Charge transport and shot noise on the surface of a topological insulator with a magnetic modulation*

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In this paper, we investigate the transport features and the Fano factor of Dirac electrons on the surface of a three-dimensional topological insulator with a magnetic modulation. We consider a hard wall bounding condition on the edge of the topological insulator, which implies that a surface state of the topological insulator is insulating. We find that a valley of conductivity at the Dirac point is associated with a Fano factor peak, and more interestingly, this topological metal changes from insulating to metallic by controlling the effective exchange field.

Keywords: topological insulator, electronic transport, ferrimagnetic

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1. Introduction

The time-reversal invariant topological insulator (TI) is a new state of matter, distinguished from a regular band insulator by a nontrivial topological invariant, which characterizes its band structure.[1] Its theoretical[2] and experimental[3] discovery has accordingly generated a great deal of excitement in the condensed matter physics community. The most robust observable consequence of a nontrivial topological character of these materials is the presence of gapless helical edge states, whose gaplessness is protected by time-reversal symmetry and is thus robust to perturbations that do not break this symmetry.[4] Surface-sensitive experiments such as angle-resolved photoemission spectroscopy (ARPES) and scanning tunneling microscopy (STM)[1,5] have confirmed the existence of this exotic surface metal, in its simplest form, which takes a single Dirac dispersion. The recent theoretical and experimental discovery of the two-dimensional (2D) quantum spin Hall system[6−13] and its generalization to the TI in three dimensions[14−16] have established the state of matter in time-reversal symmetric systems.

The concept of a TI dates back to the work of Kane and Mele, who focused on 2D systems.[6] There has been much recent interest in topological insulators (TIs), three-dimensional insulators with metallic surface states protected by time reversal invariance.[1−25] In particular, the surface of a three-dimensional (3D) TI called topological metal (TM), such as Bi_2Se_3 or Bi_2Te_3,[17,18] is a 2D metal whose band structure consists of an odd number of Dirac cones centered at time reversal-invariant momenta in the surface Brillouin zone.[19] This corresponds to the infinite mass Rashba model,[20] where only one of the spin-split bands exists. This has been beautifully demonstrated by spin- and angle-resolved photoemission spectroscopy.[21,22] The 3D TIs are expected to show several unique properties when the time reversal symmetry is broken.[23−25] This can be realized directly by a ferromagnetic insulating (FI) layer attached to the 3D TI surface. As we know, the motion of the Dirac electrons is influenced mainly by the magnetization of the FI layer rather than its stray field. One remarkable feature of the Dirac fermions is that the Zeeman field acts like a vector potential: the Dirac Hamiltonian is transformed as $\boldsymbol{\sigma}\cdot\boldsymbol{k}\rightarrow\boldsymbol{\sigma}\cdot(\boldsymbol{k}+\boldsymbol{H})$ by the Zeeman field $\boldsymbol{H}$.[26] This is in contrast to the Schrödinger electrons in conventional semiconductor heterostructures modulated by nanomagnets.[27,28]

Currently, most works focus on searching for TI materials and novel transport properties. To the best of our knowledge, the fabrication of such TI-based nanostructure is still a challenging task. Usually such structures are fabricated by utilizing the split gate and etching technique.[29] In this work, we theoret-

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ically study the transport properties and the Fano factor (the ratio of noise power and mean current[30]) of charges on the top of a topological surface with the width W, contacted by two electrodes at a distance L. As shown in Fig. 1, an FI is put on the top of the topological insulator to induce an exchange field via the magnetic proximity effect. The easy axis of an FI stripe is usually along its length direction and thus either in parallel P or antiparallel AP with the $+y$ axis. For a finite width of TIs, we consider a hard wall bounding condition (the infinite mass confinement) in the edge of TIs, which implies that the surface state of a three-dimensional TI is insulating. Recent experiments and theories have also reported a large-gap TI class with a single Dirac cone on the surface.[17] We find that a valley of the conductivity at the Dirac point, which deeply depends on the magnetization strength rather than the magnetization direction of adjacent FI stripes, is related to a peak of the Fano factor. Moreover, a quantum limited characteristic and quantum oscillations exist for charge transport and shot noise which can be tuned by controlling the effective exchange field M_0. We note that the oscillating period and oscillate amplitude (OA) of the conductivity (or the Fano factor) are inversely proportional to the ratio $\gamma = W/L$. In particular, when the ratio γ is large enough, quantum oscillations of the conductivity (or the Fano factor) disappear, which implies that the effect of effective exchange field on the conductivity (or Fano factor) can be ignored. The limiting behavior at the Dirac point is also $\sigma \to 2$, $F \to 1/3$ in the limit $\gamma \to \infty$. More interestingly, we note that this TM changes from insulating to metallic by controlling the effective exchange field of the FI stripes.

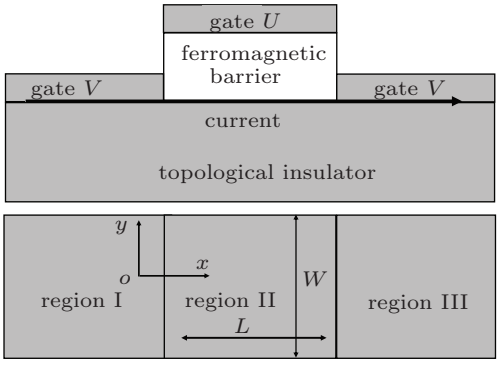


Fig. 1. Schematic diagram of the ferromagnetic barrier on the topological surface with finite width W contacted by two electrodes (leads) at a distance L. A low bias voltage source $\delta V \approx 0$ drives a current through the strip. Note that the V electrode potentials in two leads ($x > L$ and $x < 0$) and U electrostatic potential ($0 < x < L$) may be tuned by controlling the gate electrodes and a gate voltage, respectively. The potential barrier tuned by a gate voltage allows the carrier concentration in the strip to be tuned around the neutrality point.

The rest of this article is arranged as follows. In Section 2, we introduce the model and method for our calculation. In Section 3, the numerical analyses of our important issues are reported. Finally, a brief summary is given in Section 4.

2. Model and method

Now, let us consider a ferromagnetic barrier (FB) on the top of a topological surface with width W contacted by two electrodes at distance L. In order to theoretically investigate the Dirac electron confined in the domain $0 \leq y \leq W$, we introduce a mass term $m(y)$, which describes a hard wall bounding on the edge of the finite domain. We assume that the mass term $m(y)$ is zero in the domain, but rises to infinity at both edges. Thus we focus on charge transport at the Fermi level of the surface of the TIs, which is described by the 2D Dirac Hamiltonian as follows:

$$H = \upsilon_{\rm F}\boldsymbol{\sigma}\cdot\boldsymbol{P} + \boldsymbol{\tau}\cdot\boldsymbol{M} + m(y)\upsilon_{\rm F}^2\sigma_z, \qquad (1)$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are Pauli matrices, and $\boldsymbol{M} = M_y(x) = M_0$ is the effective exchange field. The ferromagnetism is induced due to the proximity effect by the ferromagnetic insulators deposited on the top, as shown in Fig. 1. In an actual experiment, one can use a magnet with very strong (soft) easy axis anisotropy to control the ferromagnetic. A low bias voltage source $\delta V \approx 0$ drives a current through the strip. In our model, we describe the behavior of an electron on the surface of TI with the width W by the Dirac equation in this FB region, where a gate potential $U_{\rm g}$ is applied. Thus this potential barrier tuned by a gate voltage allows the carrier concentration in the strip to be tuned around the neutrality point. Because of the translational invariance of the system along the y direction, the equation $H\Psi(x,y) = E\Psi(x,y)$ admits solutions of the form $\Psi(x,y) = (\Psi_1(x), \Psi_2(x))^{\rm T}\exp({\rm i}k_y y)$. We choose the zero energy such that the Fermi level is at $E = 0$. The electrostatic potential $U(x) = U_{\rm gate}$ for $0 < x < L$, varied by a gate voltage, determines the concentration of the carriers in the strip. The value $U_{\rm gate} = 0$ corresponds to charge neutrality, being the point where the electron and hole excitations are degenerate (known as the Dirac point). We model the electrodes by taking a large value $V(x) = V_\infty$ in the leads $x < 0$ and $x > L$. Thus the transmission probability can be written as: $T = q_x^2/(q_x^2\cos^2(q_x L) + U_{\rm g}^2 * \sin^2(q_x L))$. As shown

by Berry *et al.*,[31] the hard wall bounding (the infinite mass confinement) corresponds to boundary condition: $\Psi_2(x,y)/\Psi_1(x,y) = \mathrm{i}\exp(\mathrm{i}\lambda)$ where $\lambda = 0$ at $y = W$ and $\lambda = \pi$ at $y = 0$. As a result of this boundary condition, the transversal momenta are quantized as $k_{n,y} = (n + 1/2)\pi/W$ and $n = 0, \pm1, \pm2, \ldots$. The ballistic conductance G and Fano factor F at zero temperature can be expressed by summing over the modes[32–34]

$$G = g_0 \sum_{n=-N_{\max}}^{N_{\max}} T_n, \quad F = \frac{\sum_{n=-N_{\max}}^{N_{\max}} T_n(1-T_n)}{\sum_{n=-N_{\max}}^{N_{\max}} T_n}, \tag{2}$$

where $g_0 = e^2/h$,[34] and $N_{\max}$ is the maximum of propagating modes in the leads. Here, in our calculations we apply a large electrode potential V_{g} that is larger than the electrostatic potential U_{g}, so the integer $N_{\max}$ is given by $N_{\max} = \mathrm{Int}\,(k_\infty W/\pi + 1/2) - 1$ where $eV_\infty = \hbar v_{\mathrm{F}} k_\infty$.

3. Results and discussions

For convenience, we express all quantities in dimensionless units by means of the length of the basic unit a and the energy $E_0 = \hbar v_{\mathrm{F}}/L$. For a typical value of $L = 50$ nm and the Bi_2Se_3 material $v_{\mathrm{F}} = 5 \times 10^5$ m/s, one has $E_0 = 6.6$ meV. We define the ratio γ with the form $\gamma = W/L$. A typical value of induced exchange field due to the magnetic proximity effect would be 5 meV–50 meV,[35] although this depends on the interface property and the material choice of the ferromagnet. Therefore the M_0 would be chosen from -10 to 10. Next, we will study the conductivity $\sigma \equiv G \times L/W$ and Fano factor F of charges penetrating through a surface of a TI with a finite width W under the modulation of an exchange field provided by ferromagnetic insulating FI stripes.

In Fig. 2, we show conductivity (a) and Fano factor (b) as a function of the ratio γ in four different effective exchange fields $M_0 = 0$, 0.5, 2.0, 5.0 for the fixed gate voltage $U_{\mathrm{g}} = 0$, respectively. Thus, the transmission probability here has a simple form as: $T_n = 1/\cosh^2[(n + 1/2)\pi/\gamma + M_0]$. Note that the effective exchange field M_0 can be neglected in two limiting cases ($\gamma \to 0$ and $\gamma \to \infty$). The limiting behavior for $\gamma \to 0$ at the Dirac point is $\sigma \to 0$, $F \to 1$, but the limiting behavior at the Dirac point is $\sigma \to 2$, $F \to 1/3$ in the opposite limit $\gamma \to \infty$. The reasons for this are the two points: (i) the valley degeneracy is not broken for the lowest mode ($n = 0$) as we take into account the hard well bounding condition for $\gamma \to 0$; (ii) the sum of these discrete wave modes of transmission probability contributing to the conductivity (or the Fano factor) in the limit $\gamma \to \infty$ can change into a definite integral, where the effective exchange field M_0 can be removed using a mathematical replacement. More interestingly, we find that the conductivity and the Fano factor depend deeply on the effective exchange field M_0 under the finite width ($\gamma \neq 0$ and ∞). We can see that the oscillating amplitude (OA) of the conductivity and the Fano factor eventually collapses by increasing the ratio γ in the case of $M_0 \neq 0$, and even disappears as the ratio is $\gamma \sim 4$–5. The larger the effective exchange field M_0, the more obvious the characteristic of OA. Nevertheless, regarding $M_0 = 0$, there is no such collapse for the OA of the conductivity (or the Fano factor). For $M_0 = 0$, we note that the conductivity increases monotonously but the Fano factor decreases monotonously with increasing γ ratio. In order to open out the reason for the OA characteristic, we find that the peak for the conductivity (the valley for the Fano factor) corresponding to a certain γ must satisfy the relation $m_0 - \pi/2\gamma \equiv m$, where m is an integer. If the relation is satisfied, the valley degeneracy is broken for the lowest mode ($n = 0$), while all higher modes ($n = 1, 2, \ldots$) retain the twofold valley degeneracy. In other words, the transversal momenta here are quantized as $k_{n,y} = n\pi/W$ and $n = 0, \pm1, \pm2, \ldots$. Thus the presence of the effective exchange field can change the strip of TM from insulating ($M_0 = 0$) to metallic. On second thoughts, we find that there is a general form of the transversal momenta in the presence of the effective exchange field M_0, satisfying the relation $k_{n,y} = (n + \alpha)\pi/W$, where n is an integer and α belongs to the domain from 0 to 1/2. Thus we find that the value α appears continuously and repeats from 0 to 1/2 for the fixed effective exchange field M_0 with increasing γ ratio. When the $\alpha = 0$ value appears, a peak of the conductivity (or a valley for the Fano factor) exists. However, when the value $\alpha = 1/2$ ($M_0 = 0$) appears, a valley of the conductivity (a peak for the Fano factor) exists [see the two solid lines in Fig. 2(a)]. We find that the conductivity increases monotonously but the Fano factor decreases monotonously with increasing γ ratio for $\alpha = 1/2$ ($M_0 = 0$). Nevertheless, the conductivity decreases monotonously but the Fano factor increases monotonously with increasing γ ratio for $\alpha = 0$. These reasons should be noted so that we easily understand why OA appears and eventually collapses with an increase in γ.

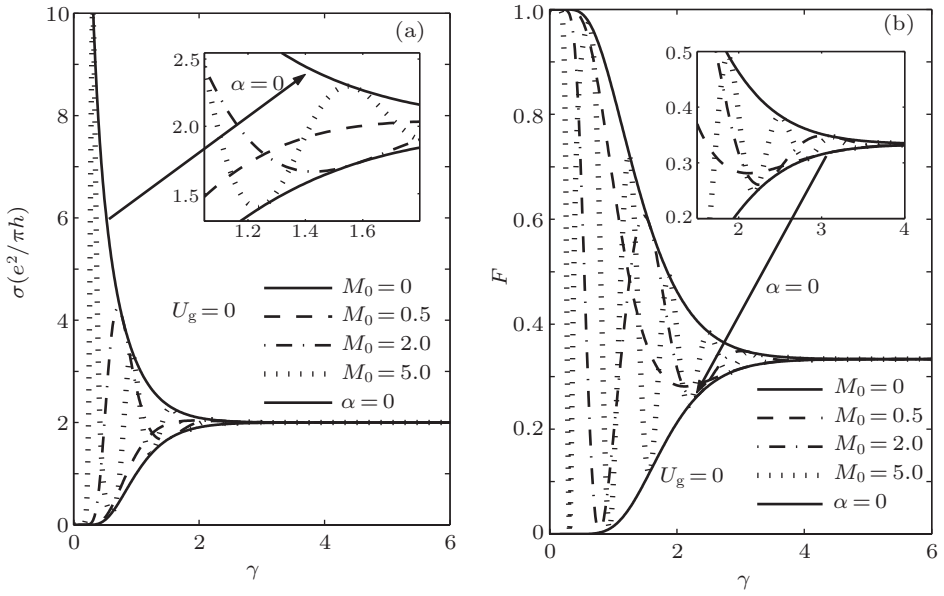


Fig. 2. Conductivity (a) and Fano factor (b) as a function of the ratio γ in four different effective exchange fields, M_0, for the fixed gate voltage $U_g = 0$, respectively. The inset in panels (a) and (b) are magnifications of the quantum oscillation regions of the conductivity and Fano factor, respectively. The arrows corresponding to the two solid lines denote the $\alpha = 0$.

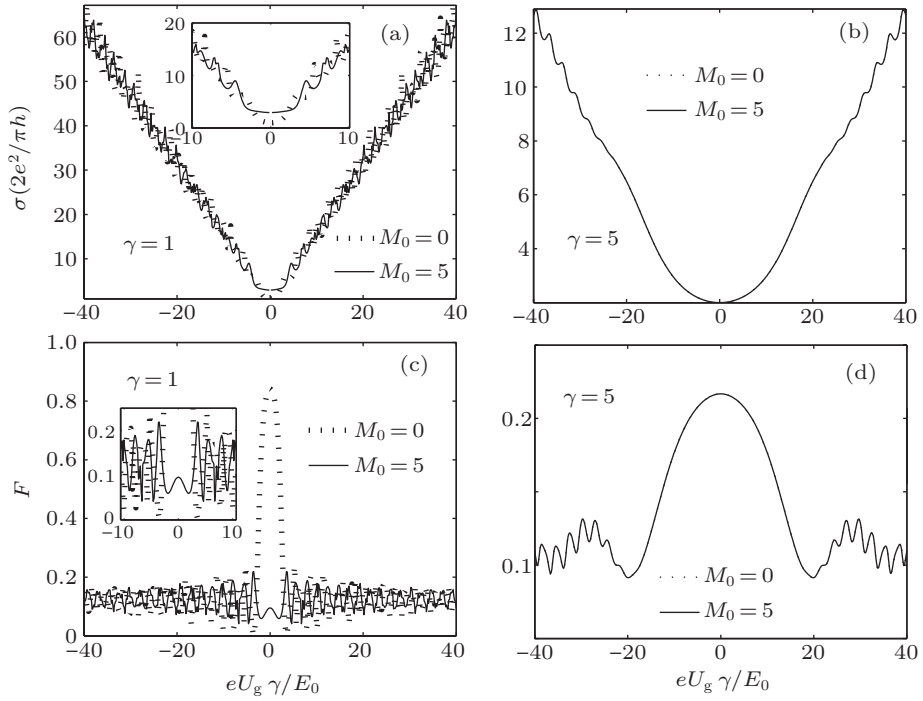


Fig. 3. Gate voltage dependence of the conductivity [panels 3(a) and 3(b)] and the Fano factor [panels 3(c) and 3(d)] for two different effective exchange fields, M_0 ($M_0 = 0$ and 5), respectively. In the left panel we take the $\gamma = 1$ ratio and in the right panel we take the $\gamma = 5$ ratio. The inset in panels 3(a) and 3(c) are magnifications of the quantum oscillation regions of conductivity and Fano factor, respectively.

In order to observe the gate voltage U_g dependence of the conductivity and the Fano factor, we show in Fig. 3 the conductivity [panels 3(a) and 3(b)] and Fano factor [panels 3(c) and 3(d)] as a function of gate voltage, U_g, for two different effective exchange fields, M_0 ($M_0 = 0$ and 5), respectively. In the left panel we take the $\gamma = 1$ ratio and in the right panel we take the $\gamma = 5$ ratio. We find that the conductivity and the Fano factor show a series of evenly spaced sharp spikes. It is clearly seen that the minimum conduc-

tivity at the Dirac point in the absence of the effective exchange field M_0 is in relation to a maximum of the Fano factor. The effective exchange field M_0 can enhance the minimum conductivity, but can suppress the maximum of the Fano factor. But when the value of γ is large enough, such as $\gamma = 5$, the effective exchange field M_0 can be ignored. Here we find that there exists a quantum limited characteristic for charge transport and shot noise, namely, $\sigma \to 2$ and $F \to 1/3$. More interestingly to us, as the gate voltage arrives at a certain value, quantum oscillations occur in the calculation of the conductivity and the Fano factor. The observation of quantum conductance oscillations in mesoscopic systems has traditionally required the confinement of the carriers to a phase space of reduced dimensionality.[36] Factually, electrons, which are injected from one lead to another across the surface of TI, are inevitably accompanied by dissipation and noise at the contacts. This dissipation result inside the leads also originates from stochastic processes of relaxation in the leads near the contacts.[37,38] This also explains why the oscillations appear in Fig. 3.

In Fig. 4, we show the effective exchange field M_0 dependence of the conductivity (a) and the Fano factor (b) for the fixed gate voltage $U_g = 0$ in the four different γ ratios ($\gamma = 1$, 2, 5, and ∞), respectively. We find that the conductivity and the Fano factor have a fixed oscillation period. The reason for this is that the effective exchange field M_0 can change the transversal momenta, which are quantized when we take into account the hard well bounding condition. As interpreted in Fig. 2, a general form of the transversal momenta in the presence of the effective exchange field M_0 satisfies the relation $k_{n,y} = (n+\alpha)\pi/W$, where n is an integer and α belongs to the domain from 0 to 1/2. The α value appears periodically with an increase in the effective exchange field M_0. When the value $\alpha = 0$ appears, which corresponds to $M_0\gamma/\pi = (2m+1)/2$ where m is an integer, there exists a peak of the conductivity (a valley for the Fano factor). However, when the $\alpha = 1/2$ value appears, which corresponds to $M_0\gamma/\pi = m$ where m is an integer, a valley of conductivity (a peak for the Fano factor) exists. Thus we find that the oscillating period of the conductivity (or the Fano factor) becomes equal to $\triangle M_0 = \pi/2\gamma$. Note that the OA of the conductivity decreases with increasing γ ratio. When the γ ratio is large enough, the OA of the conductivity and the Fano factor disappear. Thus the effect of effective exchange field on the conductivity and the Fano factor can be ignored in the limit of $\gamma \to \infty$. That is to say, the oscillating period and OA of the conductivity and the Fano factor are inversely proportional to the γ ratio. More interestingly, the limiting behavior at the Dirac point is $\sigma \to 2$, $F \to 1/3$ in the limit $\gamma \to \infty$. These results are well in accordance with those in Figs. 2 and 3.

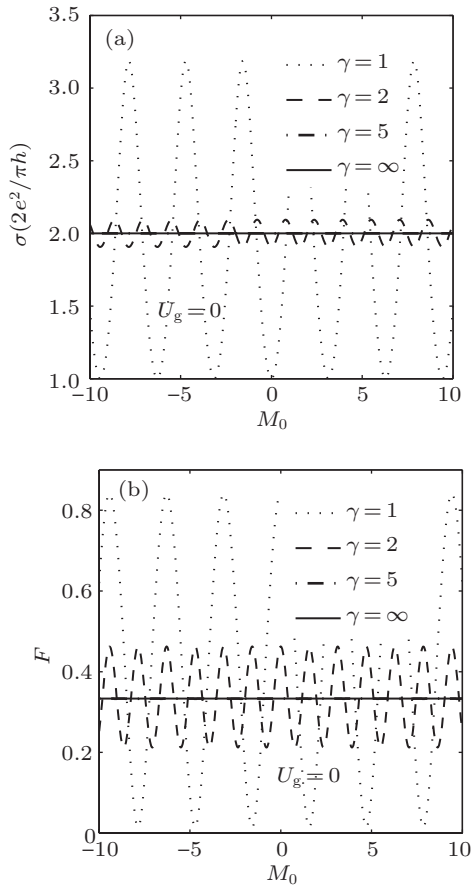


Fig. 4. The effective exchange field M_0 dependence of the conductivity (a) and Fano factor (b) for the fixed gate voltage $U_g = 0$ in the four different γ ratios ($\gamma = 1$, 2, 5, and ∞), respectively.

4. Conclusion

In summary, we theoretically investigated the transport features of Dirac electrons on the surface of a three-dimensional TI with a finite width W under the modulation of an exchange field provided by FI stripes. We consider a hard wall bounding condition (the infinite mass confinement) on the edge of the TI, which implies that the surface state of a three-dimensional TI is insulating. We also find that the valley conductivity at the Dirac point, which changes with increasing magnetization strength, corresponds to a maximum of the Fano factor. A quantum limited characteristic exists, as well as quantum oscillations, with a period $\triangle M_0 = \pi/2\gamma$ of charge transport, and shot noise appears in a finite width of the surface of a three-dimensional topological insulator TI. The oscillating period and the OA of the conductivity and

the Fano factor are inversely proportional to the γ ratio. In particular, when the γ ratio is large enough, the OA of the conductivity and the Fano factor disappear, which implies that the effect of the effective exchange field on the conductivity and the Fano factor can be ignored. The limiting behavior at the Dirac point is $\sigma \to 2$, $F \to 1/3$ in the limit $\gamma \to \infty$. More interestingly, we note that this TM changes from insulating to metallic by controlling the effective exchange field of the FI stripes.

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