The influences of model parameters on the characteristics of memristors*

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(Received 24 July 2011; revised manuscript received 30 September 2011)

As the fourth passive circuit component, a memristor is a nonlinear resistor that can "remember" the amount of charge passing through it. The characteristic of "remembering" the charge and non-volatility makes memristors great potential candidates in many fields. Nowadays, only a few groups have the ability to fabricate memristors, and most researchers study them by theoretic analysis and simulation. In this paper, we first analyse the theoretical base and characteristics of memristors, then use a simulation program with integrated circuit emphasis as our tool to simulate the theoretical model of memristors and change the parameters in the model to see the influence of each parameter on the characteristics. Our work supplies researchers engaged in memristor-based circuits with advice on how to choose the proper parameters.

Keywords: memristor, I–V characteristics, simulation program with integrated circuit emphasis

PACS: 84.32.–y, 89.20.Ff, 84.37.+q **DOI:** 10.1088/1674-1056/21/4/048401

1. Introduction

According to the predictions, traditional Si-based complementary metal–oxide semiconductor (CMOS) technology will reach its limitation of 10 nanometers in size and performance around 2020.[1] As the size decreases, the quantum effect becomes obvious. Problems such as the basic physical characteristics, techniques, and huge investment will impose great restrictions on the speed, power, integration, cost, homogeneity, and reliability of micro-electronics. To overcome these restrictions, nano-electronics is the best option, and as a new nano-electronic component, the memristor[2,3] will be widely used in the future.

A memristor, short for memory resistor,[2] is an electronic device, of which the resistance (memristance) can be programmed by a charge passing through the device. The memristance increases when charge passes the memristor in one direction, and decreases in the opposite direction. When there is no charge passing through, the memristor "remembers" its current resistance and presents this resistance when charged again.

More and more researchers have been paying attention to memristors since they were found by Strukov *et al.*[3] in 2008. Many groups studied the modelling of memristors. In 2009, Biolek *et al.*[4] used a simulation program with integrated circuit emphasis (SPICE) to model the nonlinear drifting process inside the memristor to study its characteristics. Benderli and Wey[5] proposed to use the SPICE macro model to simulate the behaviour of memristors. Using the SPICE, Batas and Fiedler[6] implemented and discussed the model of charge controlled memristors and that of magnetically controlled memristors, respectively. Zhang *et al.*[7] studied the mathematical model and properties of memristors with border constraint.

Memristor-based applications also develop quickly. In 2009, Pershin *et al.*[8] proposed to use circuits with memristors to imitate amoeba learning, and Bao *et al.*[9] analysed the dynamical behaviours of memristor chaotic oscillators. In 2010, Jo *et al.*[10] used memristors to simulate neuromorphic synapses, and Vontobel *et al.*[11] implemented and analysed writing and reading operations on a nano-scale crossbar memory based on memristors.

Regardless of what applications memristors are used in, either memory or neuromorphic networks, the appropriate parameters for the model are needed. However, there has been no detailed discussion about the influence of the parameters on the characteristics

*Project supported by the Young Scientists Fund of the National Natural Science Foundation of China (Grant No. 61003082) and the Science Fund for Creative Research Groups of the National Natural Science Foundation of China (Grant No. 60921062).

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of memristors in the existing research. So in this paper we study the memristor model proposed by Strukov *et al.*[3] and analyse the influence of the model parameters on the I–V characteristics of the memristor through HSPICE simulation, so as to give some suggestions for future research.

2. Memristors

In 1971, Chua[2] speculated that there existed a "missing circuit element" from the electromagnetic characteristics and necessity of symmetry reasons.

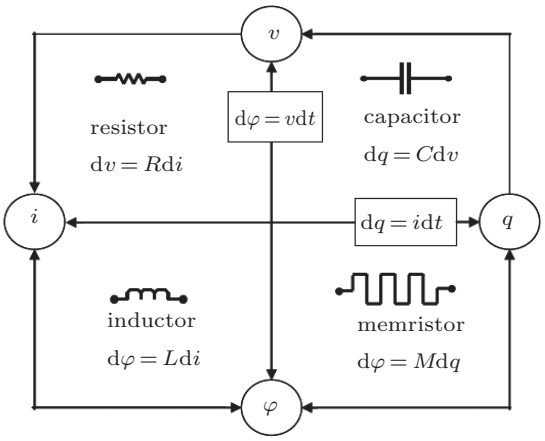


Fig. 1. Four circuit properties and their relations.

As shown in Fig. 1, there are four circuit variables, voltage v, current i, charge q, and magnetic flux φ, and three known circuit elements, resistor R, capacitor C, and inductor L. The relations between these variables are as follows:

$$\begin{cases} q(t) = \int_{-\infty}^{t} i(\tau)\mathrm{d}\tau, \\ \varphi(t) = \int_{-\infty}^{t} v(\tau)\mathrm{d}\tau, \end{cases} \quad \begin{cases} \mathrm{d}v = R\mathrm{d}i, \\ \mathrm{d}q = C\mathrm{d}v, \\ \mathrm{d}\varphi = L\mathrm{d}i. \end{cases} \tag{1}$$

From the necessity of symmetry, Chua suggested that some relation between magnetic flux φ and charge q should exist, and called the device the memristor, denoted by M, which is shown in the bottom-right part of Fig. 1, and expressed as

$$M = \frac{\mathrm{d}\varphi}{\mathrm{d}q}. \tag{2}$$

Due to the absence of a physical device, not much attention was paid to the memristor until 2008, when researchers[3] claimed that they had obtained a physical memristor. Since then, memristors have gradually attracted many researchers. Researchers[12] used a nano-scale $Pt/TiO_2/Pt$ structure to implement a memristor and found from experimental results that the I–V curve of the memristor could be expressed approximately as

$$I = \omega^n \beta \sinh(\alpha V) + \chi(\exp(\gamma V) - 1), \tag{3}$$

where ω is the state variable of the memristor[3] and the constants α, β, χ, and γ depend on the physical properties of the materials of the memristor, and n is the exponent that determines how the state variable ω affects the I–V characteristics. The time derivative of the state variable ω is modeled as

$$\frac{\mathrm{d}w}{\mathrm{d}t} = a \cdot f(w) \cdot g(V), \tag{4}$$

where a is a constant, $f : [0,1] \to R$ is called the window function, and $g : R \to R$ is a function of the real number. According to the simulation of the behaviour of the memristor, it was proposed in Ref. [3] that g must be a nonlinear, odd, and monotonically increasing function, while in Ref. [13] it was indicated that based on the experimental results, g could be supposed to be a polynomial function and expressed as

$$g(V) = V^q, \tag{5}$$

where q is a positive odd number.

3. The SPICE memristor model

SPICE was originally developed at the Electronics Laboratory of the University of California, Berkeley in 1975 by Nagel.[14] Among the commercial SPICE simulators, HSPICE is the most widely adopted and capable of analysing steady state, transience and frequency.[15,16] In the present paper, we adopt HSPICE to simulate the behaviours of memristors.

Figure 2 shows the SPICE model of the memristor, which is simulated as a two-end subcircuit with two parts. The left part is composed of a voltage controlled current source, G_{mem}, and a small resistance, R_{series}. G_{mem}, controlled by the voltage at point x in the right part, generates different currents and the I–V relation is as equation (4) shows. The right part consists of the voltage controlled current source, G_x, capacitor, C_x, and resistor, R_x, which together simulate the time derivative of the state variable. The current of G_x is controlled by the voltage over the memristor and the window function, and the I–V relation is shown in Eq. (4).

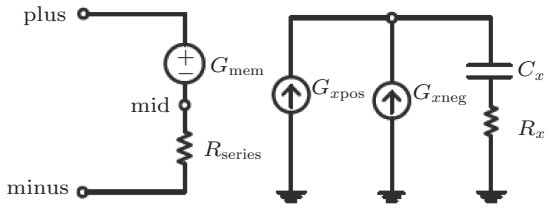


Fig. 2. The SPICE memristor model.

Figure 3 shows the HSPICE program for the memristor described in Eq. (3). With the parameters given in Fig. 3, the I and V curves of the memristor are shown in Fig. 4(a). In Fig. 4(b), the I–V curve of the memristor becomes a twisted hysteresis. In order to describe the curve, we call the part with a very small slope the "high resistance part" and that with a larger slope the "low resistance part". Under the given parameters, and with the currents and voltages from the I and V curves, respectively, we can compute the high resistance and the low resistance to be 189.4 MΩ and 4.7 MΩ, and the maximum currents in the negative direction and the positive direction to be −200 nA and 250 nA, respectively, as shown in Fig. 4(b). To simplify the description, we denote the resistance in the high resistance part as $R_{\rm high}$ and that in the low resistance part as $R_{\rm low}$.

```
***MEMRISTOR***
.SUBCKT memristor plus minus
.PARAM a=2 b=9 c=0.01 h=4 n=4 q=13 p=1 D=10n rratio=0.01 lin_max=0.95 lin_min=0.05
*window function
.PARAM f(x,p)='1-(2*x-1)^(2*p)'
.PARAM sign22(x)='(x>0)?1:0'
* DIFFERENTIAL EQUATION MODELING *
Gxpos 0 x value='sign22(-V(x)+lin_max) * (1-sign22(V(minus,plus))) * (( V(plus,minus) )^q) * f(V(x),p)'
Gxneg 0 x value='sign22(V(x)-lin_min) * sign22(V(minus,plus)) * (( V(plus,minus) )^q) * f(V(x),p)'
Cx x 0 0.00008
Rx x 0 0.01T
.IC V(x) 0.15
* RESISTIVE PORT OF THE MEMRISTOR *
Rseries mid minus 0.01meg
Gmem plus mid value='((((V(x))^n)+rratio)*b*sinh(a*V(plus,minus))+ c* ( exp(h*V(plus,minus))-1 ) ) * D'
.ENDS memristor
```

Fig. 3. The HSPICE code of the memristor model.

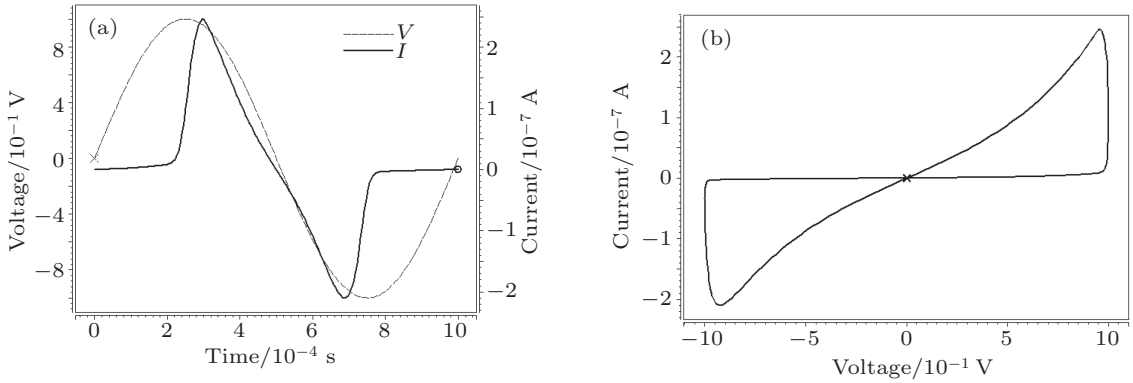


Fig. 4. (a) The voltage (dashed) and current (solid) curves of the memristor; (b) the I–V curve of the memristor.

4. Analysis of parameter influence on memristor characteristics

In this section, we discuss the influences of the parameters in Eqs. (3) and (4), and the HSPICE code on the memristor characteristics. To analyse the exact effects of each parameter on the memristor characteristics, when one parameter is considered, other parameters take the default values that are used in the HSPICE code in Fig. 3. In the following experiments we use an ideal sinusoidal voltage with a frequency of 1 kHz and a peak value of 1 V. And the high resistances and low resistances are computed under a voltage of 0.7 V.

4.1. The α parameter

The influence of the a value in the HSPICE code is examined in Fig. 3. With the values of a taken to be 1, 2, 3, 4, 5, and 6, the I–V curves are shown in Figs. 5(a)–5(f), respectively.

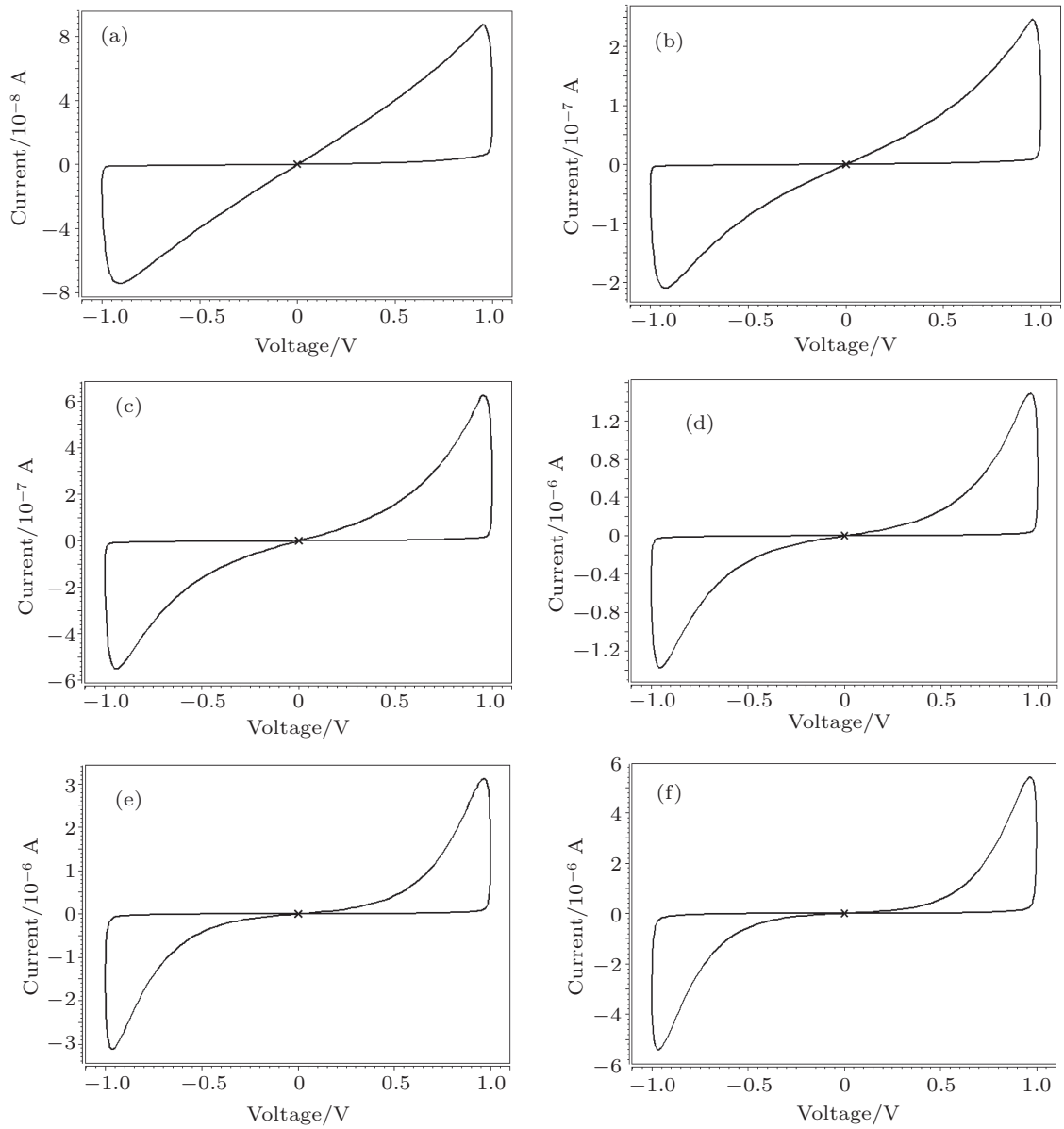


Fig. 5. The I–V curves of memristors with different values of parameter a: $a = 1$ (a), $a = 2$ (b), $a = 3$ (c), $a = 4$ (d), $a = 5$ (e), and $a = 6$ (f).

From Fig. 5, it can be found that when the value of a is larger, the maximum current is larger, and the low resistance part of the I–V curve has a larger curvature. For example, when $a = 1$, the negative maximum current is −72 nA and the positive one is 90 nA, while they are −5.5 μA and 5.5 μA, respectively, when $a = 6$. The variance in current reflects the change in resistances, as shown in Table 1.

Table 1. High and low resistances under different values of parameter a.

	$a = 1$	$a = 2$	$a = 3$	$a = 4$	$a = 5$	$a = 6$
$R_{\rm high}$/MΩ	276.7	189.4	116.8	64.5	32.8	15.8
$R_{\rm low}$/MΩ	11.9	4.7	2.2	1.0	0.5	0.3

4.2. The β parameter

Now we examine the influence of the $b$value in the HSPICE code. The values of b are taken to be 5, 7, 9, and 11, and the I–V curves are shown in Figs. 6(a)–6(d), respectively.

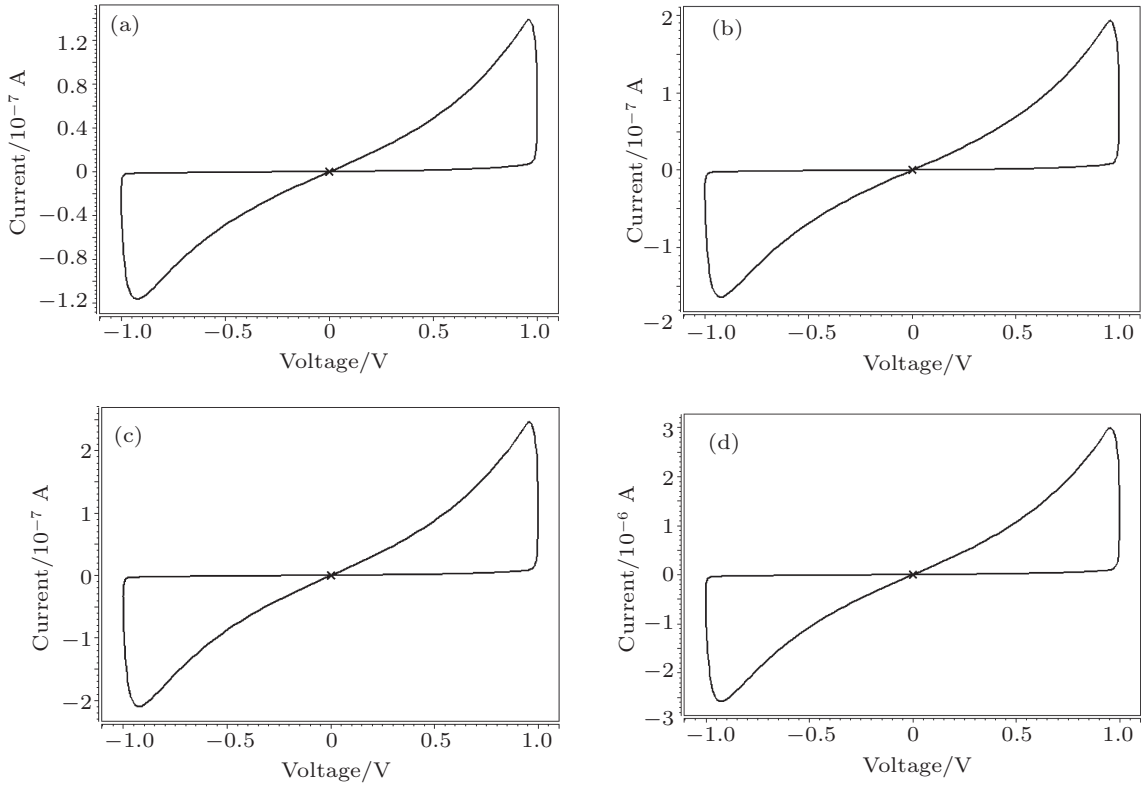


Fig. 6. The I–V curves of the memristor with different values of parameter b: $b = 5$ (a), $b = 7$ (b), $b = 9$ (c), and $b = 11$ (d).

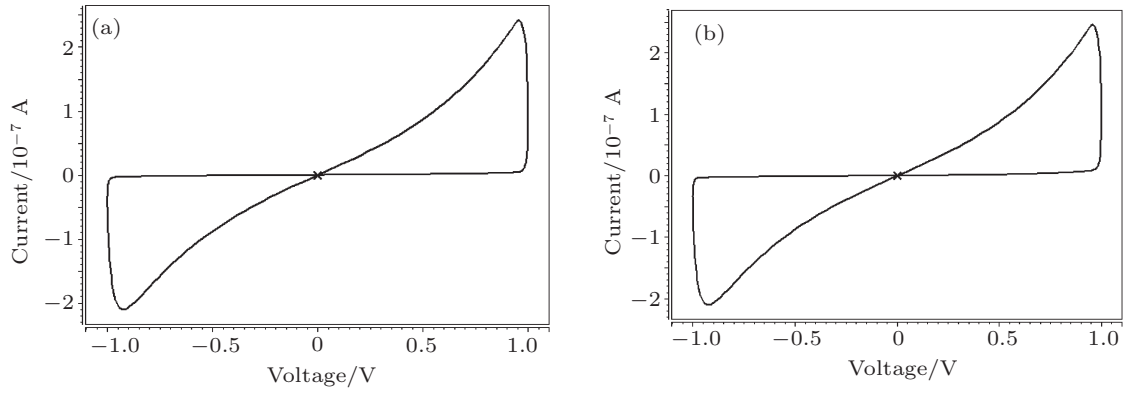
From Fig. 6, we can find that when the value of b is larger, the maximum current is also larger, which is consistent with Eq. (3). For example, when $b = 5$, 7, 9, 11, the positive maximum currents are 140 nA, 190 nA, 250 nA, and 300 nA, respectively, which reflects the change in resistances, as shown in Table 2.

Table 2. High and low resistances under different values of parameter b.

	$b = 5$	$b = 7$	$b = 9$	$b = 11$
$R_{\rm high}/{\rm M}\Omega$	245.5	213.9	189.4	170
$R_{\rm low}/{\rm M}\Omega$	8.5	6.1	4.7	3.9

4.3. The χ parameter

Now we examine the influence of the $c$value in the HSPICE code. The values of c are taken to be 0.001, 0.01, 0.1, and 1, and the I–V curves are shown in Figs. 7(a)–7(d), respectively.



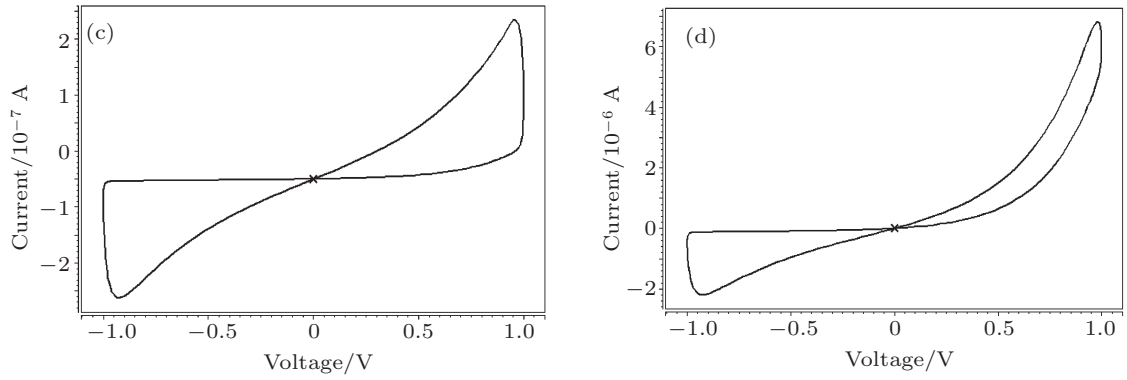


Fig. 7. The I–V curves of the memristor with different values of parameter c: $c = 0.001$ (a), $c = 0.01$ (b), $c = 0.1$ (c), and $c = 1$ (d).

As figure 7 shows, when the value of c is small, e.g. 0.001 and 0.01, there is little effect on the I–V curve; when the value of c is large, e.g. 0.1 and 1, there is an obvious deviation of the positive part of the I–V curve, or say, the high resistance part comes close to the low resistance part. We can explain this from Eq. (3): as c increases, the current expressed by the second part of Eq. (3) increases dramatically, which makes the memristor present low resistance. The resistances in Table 3 show the same trend in variance.

Table 3. High and low resistances under different values of parameter c.

	c=0.001	c=0.01	c=0.1	$c = 1$
$R_{\rm high}$/MΩ	336.7	189.4	35.3	3.9
$R_{\rm low}$/MΩ	4.8	4.7	4.3	2.1

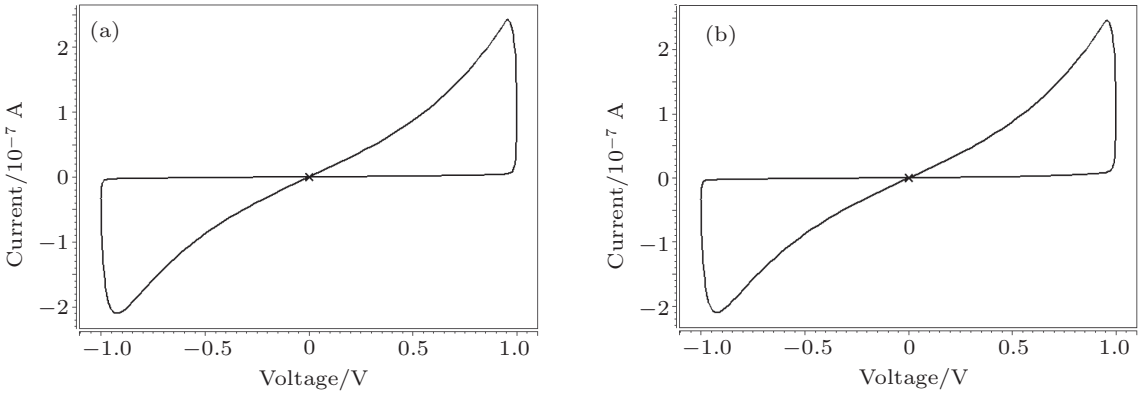
4.4. The γ parameter

Now we examine the influence of the h value in the HSPICE code. The values of h are taken to be 2, 4, 6, 8, 9, and 10, and the I–V curves are shown in Figs. 8(a)–8(f), respectively.

As shown in Fig. 8, parameter h mainly affects the positive part of the I–V curve. When the value of h is small, e.g. 2 and 4, there is little effect on the I–V curve; when the value of h is larger, e.g. 6, the slope of the high resistance part in the positive area increases, and the maximum current slightly increases; when the value of h is much larger, e.g. 8, 9 and 10, there is a large deviation in the positive high resistance part and the maximum current increases dramatically. We can explain this from Eq. (3): as h increases, the current expressed by the second part of Eq. (3) increases dramatically, which makes the memristor present low resistance. The effect of the γ parameter (also h) on the I–V curve is similar to that of the χ parameter (also c); however, as γ is at the exponent position, its effect is more dramatic. Table 4 shows the values of memristor resistance under different values of h.

Table 4. High and low resistances under different values of parameter h.

	$h = 2$	$h = 4$	$h = 6$	h=8	h=9	$h = 10$
$R_{\rm high}$/MΩ	315.0	189.4	65.6	15.7	7.3	3.4
$R_{\rm low}$/MΩ	4.8	4.7	4.5	3.7	2.9	2.4



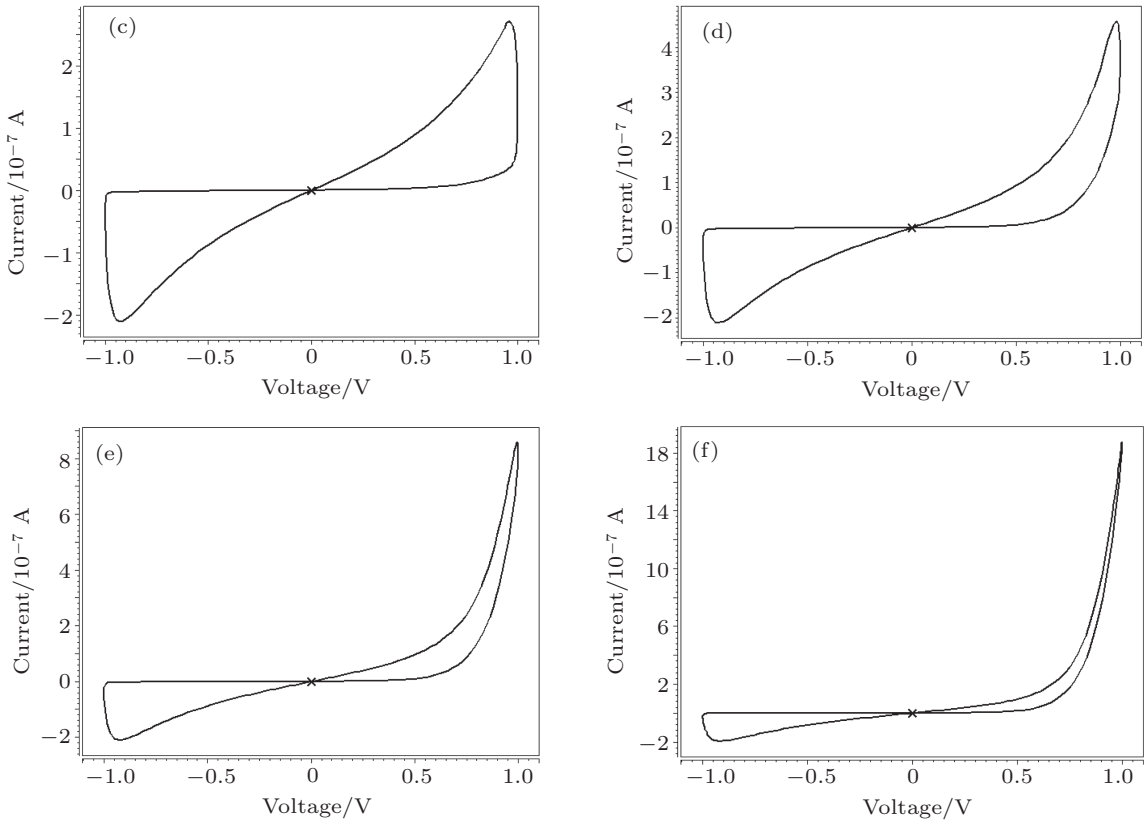
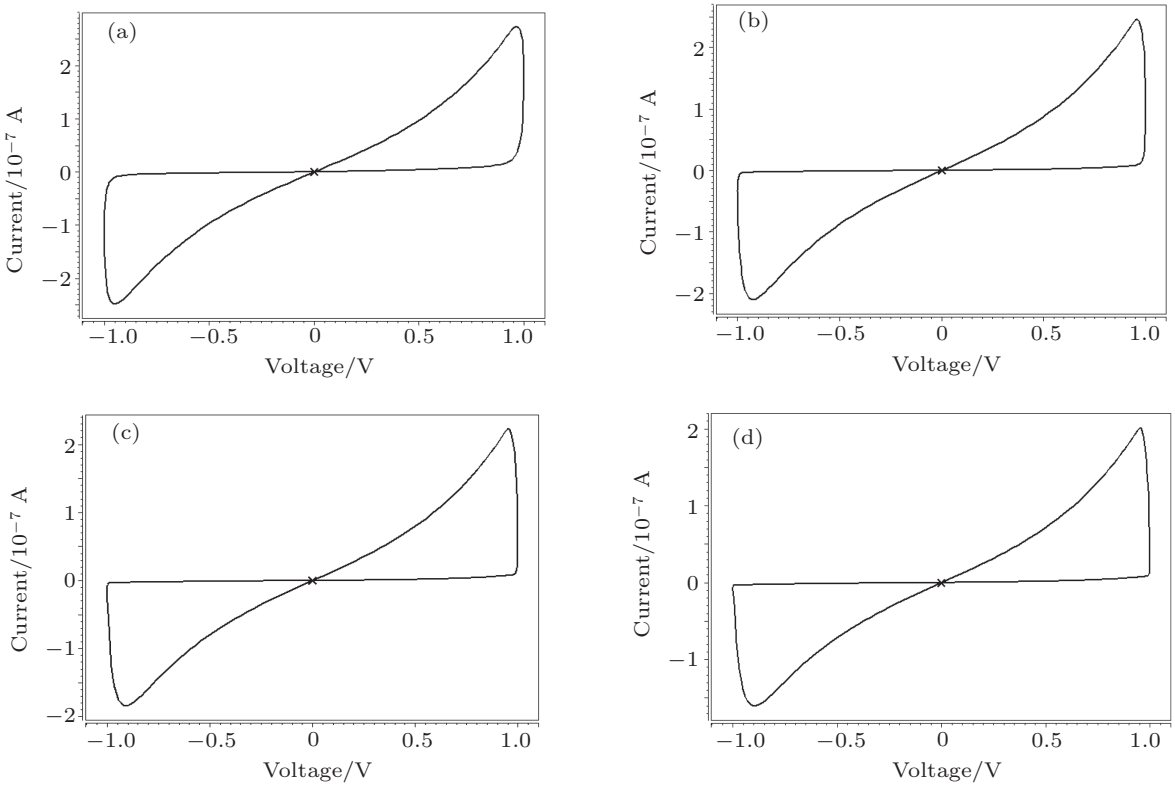


Fig. 8. The I–V curves of the memristor with different values of parameter h: (a) $h = 2$ (a), $h = 4$ (b), $h = 6$ (c), $h = 8$ (d), $h = 9$ (e), and $h = 10$ (f).

4.5. The n parameter

Now we examine the influence of the n value in the HSPICE code. The values of n are taken to be 2, 4, 6, 8, 10, and 12, and the I–V curves are shown in Figs. 9(a)–9(f), respectively.



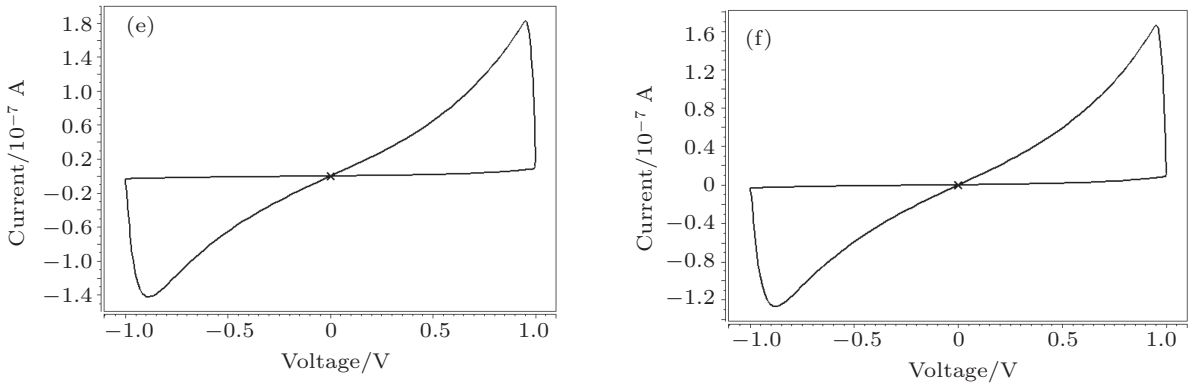


Fig. 9. The I–V curves of the memristor with different values of parameter n: $n = 2$ (a), $n = 4$ (b), $n = 6$ (c), $n = 8$ (d), $n = 10$ (e), and $n = 12$ (f).

From Fig. 9, we find that the influence of n on the I–V curve is much weaker than those of the parameters discussed above. It mainly affects the value of the maximum current as well as the sharpness of the transition from the high resistance part to the low resistance part. The bigger the value of n, the smaller the maximum current is and the sharper the transition of the curve. Table 5 shows high and low resistances under different values of parameter n.

Table 5. High and low resistances under different values of parameter n.

	$n = 2$	$n = 4$	$n = 6$	n=8	$n = 10$	$n = 12$
$R_{\rm high}$/MΩ	86.2	189.4	195.4	195.6	195.6	195.6
$R_{\rm low}$/MΩ	4.1	4.7	5.2	5.8	5.7	5.6

4.6. The q parameter

Now we examine the influence of the q value in Eq. (4). The values of q are taken to be 11, 13, 15, and 17, and the I–V curves are shown in Figs. 10(a)–10(d), respectively.

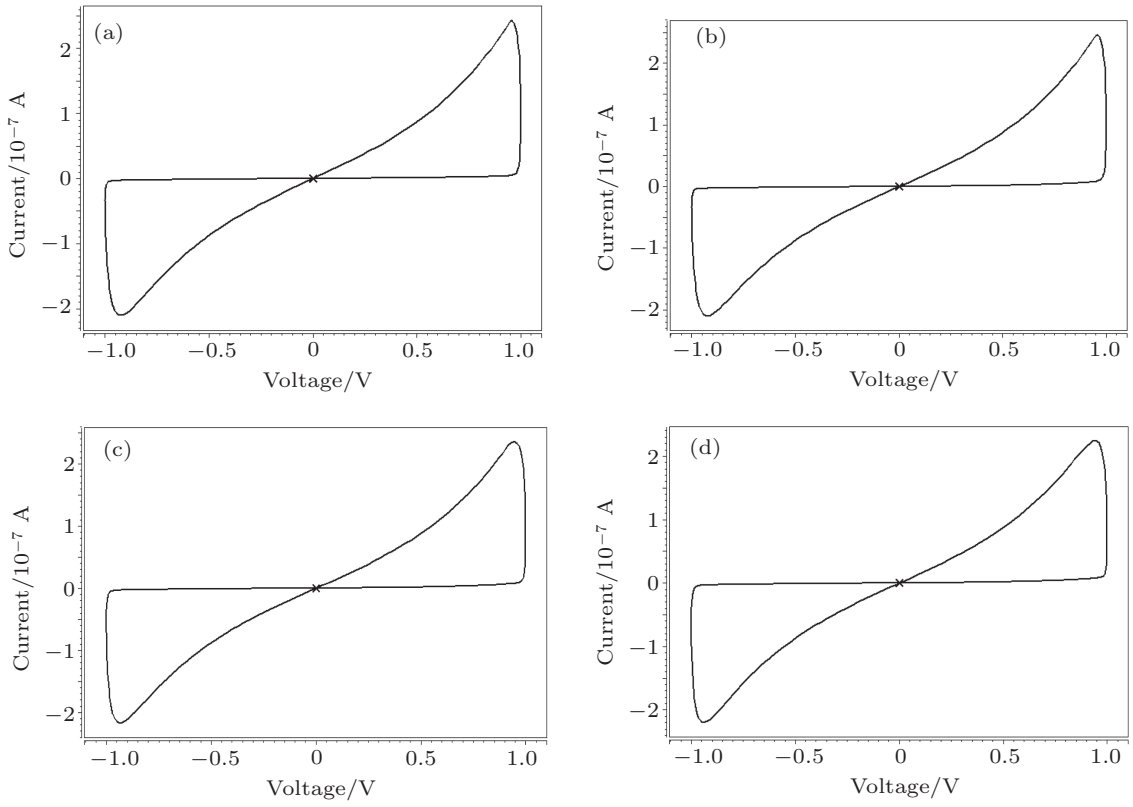


Fig. 10. The I–V curves of the memristor with different values of parameter q: $q = 11$ (a), $q = 13$ (b), $q = 15$ (c), and $q = 17$ (d).

Table 6. High and low resistances under different values of parameter q.

	$q = 11$	$q = 13$	$q = 15$	$q = 17$
$R_{\rm high}/{\rm M}\Omega$	188.5	189.4	189.9	190.2
$R_{\rm low}/{\rm M}\Omega$	3.9	4.7	4.6	4.6

As shown in Fig. 10, the q parameter mainly affects the value of the maximum current. As q increases, the maximum current of the positive part decreases while the maximum current of the negative part increases slightly. From Table 6, we find that the resistance of the memristors changes little with the variation in parameter q.

4.7. The *rratio* parameter

Now we examine the influence of the *rratio* parameter used in the HSPICE code. The values of *rratio* are taken to be 0.001, 0.01, 0.1, and 1, and the I–V curves are shown in Figs. 11(a)–11(d), respectively.

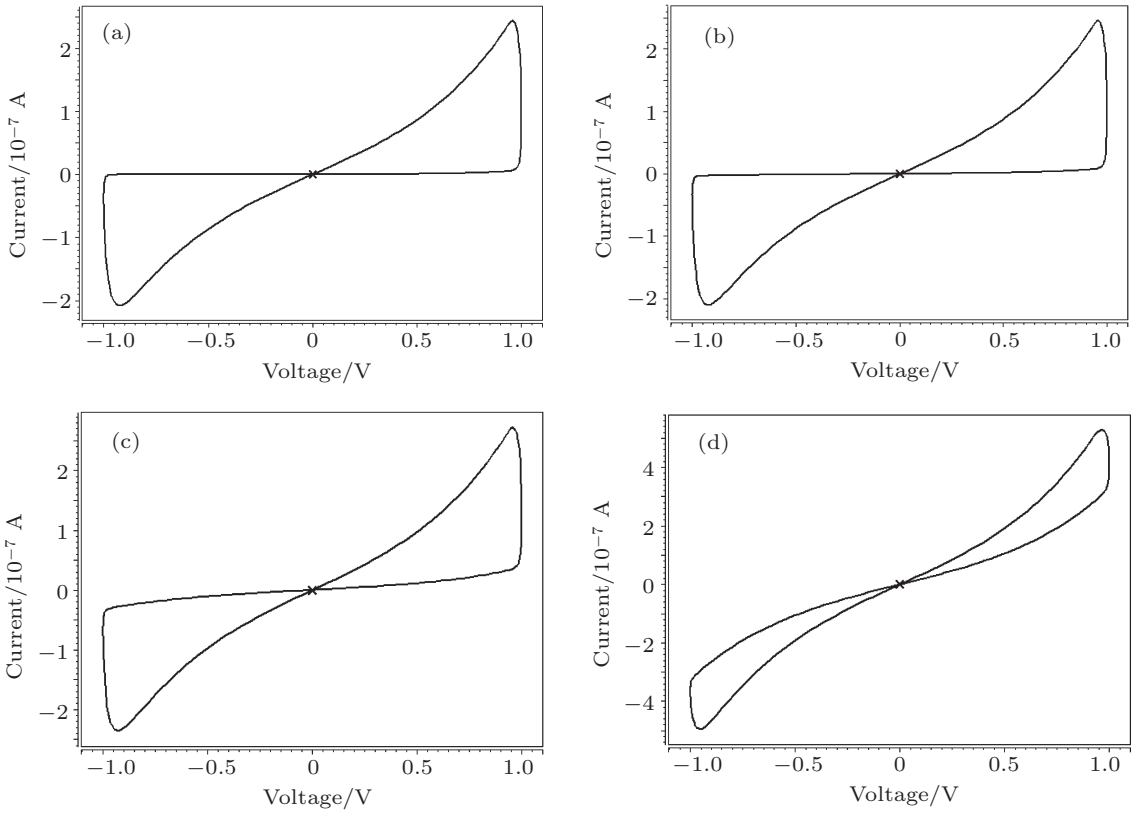


Fig. 11. The I–V curves of the memristor with different values of parameter *rratio*: *rratio*=0.001 (a), *rratio*=0.01 (b), *rratio*=0.1 (c), and *rratio*=1 (d).

When *rratio* is small, e.g. 0.001 and 0.01, there is little influence on the I–V curve; when *rratio* becomes larger, e.g. 0.1 and 1, the slope of the high resistance part of the I–V curve increases, that is, the resistance of the high resistance part decreases. From the HSPICE code, we can find that as *rratio* increases, the current through the memristor increases, which makes the resistance decrease as shown in Table 7.

Table 7. High and low resistances under different values of parameter *rratio*.

	rratio=0.001	*rratio*=0.01	*rratio*=0.1	*rratio*=1
$R_{\rm high}/{\rm M}\Omega$	334.8	189.4	35.5	3.9
$R_{\rm low}/{\rm M}\Omega$	4.8	4.7	4.3	2.2

5. Conclusions

From the experimental analyses in Section 4, we know that the memristor model from Ref. [12] presents different I–V characteristics, with different parameters adopted. With the above analyses, researchers will set appropriate parameters to satisfy the requirements for their research goals, which is also our future work.

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