The effects of relativistic broadening and frequency down-shift on electron–cyclotron emission measurements in EAST*

Liu Yong(刘 永)†, Han Xiang(韩 翔), Ti Ang(提 昂), Wang Yu-Min(王嵎民), Ling Bi-Li(凌必利), Hu Li-Qun(胡立群), and Gao Xiang(高 翔)

Institute of Plasma Physics, Chinese Academy of Sciences, Hefei 230031, *China*

(Received 24 June 2011; revised manuscript received 16 September 2011)

This paper presents a theoretical calculation of the effects of relativistic broadening and frequency down-shift on the electron cyclotron emission measurements for a wide range of plasma parameters in the Experimental Advanced Superconducting Tokamak (EAST). The calculation is based on the radiation transfer equation, with the reabsorption and reemission processes taken into account. The broadening effect contributes to the radial resolution of the measurement, and the calculation results indicate that it is ~ 2 cm in the case of the central electron temperature 10 keV. A pseudo radial displacement of the obtained electron temperature profile occurs if the relativistic frequency down-shift effect is not taken into account in the determination of the emission layer position. The shift could be a few centimeters as the electron temperature increases, and this effect should be taken into account.

Keywords: electron–cyclotron emission, relativistic effect, experimental advanced superconducting tokamak

PACS: 52.25.Os, 52.70.Gw **DOI:** 10.1088/1674-1056/21/4/045201

1. Introduction

Electron–cyclotron emission (ECE) measurements have been a powerful diagnostic tool in magnetic-confined plasmas since the 1960s.[1] It can provide important information on the electron temperature profile with a fairly good spatial and temporal resolution.[2−6] Since ECE is radiated from the plasma continuously, it is an ideal tool for MHD[7] and electron heat transport research.[8,9] Generally, the second harmonic extraordinary mode (X-mode) ECE is of physics interest, and the radiance is proportional to $T_{\rm e}(1-{\rm e}^{-\tau})$, where $T_{\rm e}$ and τ are the electron temperature and the optical depth, respectively. In tokamaks, the toroidal magnetic field B is inversely proportional to the radial position R, hence there is one-to-one mapping between the ECE frequency ω_n and the radial position,

$$\omega_{\rm n} = n\frac{e}{m_{e0}}\frac{B_0R_0}{R}\frac{\sqrt{1-\beta^2}}{1-\beta_{\parallel}\cos\theta},$$

where n is the harmonics of the emission, θ is the angle of propagation of the wave phase surface with respect to the magnetic field. The term $1-\beta_{\parallel}\cos\theta$ represents the Doppler effect, and it broadens the ECE spectra. The term $\sqrt{1-\beta^2}$ is attributed to the relativistic effect, which broadens and down-shifts the ECE spectra. Normally the radial position for a particular ECE frequency is determined from the non-relativistic ECE frequency ($n\frac{e}{m_{e0}}\frac{B_0R_0}{R}$) in the ECE measurements. Given the relativistic frequency down-shift effect, the actual position of a particular frequency is different from the one deduced from the non-relativistic ECE frequency. Hence this will lead to a pseudo radial displacement of the obtained electron temperature profile if the radial position is deduced from the non-relativistic ECE frequency. There is no doubt that the relativistic effect should be taken into account in the ECE measurements in a reactor-relevant high temperature plasma. Recently, it was pointed out that this effect is not negligible even in medium temperature plasma.[10,11]

The Experimental Advanced Superconducting Tokamak (EAST)[12] is a full superconducting tokamak with a non-circular cross-section vacuum vessel. The scientific and engineering missions of the EAST project have been defined to study physics issues of the advanced steady-state tokamak operation and to

*Project supported by the Knowledge Innovation Program of the Chinese Academy of Sciences (Grant No. Y05FCQ0125) and the National Magnetic Confinement Fusion Science Program of China (Grant No. 2011GB107001).

†Corresponding author. E-mail: liuyong@ipp.ac.cn

© 2012 Chinese Physical Society and IOP Publishing Ltd

establish the technological basis of full superconducting tokamaks. The main parameters of EAST is listed in Table 1. The electron temperature is ~ 1 keV in the Ohmic phase, and it can reach the order of 10 keV once the auxiliary heating is achieved. In this paper, we investigate the relativistic broadening and frequency down-shift effects on the ECE measurements for a wide range of plasma parameters in EAST.

Table 1. Main parameters of EAST.

Parameters	Value and units
Toroidal field B_0	3.5 T
Plasma current I_p	1 MA
Major radius R_0	1.7 m
Minor radius a	0.4 m
Elongation K_x	1.6–2
Triangularity δ_x	0.6–0.8
Heating and driving	
ICRH	3 MW
LHCD	3.5 MW
ECRH	0.5 MW

2. Method of calculation

The emission and the absorption processes in a plasma are described by the following radiation transfer equation[13]

$$N_\mathrm{r}(\omega,s)^2\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{I(\omega,s)}{N_\mathrm{r}(\omega,s)^2}\right) = j(\omega,s)-\alpha(\omega,s)I(\omega,s), \quad (1)$$

where N_r is the ray refractive index, ω is the angular frequency of the wave, s is the distance along the ray trajectory, j and I are respectively the emissivity and the radiance, and α is the absorption coefficient. The solution of Eq. (1) is

$$I(\omega,s)=I_0\,\mathrm{e}^{-\tau(s)}+\int\frac{j(\omega,s)}{N_\mathrm{r}(\omega,s)^2}\,\mathrm{e}^{-\tau(s)}\,\mathrm{d}s, \quad (2)$$

where I_0 is the intensity of the incident radiation, and the first term is negligible in the case of the ECE measurements, $\tau(s)=\int\alpha(\omega,s)\mathrm{d}s$ is the optical depth. When the plasma is in thermodynamic equilibrium, we have

$$\frac{j(\omega,s)}{N_\mathrm{r}(\omega,s)^2}=\frac{\omega^2}{8\pi^3c^2}kT_\mathrm{e}(s)\alpha(\omega,s)$$

for $\hbar\omega \ll kT_\mathrm{e}(s)$ (this condition is satisfied in the ECE measurement) according to Kirchoff's law. By definition of the emissivity function $G(s)=T_\mathrm{e}(s)\alpha(\omega,s)\,\mathrm{e}^{-\tau(s)}$, we obtain

$$I(\omega,s)=\frac{\omega^2}{8\pi^3c^2}k\int G(s)\mathrm{d}s. \quad (3)$$

The emissivity function describes the profile of the emission layer for a particular frequency. In our calculation, the equations for the absorption coefficient from Refs. [14] and [15] are employed. The equations for the X-mode wave are given as follows:

$$\alpha_n^{(\mathrm{X})}=\left(\frac{\omega_\mathrm{p}}{\omega}\right)^2\frac{\omega}{c}\frac{1}{N_\perp^{(\mathrm{X})}}\left(\frac{c}{v_\mathrm{t}}\right)^2P_n^{(\mathrm{X})}(Z_n), \quad (4)$$

$$\begin{aligned}P_n^{(\mathrm{X})}(Z_n)&=\frac{n^2}{2^n n!}\left(nN_\perp^{(\mathrm{X})}\frac{v_\mathrm{t}}{c}\right)^{2(n-1)}\\&\quad\times(1+A)^2\sum_{k=0}^{\infty}C_{n,k}D_{n,k}\left(nN_\perp^{(\mathrm{X})}\frac{v_\mathrm{t}}{c}\right)^{2k}\\&\quad\times\left[-G''_{n+k+3/2}(Z_n)\right],\quad n\geq 2,\\C_{n,k}&\equiv\frac{(-1)^k n![2(n+k)!]}{2^k k!(n+k)!(2n+k)!},\\D_{n,k}&\equiv 1+\frac{2k}{n}\left[\frac{1}{1+A}+\frac{(k-1)(n+k)}{n(2n+2k-1)}\frac{1}{(1+A)^2}\right],\\G''_q(Z_n)&\equiv\left(\frac{\omega}{n\omega_\mathrm{c}}\right)^{2q-5}\left[\frac{1}{2}\left(\frac{n\omega_\mathrm{c}}{\omega}+1\right)\right]^{q-1}F''_q(Z_n),\\F''_q(Z_n)&\equiv-\frac{\pi}{\Gamma(q)}Z_n^{q-1}\exp(-Z_n),\\Z_n&\equiv\left(\frac{c}{v_\mathrm{t}}\right)^2\frac{n\omega_\mathrm{c}-\omega}{\omega},\end{aligned}$$

where ω_p is the plasma frequency; $N_\perp=N\sin\theta$, N is the refractive index; θ is the angle between the wave vector and the magnetic field; $v_\mathrm{t}=\sqrt{T_\mathrm{e}/m_\mathrm{e}}$ is the electron thermal velocity.

3. Results and discussion

For all the calculations in this paper, the electron temperature profile is taken to be

$$T_\mathrm{e}=(T_{\mathrm{e}0}-T_\mathrm{ea})\left(1-\left(\frac{R-R_0}{a}\right)^2\right)^2+T_\mathrm{ea},$$

where $T_{\mathrm{e}0}$ and $T_\mathrm{ea}=0.1T_{\mathrm{e}0}$ are the electron temperatures at the plasma center and edge, R_0 and a are 1.85 m and 0.4 m, respectively. The electron density profile is supposed to be uniform. The relation between the toroidal magnetic field and the toroidal coils current is

$$B[\mathrm{T}]=4.16\times10^{-4}\frac{It[\mathrm{A}]}{R[\mathrm{m}]},$$

which is derived from the results of the field simulation.[16] Figure 1 illustrates the relativistic broadening and frequency down-shift effects for a single 2nd harmonic X-mode frequency of 124 GHz in the case of $T_{\mathrm{e}0}=10$ keV, $n_\mathrm{e}=3\times10^{19}$ m^{-3}, and

$It = 10000$ A (2.25 T@1.85 m). The dashed line indicates the position deduced from the non-relativistic ECE frequency. As Fig. 1(a) shows, the width of the absorption region is quite large due to the relativistic broadening effect, and it is ~ 15 cm in this case. The optical depth is the integral of absorption coefficient along the ray path and reaches a value in this case, ~ 68, as shown in Fig. 1(b). Taking into account the reabsorption and reemission of the radiation, the actual emission layer is determined by the emissivity function $G(s)$ shown in Fig. 1(c). To obtain the position and the width of the emission layer, the emissivity function is integrated. The width of the emission layer due to relativistic broadening is defined as the distance between 5% and 95% of the emission level, and the center of the emission is determined by the 50% radius. In this case, the width of the emission layer is ~ 2.4 cm, and the actual position shifts to the high-field side (HFS) by ~ 2 cm.

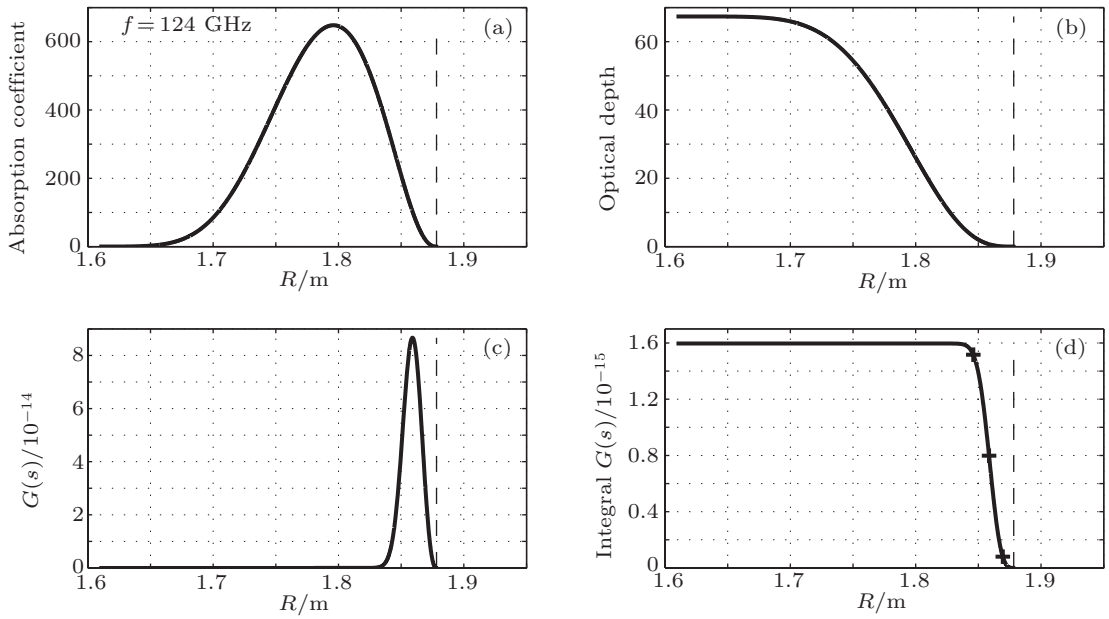


Fig. 1. Profiles of (a) absorption coefficient, (b) optical depth, and (c) emissivity function for a single 2nd harmonic X-mode frequency of 124 GHz in EAST in the case of $T_{e0} = 10$ keV, $n_e = 3 \times 10^{19}$ m^{-3}, and $It = 10000$ A. In panel (d) the emission width is determined as the distance between 5% and 95% emission levels, and the center of the emission is demarked by the 50% radius.

Figure 2 summarizes the calculated emission layer widths due to the relativistic broadening effect at different electron temperatures and electron densities. For the case of the central electron temperature $T_{e0} = 1$ keV, the width of the emission layer at the plasma core decreases from ~ 1.4 cm to ~ 0.4 cm as the electron density goes up from 1×10^{19} m^{-3} to 5×10^{19} m^{-3}. The contribution to the radial resolution of the ECE measurements is similar to the value attributed to the Doppler broadening effect and the spectral resolution of the measurement system. The emission layer width increases as the electron temperature increases. It is $\sim$1.8 cm for the case $T_{e0} = 10$ keV, $n_e = 5 \times 10^{19}$ m^{-3}. In this case, the relativistic broadening effect dominates the effects which contribute to the radial resolution of the ECE measurements.

Figure 3 illustrates the difference between the emission layer position obtained from the non-relativistic frequency and the actual position at different electron temperature and electron density. Normally, this shift is negligible for low electron temperature and high electron density conditions. We calculated the shift for a wide range of plasma parameters, $T_{e0} = 1, 2, 5, 10$ keV, $n_e = (1, 2, 3, 4, 5) \times 10^{19}$ m^{-3}, $It = 10000$ A (2.25 T@1.85 m). The shift of the emission layer at the plasma core is ~ 0.7 cm for the case $T_{e0} = 1$ keV, $n_e = 1 \times 10^{19}$ m^{-3}; and it decreases by a factor of around 2 when the electron density increases by 5 times. In the case of $T_{e0} = 10$ keV, the shift ranges ~ 3 cm to ~ 1.6 cm for $n_e = 1 \times 10^{19}$ m^{-3} and $n_e = 5 \times 10^{19}$ m^{-3}. Figure 4 shows the electron temperature profile assumed (solid line) and obtained from the non-relativistic ECE frequency in the case of $It = 10000$ A (2.25 T@1.85 m), $T_{e0} = 10$ keV and $n_e = (1, 2, 3, 4, 5) \times 10^{19}$ m^{-3} (dashed line). The pseudo electron temperature profile shifts to the low-field side (LFS) by a few centimeters, and this effect will lead to an overestimation in the LFS and underestimation in the HFS by a few percentage.

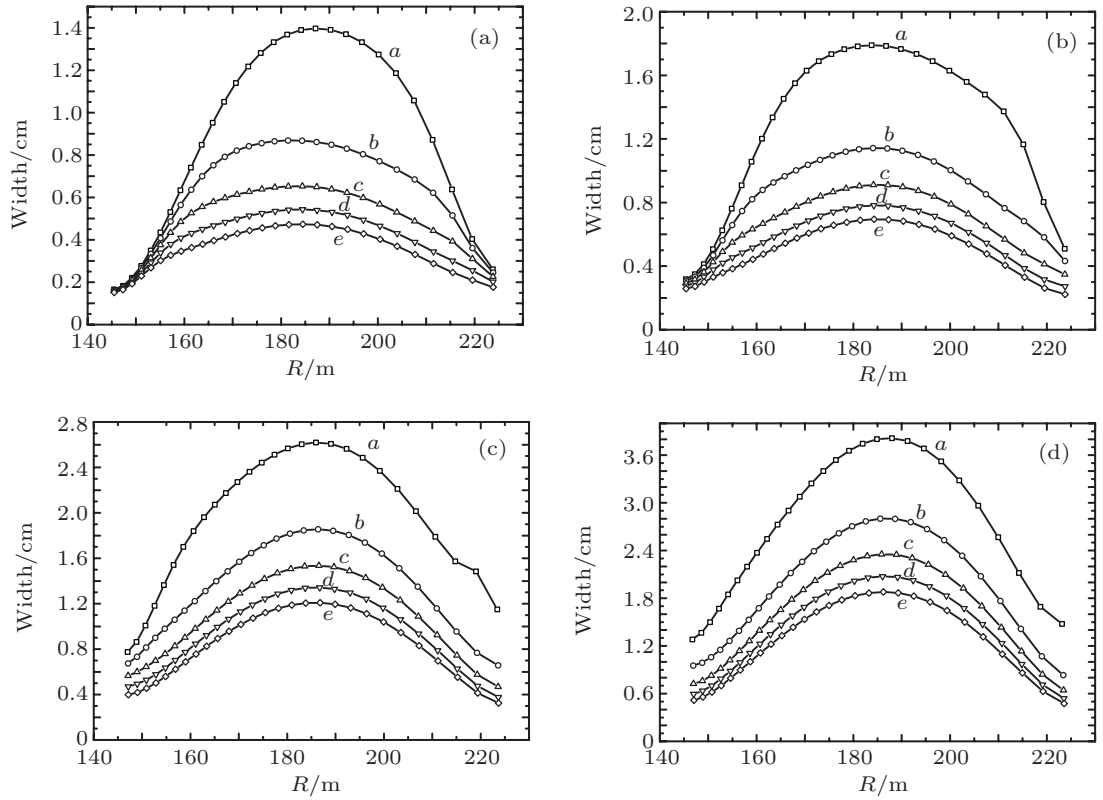


Fig. 2. Dependence of the emission layer width due to the relativistic broadening effect on the electron temperature and electron density for $T_{e0} = 1$ keV (a), $T_{e0} = 2$ keV (b), $T_{e0} = 5$ keV (c), and $T_{e0} = 10$ keV (d). The electron density is taken as $n_e = 1 \times 10^{19}$ m^{-3} (curve *a*), 2×10^{19} m^{-3} (curve *b*), 3×10^{19} m^{-3} (curve *c*), 4×10^{19} m^{-3} (curve *d*), and 5×10^{19} m^{-3} (curve *e*) respectively.

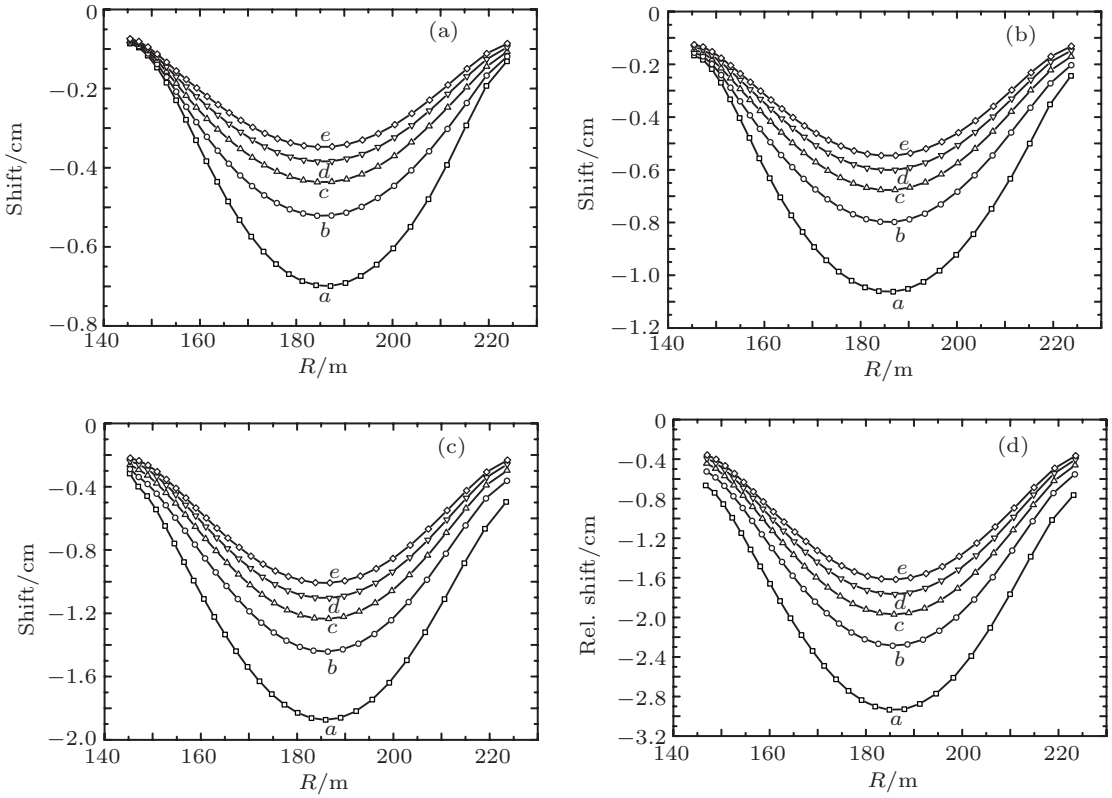


Fig. 3. Dependence of the difference between the emission layer position obtained from the non-relativistic frequency and the actual position on the electron temperature and electron density for $T_{e0} = 1$ keV (a), $T_{e0} = 2$ keV (b), $T_{e0} = 5$ keV (c), and $T_{e0} = 10$ keV (d). The electron density is taken as $n_e = 1 \times 10^{19}$ m^{-3} (curve *a*), 2×10^{19} m^{-3} (curve *b*), 3×10^{19} m^{-3} (curve *c*), 4×10^{19} m^{-3} (curve *d*), and 5×10^{19} m^{-3} (curve *e*), respectively.

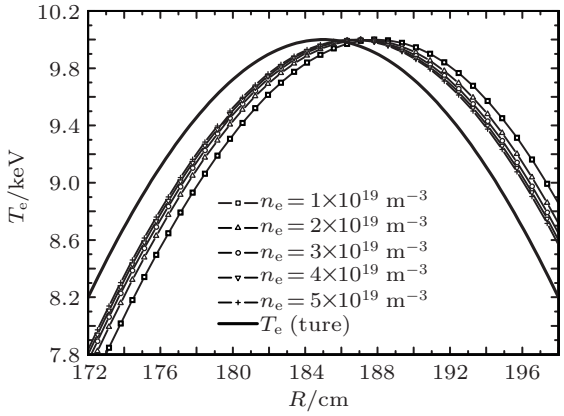


Fig. 4. The electron temperature profile assumed and obtained from the non-relativistic frequency in the case of It = 10000 A (2.25 T@1.85 m), T_{e0} = 10 keV and $n_e = (1,\ 2,\ 3,\ 4,\ 5) \times 10^{19}$ m^{-3}.

4. Conclusion

The relativistic broadening and frequency down-shift effects on the ECE measurements in EAST has been studied for a wide range of plasma parameters, $T_{e0} = 1, 2, 5, 10$ keV, $n_e = (1, 2, 3, 4, 5) \times 10^{19}$ m^{-3}, It = 10000 A (2.25 T@1.85 m). The calculation is based on the radiation transfer equation, with the reabsorption and reemission processes taken into account. The relativistic broadening effect contributes to the radial resolution of the ECE measurements. It is comparable to the contribution from the Doppler broadening effect and the spectral resolution of the measurement system in the case of $T_{e0} = 1$ keV, and it dominates when the electron temperature increases. Due to the relativistic down-shift effect, the obtained electron temperature profile, for which the radial position is deduced from the non-relativistic frequency, shifts from the actual profile. The shift could be a few centimeters, and this effect should be taken into account in the ECE measurements.

References

[1] Lichtenberg A J, Sesnic S and Trivelpiece A W 1964 *Phys. Rev. Lett.* **13** 387

[2] TFR Group 1975 *in Controlled Fusion and Plasma Physics (Proc. 7th Europ. Conf. Lausanne, 1975)* **Vol. 1** 14b

[3] Hutchinson I H and Komm D S 1977 *Nucl. Fusion* **17** 1077

[4] Kato K and Hutchinson I H 1986 *Phys. Rev. Lett.* **56** 340

[5] Kato K and Hutchinson I H 1987 *Phys. Fluids* **30** 3809

[6] Sozzi C, Luna E D L, Farina D, Fessey J, Figini L, Garavaglia S, Grossetti G, Nowak S, Platania P, Simonetto A, Zerbini M and JET EFDA contributors 2008 *AIP Conf. Proc. 988, Buring Plasma Diagnostics* p. 73

[7] Isayama A, Kamada Y, Ozeki T and Isei N 1999 *Plasma Phys. Control. Fusion* **41** 35

[8] Ryter F, Angioni C, Beurskens M, Cirant S, Hoang G T, Hogeweij G M D, Imbequx F, Jacchia A, Mantica P, Suttrop W and Tardini G 2001 *Plasma Phys. Control. Fusion* **43** A323

[9] Li E Z, Ling B L, Liu Y, Ti A, Hu L Q and Gao X 2010 *Chin. Phys.* B **19** 035203

[10] Sato M, Isei N and Ishida S 1995 *Jpn. J. Appl. Phys.* **34** L708

[11] Sato M, Isei N, Ishida S and Isayama A 1998 *J. Phys. Soc. Jpn.* **67** 3090

[12] Songtao W and the EAST Team 2007 *Fusion Engineering and Design* **82** 463

[13] Bekefi G 1966 *Radiation Processes in Plasmas* (New York: John Wiley and Sons) Chap. 1

[14] Bornatici M, Cano R, Barbieri O D and Engelmann F 1983 *Nucl. Fusion* **23** 1153

[15] Bornatici M, Engelmann F and Ruffina U 1983 *Sov. J. Quantum Electron* **13** 68

[16] Private Communication with proffesor Weiyue Wu (Tokamak Design Division, Institute of Plasma Physics)