - \bigwedge\limits_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(y_j(s)) \mathrm{d}s \\ \qquad + \bigvee\limits_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s)) \mathrm{d}s - \bigvee\limits_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(y_j(s)) \mathrm{d}s, \quad i \in \Lambda, \\ e_i(s) = \phi_i(s) - \varphi_i(s), \quad s \in (-\infty, 0]. \end{cases} \tag{5}$$

Remark 1 Note that the assumptions A1 and A2 on activation function and time-varying delay in this paper are weaker than those generally used in the literature,[21,24] namely, the boundedness of the activation function f_j. Further, the differentiability of the time varying delay $\tau(t)$ is not required in this paper.

Now, we state the following a few lemmas which will be used in the following discussion.

Lemma 1 (Schur complement[25]) The linear matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^{\mathrm{T}}(x) & R(x) \end{bmatrix} > 0,$$

where $Q(x) = Q^{\mathrm{T}}(x)$, $R(x) = R^{\mathrm{T}}(x)$ is equivalent to

$$R(x) > 0 \quad \text{and} \quad Q(x) - S(x) R^{-1}(x) S^{\mathrm{T}}(x) > 0.$$

Lemma 2 For any $x, y \in \mathbb{R}^n$, $\epsilon > 0$, and positive-definite matrix $Q \in \mathbb{R}^{n \times n}$, the following matrix inequality holds:

$$2x^{\mathrm{T}} y \le \epsilon x^{\mathrm{T}} Q x + \epsilon^{-1} y^{\mathrm{T}} Q^{-1} y.$$

Lemma 3[26] Let z, z' be two states of system (1), then we have

$$\begin{aligned}&\left|\bigwedge_{j=1}^{n}\alpha_{ij}f_j(z)-\bigwedge_{j=1}^{n}\alpha_{ij}f_j(z')\right|\\&\leq\sum_{j=1}^{n}|\alpha_{ij}||f_j(z)-f_j(z')|,\\&\left|\bigvee_{j=1}^{n}\beta_{ij}f_j(z)-\bigvee_{j=1}^{n}\beta_{ij}f_j(z')\right|\\&\leq\sum_{j=1}^{n}|\beta_{ij}||f_j(z)-f_j(z')|.\end{aligned}$$

Lemma 4[27] For any $x\in\mathbb{R}^n$, any constant matrix $A=[a_{ij}]_{n\times n}$ with $a_{ij}\geq 0$, the following matrix inequality holds:

$$x^{\mathrm{T}}A^{\mathrm{T}}Ax\leq nx^{\mathrm{T}}A_s^{\mathrm{T}}A_sx,$$

where

$$A_s=\mathrm{diag}\left(\sum_{i=1}^{n}a_{i1},\sum_{i=1}^{n}a_{i2},\ldots,\sum_{i=1}^{n}a_{in}\right).$$

Lemma 5[28] Given any real matrix $M=M^{\mathrm{T}}>0$ of appropriate dimension, a scalar $\eta>0$, and a vector function $\omega(\cdot):[a,b]\to\mathbb{R}^n$, such that the integrations concerned are well defined, then

$$\begin{aligned}&\left[\int_a^b\omega(s)\mathrm{d}s\right]^{\mathrm{T}}M\left[\int_a^b\omega(s)\mathrm{d}s\right]\\&\leq(b-a)\int_a^b\omega^{\mathrm{T}}(s)M\omega(s)\mathrm{d}s.\end{aligned}$$

3. Main results

Theorem 1 Assume that assumptions A1–A3 hold. The error dynamical system (3) is globally asymptotically stable, if there exist $n\times n$ positive diagonal matrices P, R, U_1, and U_2, some $n\times n$ positive-definite symmetric matrices Q, W, M_1, and M_2, two scalars $\mu_1>0$, $\mu_2>0$, and a $2n\times 2n$ matrix $\left(\begin{smallmatrix}T_{11}&T_{12}\\ \star&T_{22}\end{smallmatrix}\right)>0$ such that the following LMI has feasible solution:

$$\Omega=\begin{bmatrix}\Omega_{i,j}&\Gamma_1^{\mathrm{T}}&\Gamma_2^{\mathrm{T}}\\ \star&-\mu_1n^{-1}I&0\\ \star&\star&-\mu_2n^{-1}I\end{bmatrix}<0,\qquad(6)$$

where $i,j=1,2,\ldots,10$ with

$$\begin{aligned}&\Omega_{1,1}=-2PD-Q^{\mathrm{T}}-\frac{1}{\tau}M_1-2M_2-U_1\Sigma_1,\\&\Omega_{1,2}=\frac{1}{\tau}M_1,\ \ \Omega_{1,3}=T_{12}^{\mathrm{T}},\ \Omega_{1,4}=-D^{\mathrm{T}}P^{\mathrm{T}}-Q^{\mathrm{T}},\\&\Omega_{1,5}=0,\ \Omega_{1,6}=\frac{2}{\tau}M_2,\ \Omega_{1,7}=\frac{2}{\tau}M_2,\\&\Omega_{1,8}=PA+U_1\Sigma_2,\ \Omega_{1,9}=PB,\ \Omega_{1,10}=0,\\&\Omega_{2,2}=-Q-\frac{1}{\tau}M_1,\ \Omega_{2,3}=0,\ \Omega_{2,4}=0,\ \Omega_{2,5}=0,\\&\Omega_{2,6}=0,\ \Omega_{2,7}=0,\ \Omega_{2,8}=0,\ \Omega_{2,9}=0,\\&\Omega_{2,10}=0,\ \Omega_{3,3}=\tau T_{11}-2T_{12}^{\mathrm{T}}-U_2\Sigma_1,\ \Omega_{3,4}=0,\\&\Omega_{3,5}=0,\ \Omega_{3,6}=0,\ \Omega_{3,7}=0,\ \Omega_{3,8}=0,\\&\Omega_{3,9}=U_2\Sigma_2,\ \Omega_{3,10}=0,\\&\Omega_{4,4}=-2P+W+\tau M_1+\frac{\tau^2}{2}M_2+\tau T_{22},\\&\Omega_{4,5}=0,\ \Omega_{4,6}=0,\ \ \Omega_{4,7}=0,\\&\Omega_{4,8}=PA,\ \Omega_{4,9}=PB,\ \Omega_{4,10}=0,\\&\Omega_{5,5}=-W,\ \Omega_{5,6}=0,\ \Omega_{5,7}=0,\ \Omega_{5,8}=0,\\&\Omega_{5,9}=0,\ \Omega_{5,10}=0,\ \Omega_{6,6}=-\frac{2}{\tau^2}M_2,\\&\Omega_{6,7}=-\frac{2}{\tau^2}M_2,\ \Omega_{6,8}=0,\ \Omega_{6,9}=0,\ \Omega_{6,10}=0,\\&\Omega_{7,7}=-\frac{2}{\tau^2}M_2,\ \Omega_{7,8}=0,\ \Omega_{7,9}=0,\ \Omega_{7,10}=0,\\&\Omega_{8,8}=RK-U_1,\ \Omega_{8,9}=0,\ \Omega_{8,10}=0,\\&\Omega_{9,9}=-U_2,\ \Omega_{9,10}=0,\ \Omega_{10,10}=\mu_1I+\mu_2I-R,\\&K=\mathrm{diag}\{\tilde{k}_1\hat{k}_1,\ldots,\tilde{k}_n\hat{k}_n\},\\&|\alpha|_s=\mathrm{diag}\Big\{\sum_{i=1}^{n}|\alpha_{i1}|,\sum_{i=1}^{n}|\alpha_{i2}|,\ldots,\sum_{i=1}^{n}|\alpha_{in}|\Big\},\\&|\beta|_s=\mathrm{diag}\Big\{\sum_{i=1}^{n}|\beta_{i1}|,\sum_{i=1}^{n}|\beta_{i2}|,\ldots,\sum_{i=1}^{n}|\beta_{in}|\Big\},\\&S=|\alpha|_s+|\beta|_s,\ \Gamma_1^{\mathrm{T}}=\Big[\,(WS)^{\mathrm{T}}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\,\Big]^{\mathrm{T}},\\&\Gamma_2^{\mathrm{T}}=\Big[\,0\ 0\ 0\ (WS)^{\mathrm{T}}\ 0\ 0\ 0\ 0\ 0\ 0\,\Big]^{\mathrm{T}}.\end{aligned}$$

Moreover, the estimation gain $H=P^{-1}Q$.

Proof Consider the following Lyapunov–Krasovskii functional

$$V(t)=\sum_{i=1}^{7}V_i(t),\qquad(7)$$

where

$$\begin{aligned}V_1(t)&=e^{\mathrm{T}}(t)Pe(t)=\sum_{i=1}^{n}p_ie_i^2(t),\\V_2(t)&=\int_{t-\tau}^{t}e^{\mathrm{T}}(s)Qe(s)\mathrm{d}s+\int_{t-\tau}^{t}\dot{e}^{\mathrm{T}}(s)W\dot{e}(s)\mathrm{d}s,\\V_3(t)&=\sum_{j=1}^{n}r_j\tilde{k}_j\int_0^{\infty}k_j(\theta)\int_{t-\theta}^{t}g_j^2(e_j(s))\mathrm{d}s\mathrm{d}\theta,\end{aligned}$$

$$V_4(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)M_1\dot{e}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$V_5(t) = \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{e}^{\mathrm{T}}(s)M_2\dot{e}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta,$$

$$V_6(t) = \int_{0}^{t}\int_{\theta-\tau(\theta)}^{\theta} \begin{bmatrix} e(\theta-\tau(\theta)) \\ \dot{e}(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix} \times \begin{bmatrix} e(\theta-\tau(\theta)) \\ \dot{e}(s) \end{bmatrix} \mathrm{d}s\mathrm{d}\theta,$$

$$V_7(t) = \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)T_{22}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta.$$

From Lemma 3, we obtain

$$\begin{aligned} &\Big|\bigwedge_{j=1}^{n}\alpha_{ij}\int_{-\infty}^{t}k_j(t-s)f_j(x_j)\mathrm{d}s - \bigwedge_{j=1}^{n}\alpha_{ij}\int_{-\infty}^{t}k_j(t-s)f_j(y_j)\mathrm{d}s\Big| \\ &\le \sum_{j=1}^{n}|\alpha_{ij}|\Big|\int_{-\infty}^{t}k_j(t-s)\Big(f_j(x_j)-f_j(y_j)\Big)\mathrm{d}s\Big| \\ &= \sum_{j=1}^{n}|\alpha_{ij}|\Big|\int_{-\infty}^{t}k_j(t-s)g_j(e_j(s))\mathrm{d}s\Big|. \end{aligned}$$

By calculating the time derivation of $V_i(t)$ along the trajectory of system (5), we obtain

$$\begin{aligned} \dot{V}_1(t) =\ & 2\sum_{i=1}^{n}p_ie_i(t)\Big\{(-d_i-h_{ij})e_i(t)+\sum_{j=1}^{n}a_{ij}g_j(e_j(t))+\sum_{j=1}^{n}b_{ij}g_j(e_j(t-\tau(t))) \\ &+\bigwedge_{j=1}^{n}\alpha_{ij}\int_{-\infty}^{t}k_j(t-s)f_j(x_j)\mathrm{d}s-\bigwedge_{j=1}^{n}\alpha_{ij}\int_{-\infty}^{t}k_j(t-s)f_j(y_j)\mathrm{d}s \\ &+\bigvee_{j=1}^{n}\beta_{ij}\int_{-\infty}^{t}k_j(t-s)f_j(x_j)\mathrm{d}s-\bigvee_{j=1}^{n}\beta_{ij}\int_{-\infty}^{t}k_j(t-s)f_j(y_j)\mathrm{d}s\Big\} \\ \le\ & 2e^{\mathrm{T}}(t)P\Big[(-D-H)e(t)+Ag(e(t))+Bg(e(t-\tau(t)))\Big]+2e^{\mathrm{T}}(t)P(|\alpha|+|\beta|)\Big|\int_{-\infty}^{t}k(t-s)g(e(s))\mathrm{d}s\Big| \\ \le\ & -2e^{\mathrm{T}}(t)PDe(t)-2e^{\mathrm{T}}(t)Qe(t)+2e^{\mathrm{T}}(t)PAg(e(t))+2e^{\mathrm{T}}(t)PBg(e(t-\tau(t))) \\ &+\mu_1^{-1}ne^{\mathrm{T}}(t)P(|\alpha|_s+|\beta|_s)(|\alpha|_s+|\beta|_s)^{\mathrm{T}}Pe(t) \\ &+\mu_1\Big(\int_{-\infty}^{t}k(t-s)g(e(s))\mathrm{d}s\Big)^{\mathrm{T}}\Big(\int_{-\infty}^{t}k(t-s)g(e(s))\mathrm{d}s\Big)+2\dot{e}^{\mathrm{T}}(t)P\Big[-\dot{e}(t)+\dot{e}(t)\Big] \\ \le\ & -2e^{\mathrm{T}}(t)PDe(t)-2e^{\mathrm{T}}(t)Qe(t)+2e^{\mathrm{T}}(t)PAg(e(t))+2e^{\mathrm{T}}(t)PBg(e(t-\tau(t))) \\ &+\mu_1^{-1}ne^{\mathrm{T}}(t)P(|\alpha|_s+|\beta|_s)(|\alpha|_s+|\beta|_s)^{\mathrm{T}}Pe(t) \\ &+\mu_1\Big(\int_{-\infty}^{t}k(t-s)g(e(s))\mathrm{d}s\Big)^{\mathrm{T}}\Big(\int_{-\infty}^{t}k(t-s)g(e(s))\mathrm{d}s\Big)-2\dot{e}^{\mathrm{T}}(t)P\dot{e}(t) \\ &-2\dot{e}^{\mathrm{T}}(t)PDe(t)-2\dot{e}^{\mathrm{T}}(t)Qe(t)+2\dot{e}^{\mathrm{T}}(t)PAg(e(t))+2\dot{e}^{\mathrm{T}}(t)PBg(e(t-\tau(t))) \\ &+\mu_2^{-1}n\dot{e}^{\mathrm{T}}(t)P(|\alpha|_s+|\beta|_s)\ (|\alpha|_s+|\beta|_s)^{\mathrm{T}}P\dot{e}(t) \\ &+\mu_2\Big(\int_{-\infty}^{t}k(t-s)g(e(s))\mathrm{d}s\Big)^{\mathrm{T}}\Big(\int_{-\infty}^{t}k(t-s)g(e(s))\mathrm{d}s\Big), \end{aligned} \tag{8}$$

$$\dot{V}_2(t) = e^{\mathrm{T}}(t)Qe(t)-e^{\mathrm{T}}(t-\tau)Qe(t-\tau)+\dot{e}^{\mathrm{T}}(t)W\dot{e}(t)-\dot{e}^{\mathrm{T}}(t-\tau)W\dot{e}(t-\tau), \tag{9}$$

$$\begin{aligned} \dot{V}_3(t) =\ & \sum_{j=1}^{n}r_j\tilde{k}_j\int_{0}^{\infty}k_j(\theta)g_j^2(e_j(t))\mathrm{d}\theta-\sum_{j=1}^{n}r_j\tilde{k}_j\int_{0}^{\infty}k_j(\theta)g_j^2(e_j(t-\theta))\mathrm{d}\theta \\ \le\ & g^{\mathrm{T}}(e(t))RKg(e(t))-\sum_{j=1}^{n}r_j\int_{0}^{\infty}k_j(\theta)\mathrm{d}\theta\int_{0}^{\infty}k_j(\theta)g_j^2(e_j(t-\theta))\mathrm{d}\theta \\ \le\ & g^{\mathrm{T}}(e(t))RKg(e(t))-\sum_{j=1}^{n}r_j\Big(\int_{0}^{\infty}k_j(\theta)g_j(e_j(t-\theta))\mathrm{d}\theta\Big)^2 \end{aligned}$$

$$= g^{\mathrm{T}}(e(t))RKg(e(t)) - \Big(\int_{-\infty}^{t} k(t-s)g(e(s))\mathrm{d}s\Big)^{\mathrm{T}} R\Big(\int_{-\infty}^{t} k(t-s)g(e(s))\mathrm{d}s\Big), \tag{10}$$

$$\begin{aligned}\dot{V}_4(t) &= \tau\dot{e}^{\mathrm{T}}(t)M_1\dot{e}(t) - \int_{t-\tau}^{t}\dot{e}^{\mathrm{T}}(s)M_1\dot{e}(s)\mathrm{d}s\\ &\le \tau\dot{e}^{\mathrm{T}}(t)M_1\dot{e}(t) - \frac{1}{\tau}\,e^{\mathrm{T}}(t)M_1e(t) + \frac{2}{\tau}\,e^{\mathrm{T}}(t)M_1e(t-\tau) - \frac{1}{\tau}\,e^{\mathrm{T}}(t-\tau)M_1e(t-\tau),\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{V}_5(t) &= \frac{\tau^2}{2}\,\dot{e}^{\mathrm{T}}(t)M_2\dot{e}(t) - \int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{e}^{\mathrm{T}}(s)M_2\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\\ &\le \frac{\tau^2}{2}\,\dot{e}^{\mathrm{T}}(t)M_2\dot{e}(t) - \frac{2}{\tau^2}\Big(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\Big)^{\mathrm{T}}M_2\Big(\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta\Big)\\ &= \frac{\tau^2}{2}\dot{e}^{\mathrm{T}}(t)M_2\dot{e}(t) - \frac{2}{\tau^2}\Big(\tau e(t) - \int_{t-\tau}^{t}e(s)\mathrm{d}s\Big)^{\mathrm{T}}M_2\Big(\tau e(t) - \int_{t-\tau}^{t}e(s)\mathrm{d}s\Big)\\ &= \frac{\tau^2}{2}\dot{e}^{\mathrm{T}}(t)M_2\dot{e}(t) - \frac{2}{\tau^2}\Big(\tau e(t) - \int_{t-\tau(t)}^{t}e(s)\mathrm{d}s - \int_{t-\tau}^{t-\tau(t)}e(s)\mathrm{d}s\Big)^{\mathrm{T}}M_2\\ &\quad\times\Big(\tau e(t) - \int_{t-\tau(t)}^{t}e(s)\mathrm{d}s - \int_{t-\tau}^{t-\tau(t)}e(s)\mathrm{d}s\Big)\\ &= \frac{\tau^2}{2}\dot{e}^{\mathrm{T}}(t)M_2\dot{e}(t) - 2e^{\mathrm{T}}(t)M_2e(t) + \frac{2}{\tau}e^{\mathrm{T}}(t)M_2\int_{t-\tau(t)}^{t}e(s)\mathrm{d}s\\ &\quad+\frac{2}{\tau}e^{\mathrm{T}}(t)M_2\int_{t-\tau}^{t-\tau(t)}e(s)\mathrm{d}s + \frac{2}{\tau}\int_{t-\tau(t)}^{t}e^{\mathrm{T}}(s)\mathrm{d}sM_2e(t) - \frac{2}{\tau^2}\int_{t-\tau(t)}^{t}e^{\mathrm{T}}(s)\mathrm{d}sM_2\\ &\quad\times\int_{t-\tau(t)}^{t}e(s)\mathrm{d}s - \frac{2}{\tau^2}\int_{t-\tau(t)}^{t}e^{\mathrm{T}}(s)\mathrm{d}sM_2\int_{t-\tau}^{t-\tau(t)}e(s)\mathrm{d}s + \frac{2}{\tau}\int_{t-\tau}^{t-\tau(t)}e^{\mathrm{T}}(s)\mathrm{d}sM_2e(t)\\ &\quad-\frac{2}{\tau^2}\int_{t-\tau}^{t-\tau(t)}e^{\mathrm{T}}(s)\mathrm{d}sM_2\int_{t-\tau(t)}^{t}e(s)\mathrm{d}s - \frac{2}{\tau^2}\int_{t-\tau}^{t-\tau(t)}e^{\mathrm{T}}(s)\mathrm{d}sM_2\int_{t-\tau}^{t-\tau(t)}e(s)\mathrm{d}s,\end{aligned} \tag{12}$$

$$\begin{aligned}\dot{V}_6(t) &= \int_{t-\tau(t)}^{t}\begin{bmatrix}e(t-\tau(t))\\ \dot{e}(s)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}T_{11} & T_{12}\\ * & T_{22}\end{bmatrix}\begin{bmatrix}e(t-\tau(t))\\ \dot{e}(s)\end{bmatrix}\mathrm{d}s\\ &= \tau(t)e^{\mathrm{T}}(t-\tau(t))T_{11}e(t-\tau(t)) + 2e^{\mathrm{T}}(t)T_{12}^{\mathrm{T}}e(t-\tau(t)) - 2e^{\mathrm{T}}(t-\tau(t))T_{12}^{\mathrm{T}}e(t-\tau(t))\\ &\quad+\int_{t-\tau(t)}^{t}\dot{e}^{\mathrm{T}}(s)T_{22}\dot{e}(s)\mathrm{d}s\\ &\le e^{\mathrm{T}}(t-\tau(t))\Big[\tau T_{11} - 2T_{12}^{\mathrm{T}}\Big]e(t-\tau(t)) + 2e^{\mathrm{T}}(t)T_{12}^{\mathrm{T}}e(t-\tau(t)) + \int_{t-\tau}^{t}\dot{e}^{\mathrm{T}}(s)T_{22}\dot{e}(s)\mathrm{d}s,\end{aligned} \tag{13}$$

$$\dot{V}_7(t) = \tau\dot{e}^{\mathrm{T}}(t)T_{22}\dot{e}(t) - \int_{-\tau}^{0}\dot{e}^{\mathrm{T}}(t+\theta)T_{22}\dot{e}(t+\theta)\mathrm{d}\theta = \tau\dot{e}^{\mathrm{T}}(t)T_{22}\dot{e}(t) - \int_{t-\tau}^{t}\dot{e}^{\mathrm{T}}(s)T_{22}\dot{e}(s)\mathrm{d}s. \tag{14}$$

In addition, for any $n\times n$ diagonal matrices $U_1 > 0$, $U_2 > 0$, the following inequality holds by the methods proposed in Ref. [29]:

$$\left\{\begin{bmatrix}e(t)\\ g(e(t))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-U_1\Sigma_1 & U_1\Sigma_2\\ * & -U_1\end{bmatrix}\begin{bmatrix}e(t)\\ g(e(t))\end{bmatrix} + \begin{bmatrix}e(t-\tau(t))\\ g(e(t-\tau(t)))\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-U_2\Sigma_1 & U_2\Sigma_2\\ * & -U_2\end{bmatrix}\begin{bmatrix}e(t-\tau(t))\\ g(e(t-\tau(t)))\end{bmatrix}\right\} \ge 0. \tag{15}$$

Hence, from expressions (7)–(15) we have

$$\dot{V}(t) \le \xi^{\mathrm{T}}(t)\,[\Omega_{i,j} + \Gamma_1^{\mathrm{T}}\mu_1^{-1}n\Gamma_1 + \Gamma_2^{\mathrm{T}}\mu_2^{-1}n\Gamma_2]\,\xi(t), = \xi^{\mathrm{T}}(t)\Omega\xi(t), \tag{16}$$

where

$$\xi(t) = \Big[e^{\mathrm{T}}(t),\ e^{\mathrm{T}}(t-\tau),\ e^{\mathrm{T}}(t-\tau(t)),\ \dot{e}^{\mathrm{T}}(t),\ \dot{e}^{\mathrm{T}}(t-\tau),\ \int_{t-\tau(t)}^{t} e^{\mathrm{T}}(s)\mathrm{d}s,\ \int_{t-\tau}^{t-\tau(t)} e^{\mathrm{T}}(s)\mathrm{d}s,$$
$$g^{\mathrm{T}}(e(t)),\ g^{\mathrm{T}}(e(t-\tau(t))), \int_{-\infty}^{t} k(t-s)g^{\mathrm{T}}(e(s))\mathrm{d}s\Big]^{\mathrm{T}},$$
$$\Omega = \Omega_{i,j} + \Gamma_1^{\mathrm{T}}\mu_1^{-1}n\Gamma_1 + \Gamma_2^{\mathrm{T}}\mu_2^{-1}n\Gamma_2.$$

By expression (6), it yields

$$\dot{V}(t) \leq -\xi^{\mathrm{T}}(t)\ \Omega^*\ \xi(t), \quad t > 0,$$

where $\Omega^* = -\Omega > 0$, which guarantees globally asymptotically stable of the error dynamical system (3) by Lyapunov stability theory. As a result, the chaotic FCNN with discrete and distributed delays (2) is globally synchronized with the master FCNN (1). □

Remark 2 Yu and Jiang[23] have investigated the global exponential synchronization of fuzzy cellular neural networks with delays and reaction–diffusion terms. Feng *et al.*[24] have investigated the global exponential synchronization of delayed fuzzy cellular neural networks with impulsive effects. The authors in the previous literature have tried to solve the synchronization of FCNNs with constant delays. So far no results have been reported on the synchronization of FCNNs with unbounded distributed delays using the LMI approach. It is challenging and complicated to synchronize the FCNNs model with unbounded distributed delays. By constructing a suitable Lyapunov–krasovskii functional and simple control, we derive a simple and efficient criterion in terms of LMIs for synchronization. The results obtained in this paper are simple and easy to apply in practical applications.

Let us consider the master FCNN described by the following state equation:

$$\begin{cases} \dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau(t))) + I_i \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j(t-\tau(t))) + \bigvee_{j=1}^{n} \beta_{ij} f_j(x_j(t-\tau(t))), \quad i \in \Lambda, \\ x_i(s) = \phi_i(s), \quad s \in (-\infty, 0]. \end{cases} \tag{17}$$

The corresponding response system of system (17) is given by

$$\begin{cases} \dot{y}_i(t) = -d_i y_i(t) + \sum_{j=1}^{n} a_{ij} f_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(y_j(t-\tau(t))) + I_i \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(y_j(t-\tau(t))) + \bigvee_{j=1}^{n} \beta_{ij} f_j(y_j(t-\tau(t))) + u_i(t), \quad i \in \Lambda, \\ y_i(s) = \varphi_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{18}$$

where $\phi_i(\cdot)$, $\varphi_i(\cdot)$, d_i, a_{ij}, b_{ij}, α_{ij}, β_{ij}, I_i, f_j, and $u_i(t)$ described in systems (17) and (18) are the same as those in systems (1) and (2).

Let $e(t) = \big(e_1(t), e_2(t), \ldots, e_n(t)\big) := x(t) - y(t)$ be the error state. Then, the error dynamical system between systems (17) and (18) is given by

$$\begin{cases} \dot{e}_i(t) = -d_i e_i(t) + \sum_{j=1}^{n} a_{ij} g_j(e_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(e_j(t-\tau(t))) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij} f_j(x_j(t-\tau(t))) - \bigwedge_{j=1}^{n} \alpha_{ij} f_j(y_j(t-\tau(t))) \\ \qquad + \bigvee_{j=1}^{n} \beta_{ij} f_j(x_j(t-\tau(t))) - \bigvee_{j=1}^{n} \beta_{ij} f_j(y_j(t-\tau(t))) - u_i(t), \quad i \in \Lambda, \\ e_i(s) = \phi_i(s) - \varphi_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{19}$$

where $g_j(e_j(\cdot)) = f_j(x_j(\cdot)) - f_j(y_j(\cdot))$, $j \in \Lambda$.

Theorem 2 Assume that assumptions A1 and A2 hold. The error dynamical system (19) is globally asymptotically stable, if there exist $n \times n$ positive diagonal matrices P, U_1, U_2, and some $n \times n$ positive-definite symmetric matrices Q, W, M_1, M_2, two scalars $\mu_1 > 0$, $\mu_2 > 0$, and a $2n \times 2n$ matrix $\left(\begin{smallmatrix} T_{11} & T_{12} \\ \star & T_{22} \end{smallmatrix}\right) > 0$ such that the following LMI has feasible solution:

$$\Omega = \begin{bmatrix} \Omega_{i,j} & \Gamma_1^{\mathrm{T}} & \Gamma_2^{\mathrm{T}} \\ \star & -\mu_1 n^{-1} I & 0 \\ \star & \star & -\mu_2 n^{-1} I \end{bmatrix} < 0, \tag{20}$$

where i, $j = 1, 2, \ldots, 9$ with

$$\begin{aligned}
&\Omega_{1,1} = -2PD - Q^{\mathrm{T}} - \frac{1}{\tau} M_1 - 2M_2 - U_1\Sigma_1,\\
&\Omega_{1,2} = \frac{1}{\tau} M_1,\ \Omega_{1,3} = T_{12}^{\mathrm{T}},\ \Omega_{1,4} = -D^{\mathrm{T}}P^{\mathrm{T}} - Q^{\mathrm{T}},\\
&\Omega_{1,5} = 0,\ \Omega_{1,6} = \frac{2}{\tau} M_2,\ \Omega_{1,7} = \frac{2}{\tau} M_2,\\
&\Omega_{1,8} = PA + U_1\Sigma_2,\ \Omega_{1,9} = PB,\ \Omega_{1,10} = 0,\\
&\Omega_{2,2} = -Q - \frac{1}{\tau} M_1,\ \Omega_{2,3} = 0,\ \Omega_{2,4} = 0,\\
&\Omega_{2,5} = 0,\ \Omega_{2,6} = 0,\ \Omega_{2,7} = 0,\ \Omega_{2,8} = 0,\ \Omega_{2,9} = 0,\\
&\Omega_{2,10} = 0,\ \Omega_{3,3} = \tau T_{11} - 2T_{12}^{\mathrm{T}} - U_2\Sigma_1,\\
&\Omega_{3,4} = 0,\ \Omega_{3,5} = 0,\ \Omega_{3,6} = 0,\ \Omega_{3,7} = 0,\ \Omega_{3,8} = 0,\\
&\Omega_{3,9} = U_2\Sigma_2,\ \Omega_{3,10} = 0,\\
&\Omega_{4,4} = -2P + W + \tau M_1 + \frac{\tau^2}{2} M_2 + \tau T_{22},\\
&\Omega_{4,5} = 0,\ \Omega_{4,6} = 0,\ \Omega_{4,7} = 0,\ \Omega_{4,8} = PA,\\
&\Omega_{4,9} = PB,\ \Omega_{4,10} = 0,\ \Omega_{5,5} = -W,\\
&\Omega_{5,6} = 0,\ \Omega_{5,7} = 0,\ \Omega_{5,8} = 0,\ \Omega_{5,9} = 0,\ \Omega_{5,10} = 0,\\
&\Omega_{6,6} = -\frac{2}{\tau^2} M_2,\ \Omega_{6,7} = -\frac{2}{\tau^2} M_2,\ \Omega_{6,8} = 0,\\
&\Omega_{6,9} = 0,\ \Omega_{6,10} = 0,\ \Omega_{7,7} = -\frac{2}{\tau^2} M_2,\\
&\Omega_{7,8} = 0,\ \Omega_{7,9} = 0,\ \Omega_{7,10} = 0,\\
&\Omega_{8,8} = -U_1,\ \Omega_{8,9} = 0,\ \Omega_{8,10} = 0,\\
&\Omega_{9,9} = \mu_1 I + \mu_2 I - U_2,\ \Omega_{9,10} = 0,
\end{aligned}$$

$$\begin{aligned}
&|\alpha|_s = \mathrm{diag}\Big\{\sum_{i=1}^{n} |\alpha_{i1}|, \sum_{i=1}^{n} |\alpha_{i2}|, \ldots, \sum_{i=1}^{n} |\alpha_{in}|\Big\},\\
&|\beta|_s = \mathrm{diag}\Big\{\sum_{i=1}^{n} |\beta_{i1}|, \sum_{i=1}^{n} |\beta_{i2}|, \ldots, \sum_{i=1}^{n} |\beta_{in}|\Big\},\\
&S = |\alpha|_s + |\beta|_s,\ \Gamma_1^{\mathrm{T}} = \Big[\,(WS)^{\mathrm{T}}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\,\Big]^{\mathrm{T}},\\
&\Gamma_2^{\mathrm{T}} = \Big[\,0\ 0\ 0\ (WS)^{\mathrm{T}}\ 0\ 0\ 0\ 0\ 0\,\Big]^{\mathrm{T}}.
\end{aligned}$$

Moreover, the estimation gain $H = P^{-1}Q$.

Proof Consider the following Lyapunov–Krasovskii functional

$$V(t) = \sum_{i=1}^{6} V_i(t), \tag{21}$$

where

$$\begin{aligned}
V_1(t) &= e^{\mathrm{T}}(t)Pe(t) = \sum_{i=1}^{n} p_i e_i^2(t),\\
V_2(t) &= \int_{t-\tau}^{t} e^{\mathrm{T}}(s)Qe(s)\mathrm{d}s + \int_{t-\tau}^{t} \dot{e}^{\mathrm{T}}(s)W\dot{e}(s)\mathrm{d}s,\\
V_3(t) &= \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)M_1\dot{e}(s)\mathrm{d}s\mathrm{d}\theta,\\
V_4(t) &= \int_{-\tau}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{e}^{\mathrm{T}}(s)M_2\dot{e}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta,\\
V_5(t) &= \int_{0}^{t}\int_{\theta-\tau(\theta)}^{\theta} \begin{bmatrix} e(\theta-\tau(\theta)) \\ \dot{e}(s) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} T_{11} & T_{12} \\ * & T_{22} \end{bmatrix}\\
&\quad \times \begin{bmatrix} e(\theta-\tau(\theta)) \\ \dot{e}(s) \end{bmatrix} \mathrm{d}s\mathrm{d}\theta,\\
V_6(t) &= \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{e}^{\mathrm{T}}(s)T_{22}\dot{e}(s)\mathrm{d}s\mathrm{d}\theta.
\end{aligned}$$

The proof of this Theorem is immediately follows from Theorem 1. □

4. Numerical example

Example 1 Consider the chaotic FCNN model

$$\begin{cases}
\dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau(t))) + I_i\\
\qquad\quad + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))\mathrm{d}s + \bigvee_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))\mathrm{d}s, \quad i \in \Lambda,\\
x_i(s) = \phi_i(s), \quad s \in (-\infty, 0]
\end{cases} \tag{22}$$

with parameters defined as

$$\phi(s) = (1, -0.5, -1)^{\mathrm{T}},\ s \in (-\infty, 0],\ f_j(x_j) = \frac{1}{2}\big(|x_j + 1| - |x_j - 1|\big), \quad j = 1, 2, 3,$$
$$k(s) = 0.17\,\mathrm{e}^{-s},\ I_i = 0, \quad i = 1, 2, 3, \quad \tau(t) = 0.04|\sin(t)|,$$
$$A = \begin{bmatrix} 1.25 & -3.21 & -3.2 \\ -3.2 & 1.1 & -4.4 \\ -3.2 & 4.4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4.3 & -7.5 & -3 \\ -3 & 1.2 & -5 \\ -3.2 & 4.5 & -2.3 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$\alpha = \beta = \begin{bmatrix} -1/32 & -1/32 & 1/32 \\ -1/32 & -1/32 & 1/32 \\ -1/32 & -1/32 & 1/32 \end{bmatrix}.$$

The corresponding response system is designed as

$$\begin{cases} \dot{y}_i(t) = -d_i y_i(t) + \sum\limits_{j=1}^{n} a_{ij} f_j(y_j(t)) + \sum\limits_{j=1}^{n} b_{ij} f_j(y_j(t - \tau(t))) + I_i + \bigwedge\limits_{j=1}^{n} \alpha_{ij} \int_{-\infty}^{t} k_j(t - s) f_j(y_j(s))\,\mathrm{d}s \\ \qquad\quad + \bigvee\limits_{j=1}^{n} \beta_{ij} \int_{-\infty}^{t} k_j(t - s) f_j(y_j(s))\,\mathrm{d}s + u_i(t), \quad i \in \Lambda, \\ y_i(s) = \varphi_i(s), \quad s \in (-\infty, 0], \end{cases} \tag{23}$$

where u is given by Eq. (4) and the initial condition is

$$\varphi(s) = (-4.5, 3, 4.8)^{\mathrm{T}}, \quad s \in (-\infty, 0].$$

By using the Matlab LMI toolbox to solve the LMI (6) in Theorem 1, it can be found that the LMI is feasible and

$$P = \begin{bmatrix} 0.0027 & 0 & 0 \\ 0 & 0.0039 & 0 \\ 0 & 0 & 0.0028 \end{bmatrix},$$
$$R = \begin{bmatrix} 1.5033 & 0 & 0 \\ 0 & 1.5291 & 0 \\ 0 & 0 & 1.5443 \end{bmatrix},$$
$$Q = \begin{bmatrix} 0.0027 & -0.0000 & 0.0009 \\ -0.0000 & 0.0033 & -0.0009 \\ 0.0009 & -0.0009 & 0.0036 \end{bmatrix},$$
$$W = 10^{-3} \times \begin{bmatrix} 0.4309 & -0.0492 & 0.2845 \\ -0.0492 & 0.5644 & -0.2946 \\ 0.2845 & -0.2946 & 0.7081 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} 0.0097 & -0.0012 & 0.0070 \\ -0.0012 & 0.0130 & -0.0073 \\ 0.0070 & -0.0073 & 0.0165 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} 0.9241 & -0.0053 & -0.0184 \\ -0.0053 & 1.0913 & 0.0215 \\ -0.0184 & 0.0215 & 0.8319 \end{bmatrix},$$
$$T_{11} = \begin{bmatrix} 3.7860 & -0.0035 & -0.1892 \\ -0.0035 & 4.7300 & 0.2625 \\ -0.1892 & 0.2625 & 3.3565 \end{bmatrix},$$
$$T_{12} = \begin{bmatrix} 0.3225 & 0.0007 & 0.0189 \\ 0.0021 & 0.4482 & -0.0261 \\ 0.0139 & -0.0043 & 0.3027 \end{bmatrix},$$
$$T_{22} = \begin{bmatrix} 0.0327 & -0.0006 & 0.0079 \\ -0.0006 & 0.0493 & -0.0088 \\ 0.0079 & -0.0088 & 0.0365 \end{bmatrix},$$
$$U_1 = \begin{bmatrix} 0.6533 & 0 & 0 \\ 0 & 0.7100 & 0 \\ 0 & 0 & 0.5919 \end{bmatrix},$$
$$U_2 = \begin{bmatrix} 0.2840 & 0 & 0 \\ 0 & 0.4044 & 0 \\ 0 & 0 & 0.2505 \end{bmatrix},$$
$$\mu_1 = 0.5531, \quad \mu_2 = 0.5576.$$

Consequently, the controller gain matrix H is designed as follows:

$$H = P^{-1}Q = \begin{bmatrix} 0.9949 & -0.0107 & 0.3447 \\ -0.0076 & 0.8578 & -0.2225 \\ 0.3422 & -0.3140 & 1.2978 \end{bmatrix}. \quad (24)$$

By Theorem 1, models (22) and (23) are asymptotically synchronized. The simulation results are illustrated in Figs. 1(a)–1(d) in which the controller designed in Eq. (24) is applied.

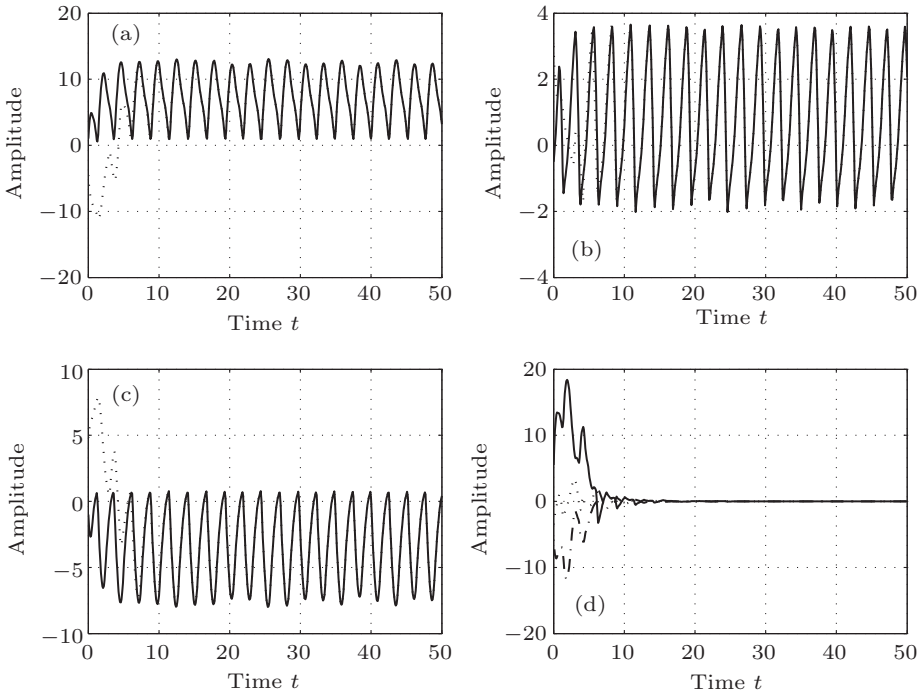


Fig. 1. State trajectories and error trajectories of drive system (22) and response system (23) with control input (24): (a) the state x_1 and its estimation, (b) the state x_2 and its estimation, (c) the state x_3 and its estimation, (d) the error states.

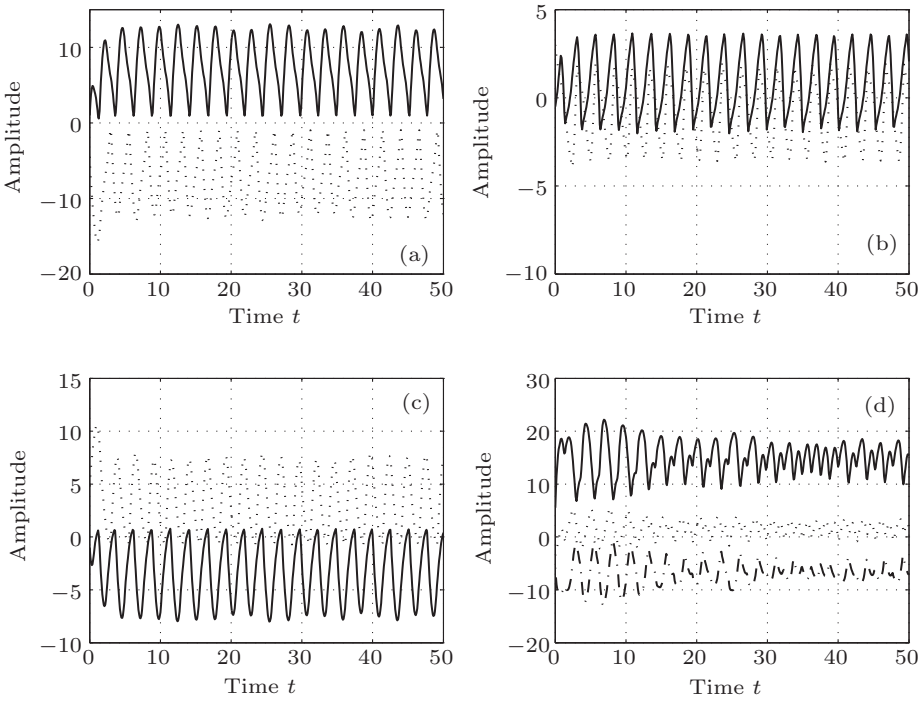


Fig. 2. State trajectories and error trajectories of drive system (22) and response system (23) without control input: (a) the state x_1 and its estimation, (b) the state x_2 and its estimation, (c) the state x_3 and its estimation, (d) the error states.

Remark 3 The simulation results can be described as follows. Figures 2(a)–2(d) provide the state trajectories and the error trajectories between the drive system (22) and the response system (23) without control input. One may observe that the drive system (22) and the response system (23) without control input cannot be synchronized. Figures 3(a) and 3(b) describe the chaotic behavior in phase space of the drive system (22) and the response system (23) without control input, respectively. Figures 1(a)–1(d) show that the state trajectories and error trajectories of the drive system (22) and the response system (23) with control input (24) to be asymptotically synchronized. In the simulations, we choose the time step size $h = 0.1$ and time segment $T = 50$. In addition, we should point out that for simplicity of our computer simulations, the delay kernel k is used as $k(s) = 0.17\,\mathrm{e}^{-s}$ for $s \in [0, 4]$ and $k(s) = 0$ for $s > 4$.

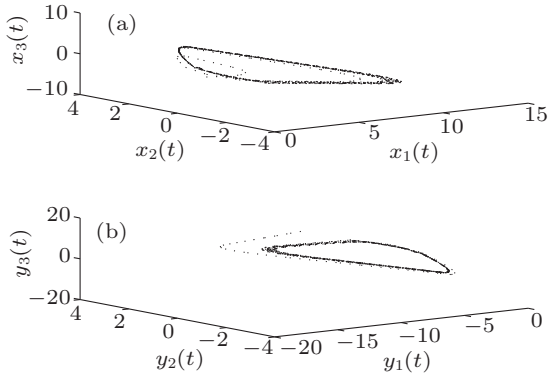


Fig. 3. (a) The chaotic behavior of drive system (22) in phase space with the initial condition $\phi(s) = (1, -0.5, -1)^{\mathrm{T}}$, $s \in (-\infty, 0]$; (b) The chaotic behavior of response system (23) in phase space without control input with the initial condition $\varphi(s) = (-4.5, 3, 4.8)^{\mathrm{T}}$, $s \in (-\infty, 0]$.

5. Conclusion

In this paper, the synchronization for FCNNs with mixed delays was considered. Mixed delays include discrete time-varying delays and unbounded distributed delays. A dynamic control scheme was proposed to achieve the synchronization between a drive network and a response network. By constructing the Lyapunov–Krasovskii functional which contains a triple-integral term and the free-weighting matrices method, we derived a simple and efficient criterion in terms of LMIs for synchronization. The controller can be easily obtained by solving the derived LMIs. The assumptions A1 and A2 on activation function and time-varying delay in this paper are weaker than those generally used in the previous literature,[21,24] namely, the boundedness of the activation function f_j. Further the differentiability of the time varying delay $\tau(t)$ is not required in this paper. A numerical example and its simulations are presented to illustrate the effectiveness of the proposed method.

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