Low group velocity in a photonic crystal coupled-cavity waveguide*

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A two-dimensional photonic crystal coupled-cavity waveguide is designed and optimized, the transmission spectrum is calculated by using the finite-difference time-domain method, and the group velocity of $c/1856$ is obtained. To our knowledge, this value of group velocity is the lowest group velocity in a photonic crystal waveguide calculated from its transmission spectrum so far. The result is confirmed by the photonic band structure calculated by using the plane wave expansion method, and it is found that the photonic crystal waveguide modes in a photonic band structure are in accordance with those in the transmission spectrum by using the finite-difference time-domain method. The mechanism of slow light in the coupled-cavity waveguide of photonic crystal is analysed.

Keywords: photonic crystal wavguide, coupled cavity, slow light

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1. Introduction

Slow light in photonic crystal (PC) has potential applications in optical storage, optical delay, lasing, etc., which is a hot topic in the PC research field.[1−8] In 2001, Notomi *et al.*[3] investigated the slow light in a conventional PC waveguide formed by removing one line of air holes. By changing the width of the waveguide, the lowest group velocity reached $c/90$.[3] In 2004, Asano *et al.*[4] measured the group velocity of $c/20$ experimentally by using the self-correlation method. Vlasov and McNab[6] found that the group velocity could decrease to $c/100$ through the coupling effect. A small group velocity of $c/200$ was obtained by changing the localization of the first line and the second line of air holes close to the PC waveguide.[9] Jagerska *et al.*[10] investigated the group velocity in a coupled-cavity PC waveguide by using the guided-mode expansion method and obtained a group velocity of $c/330$. Du *et al.*[11] designed coupled-cavity waveguide bend structures in two-dimentional photonic crystal, and realized a low group velocity of $c/170$ by calculations using the two-dimensional (2D) finite-difference time-domain (FDTD) method. In 2010, Grgic *et al.*[12] pointed out that the group velocity could reach zero ideally. However, up to date, for a practical PC waveguide in experiment, the slowest group velocity of light obtained is approximately $c/300$.[12] In this paper, a two-dimensional PC waveguide combined with coupled cavities is designed, and the characterization of the transmission and the guide mode of the PC waveguide are simulated and analysed using the plane wave expansion (PWE) method. The smallest group velocity is obtained to be up to $c/1856$ in the PC coupled-cavity waveguide.[13] This value is only 0.1∼0.2 times those evaluated from the transmission spectra reported before.[12−14]

2. Transmission spectrum and group index

The parameters of the designed triangular PC are as follows: the background refractive index n is 3.46, lattice constant a 323.71 nm, and the radius r of hole $0.275a$. Figure 1(a) shows the transmission spectrum of PC along the Γ–K direction, where the PC band gap ranges from 1.30 μm to 1.585 μm. From the cen-

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tral line along the Γ–K direction in a PC, every 8 holes are removed to form a continuous 8-hole cavity (L8 cavity), where only one hole remains separating every two L8 cavities, and then a coupled-cavity waveguide is built. The total length l_d of the waveguide is $59a$. In the following, in order to obtain a low group velocity, the radius of the hole separating each cavity is adjusted from $R = 0.17a$ to $0.50a$.

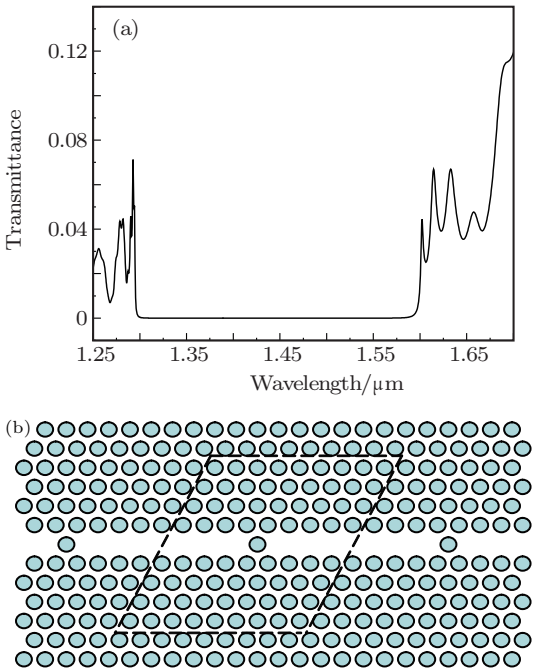


Fig. 1. (colour online) (a) Transmission spectrum for a triangular PC along Γ–K direction with TE polarization, where the lattice constant a is 323.71 nm, and radius r $0.275a$. (b) The schematic structure of the PC waveguide (part). The total length of the PC waveguide is $59a$.

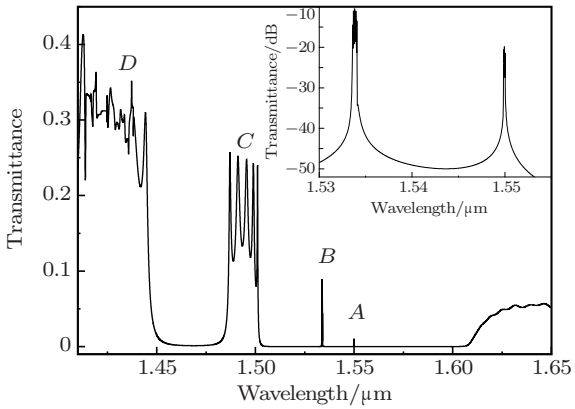


Fig. 2. The transmission spectrum for the PC waveguide with TE polarization, where the lattice constant a is 323.7nm, radius r $0.275a$, and the center hole radius R $0.23a$. The insert is the enlarged A and B bands.

The transmission spectrum of the PC waveguide with lattice constant $a = 323.71$ nm, radius $r = 0.275a$ and radius $R = 0.23a$ is shown in Fig. 2. The PC waveguide modes in the band gap are divided into several bands, as shown by A, B, C, and D in Fig. 2. The C band and D band are relatively broad, and the group refractive index in C band is smaller than 28, while light is almost not slowed down, which is indicated by the transmission spectrum in the D band. Both the A band and the B band are very narrow and their transmittances are relatively low, and it seems that they are two cavity modes. The quality factors of the A mode and the B mode in Fig. 2 reach 10000 and 3800 (by fitting into Gaussian function), respectively, while the quality factor of the central mode in the B band in Fig. 4(c) reaches 73000. The spectral resolutions in Figs. 3 and 4 are higher than those in Fig. 2. We mainly focused on the A and B bands to study the slow light of the coupled-cavity waveguide. It is shown that there are some Fabry–Perot (F–P) interference oscillation peaks (Figs. 2, 3, and 4) in these transmission spectra in bands A, B, and C. From these F–P oscillation peaks, the group velocity and index can be calculated by

$$v_{\rm g} = \frac{c}{n_{\rm g}}, \tag{1}$$

$$n_{\rm g} = \frac{\lambda^2}{2l_{\rm d}\Delta\lambda}, \tag{2}$$

where $n_{\rm g}$ is the group refractive index, c the light velocity in vacuum, λ the light wavelength, $l_{\rm d}$ the length of PC waveguide, and $\Delta\lambda$ the interval between two adjacent interfering modes.

For radius $R = 0.17a$, the transmission spectra of the A band and the B band are shown in Figs. 3(a) and 3(c). The spectra are smooth and the F–P oscillation modes are obvious. For the A band, the spectrum is around 1.55 μm, and the transmission amplitude is lower than 4%, while the group refractive indexes range from 364 to 606, which cover wavelengths from 1.5502 μm to 1.5507 μm. For the B band, the spectrum is around 1.5365 μm, the maximum transmittance reaches 17.6%, and the group refractive indexes range from 91 to 158.

When radius $R = 0.23a$, for the A band, the spectrum around wavelength 1.550 μm is shown in Fig. 4(a) and the corresponding group refractive index is in Fig. 4(b). The group index decreases with wavelength increasing, and the largest group index is 1856 at wavelength 1.54995 μm, while the smallest one is 912 at 1.55003 μm. The transmission amplitude is about 1% at maximum group index 1856, and this value of transmittance is measureable. For the B band, the spectrum and the group index each as

a function of wavelength are shown in Figs. 4(c) and 4(d), respectively, where the group index is in a range of 450–610, and the group index at the band edge is larger than that in the centre. As seen from Fig. 4(c), the transmission amplitude in B band reaches 9%, which is measurable in experiment. It can be found that with the band changing from A to B, the group index decreases, while the transmittance increases. From Figs. 3 and 4, it seems that the A band and the B band split from the cavity mode into several oscillation peaks under strong coupling between cavities in the waveguide.

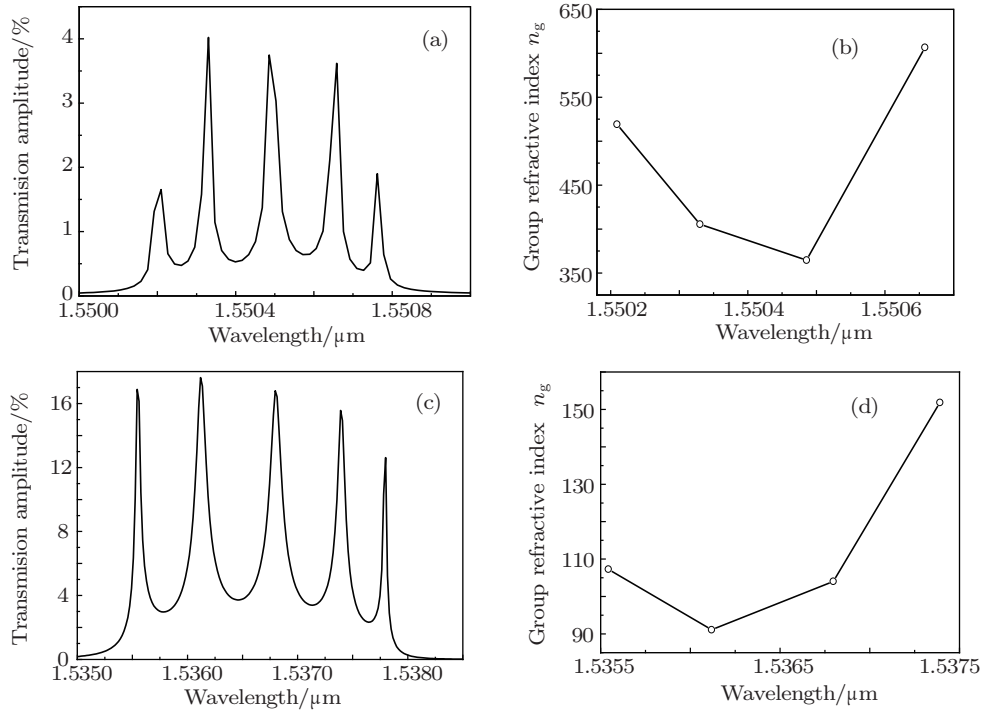


Fig. 3. For $R = 0.17a$, (a) transmission spectrum in A band (similar to that in A band in Fig. 2); (b) group index in the A band; (c) transmission spectrum in the B band; (d) group index in the B band.

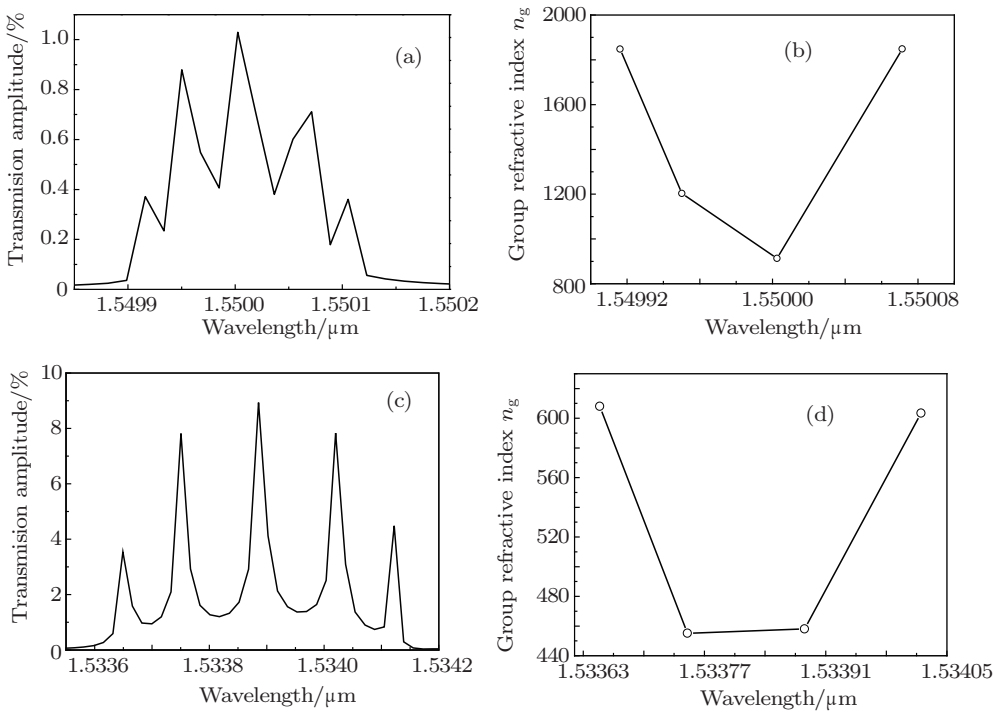


Fig. 4. For $R = 0.23a$, (a) enlargement of the spectrum in A band in Fig. 2; (b) group index in the A band; (c) enlargement of the spectrum in the B band in Fig. 2; (d) group index in the B band.

We calculate the group refractive index with radius R values ranging from $0.17a$ to $0.50a$ with the variation step of R being approximately $0.03a$. For the A band, the group refractive indices for different values of radius R are collected in Fig. 5(a). As the radius R varies from $0.17a$ to $0.50a$, the wavelength of slow light changes from 1.54966 μm to 1.55066 μm (only about 1 nm in variance). It can be found that as radius R increases from $0.17a$ to $0.275a$, the wavelength decreases from 1.55049 μm to 1.54969 μm with the corresponding group refractive index increasing from 364 to 1847. As radius R increases more from $0.29a$ to $0.50a$, the group refractive index decreases from 1848 to 519, and the corresponding wavelength decreases from 1.54992 μm to 1.55035 μm. In the short wavelength ranging from 1.54966 μm to 1.55012 μm (i.e. 0.4 nm bandwidth), the group refractive index is around 1848 and 1804, corresponding radius increases from $0.275a$ to $0.335a$; while in the long wavelength range from 1.55016 μm to 1.55066 μm, the group refractive index is smaller than 1000 (in a range from 913 to 364), corresponding to the large radius from $0.41a$ to $0.50a$ and the small radius from $0.17a$ to $0.20a$. Moreover, it can be found that from every line of the group refractive index for a defined radius, the group refractive index also varies with wavelength. For example, for an inverse-hollow-triangular line of radius R $0.245a$, the group refractive index is 1848 at wavelength 1.54983 μm but only 1222 at wavelength 1.54990 μm.

The largest group refractive index and the corresponding transmission amplitude each as a function of radius R are shown in Fig. 5(b), where the radius R ranges from $0.17a$ to $0.50a$, the group refractive index ranges from 606 to 1848, while the transmission amplitude is around 2.0%. The group index is calculated to be 913.34, 1848.96, and 1848.59 with the respective radius values R being $0.20a$, $0.23a$, and $0.26a$. As the radius R changes from $0.20a$ to $0.23a$, the group index increases from 913.34 to 1848.96, while the radius increases from $0.23a$ to $0.335a$, the group velocity remains almost unchanged. It can be found that the large group refractive indices around 1848 have large radius values ranging from $0.23a$ and $0.335a$. Moreover, the larger the group refractive index, the lower the transmittance is.

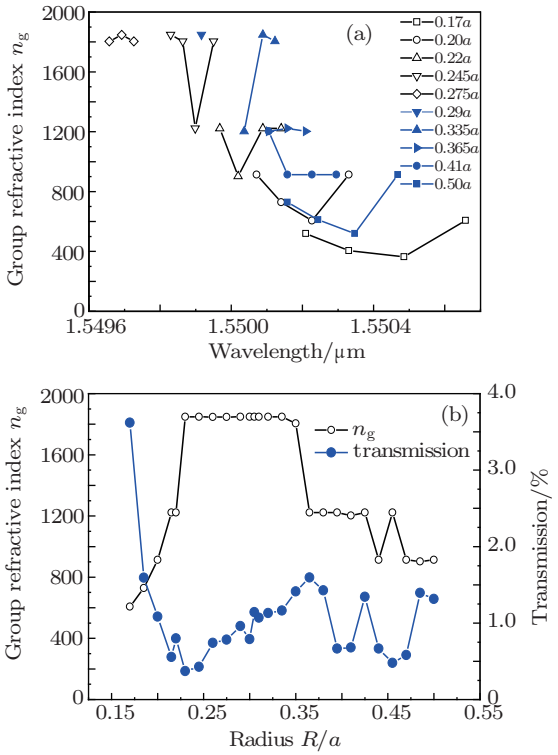


Fig. 5. (colour online) (a) Group refractive indices each as a function of wavelength for the A band at different values of radius R; (b) group refractive index and transmittance each as a function of radius R for A band.

For band B around wavelength 1.534 μm, the group refractive indices and the transmission amplitudes for different values of radius R are collected in Figs. 6(a) and 6(b). In the B band, as radius R varies from $0.17a$ to $0.50a$, the spectrum covers the wavelength range from 1.53112 μm to 1.53739 μm (6.3-nm broad), which is much broader than that in the A band. From Fig. 6(a), the open-symbol lines are for the values of small radius R varying from $0.17a$ to $0.275a$, while the solid-symbol lines are for the values of large radius R ranging from $0.29a$ to $0.50a$. The distribution of group refractive indices on the shorter wavelength side and that on the long wavelength side are almost symmetric with respect to the central wavelength around 1.534 μm, both for the solid-symbol lines and the open-symbol lines. That is, around the central wavelength 1.5431 μm, the group refractive index reaches maximum 1856 for an open-circle line with R=$0.22a$, from which position, the refractive index decreases with both wavelength and radius increasing but it decreases with wavelength and radius decreasing. Around the largest group index 1856 at the radius R of $0.22a$, the group indices for radius R values of $0.20a$, $0.23a$, and $0.26a$ are calculated to be 519.98, 608.07, and 243.01, respectively. On the other hand, the largest group index in this figure for solid-symbol

lines representing large radius values reaches 1811 at a solid-circle line with radius R 0.365a at wavelength 1.53466 μm, from which wavelength, with both wavelength and radius R increasing or decreasing, for solid-symbol lines the group refractive index decreases. The smallest group refractive index for an open symbol is only 91 at wavelength 1.53612 μm, while the smallest group refractive index for the solid-symbol line is 191 at wavelength 1.53289 μm with radius R values of 0.29a, and 140 at wavelength 1.53551 μm with radius R 0.50a.

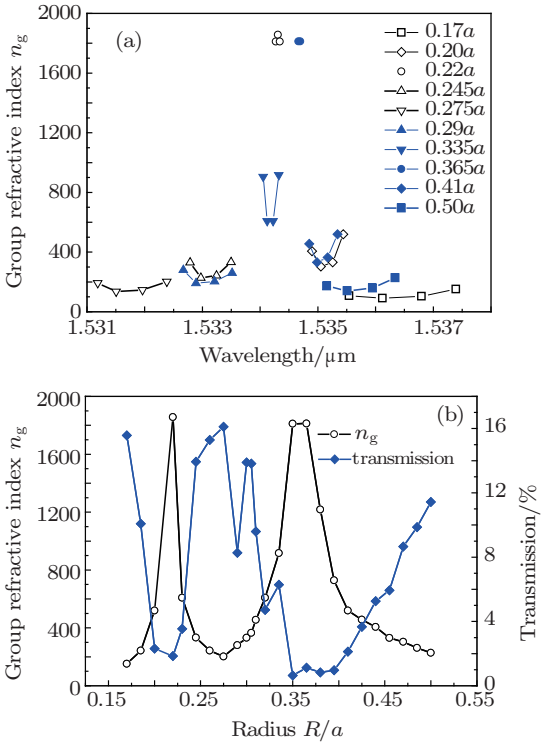


Fig. 6. (colour online) (a) Group refractive indices each as a function of wavelength for the B band at different values of radius R; (b) group refractive index and transmittance each as a function of radius R for the B band.

In Fig. 6(b), the largest group refractive index for each value of radius R has some oscillation variation in the radius R range from 0.17a to 0.50a. As radius R increases from 0.17a to 0.22a, the group refractive index rises from 151 to 1856. When radius R increases from 0.22a to 0.275a, the group refractive index decreases from 1856 to 202. As radius R changes from 0.275a to 0.35a, the group refractive index rises again from 202 to 1811; while the radius R increases from 0.35a to 0.50a, the refractive index decreases from 1811 to 227. The two peaks of the group refractive index in Fig. 6(b): 1856 at radius R 0.22a and 1811 at radius R 0.365a, just correspond to those two peaks in Fig. 6(a), represented by the open-circle line at radius R 0.22a and the solid-circle line at radius R 0.365a, respectively. The trend of transmittance varying with group refractive index is also similar to that described above, i.e., the higher the transmittance, the smaller the refractive index is. However, even the smallest group refractive index for the B band in Fig. 6(b) reaches 151 at radius R 0.17a, and the highest transmittance reaches 16%; while the largest group refractive index reaches 1856 at radius R 0.22a, and the lowest transmittance is about 0.64%.

It is shown that the PC coupled-cavity waveguide exhibits a large group refractive index and considerable transmittance at the communication wavelength with reachable parameters. This PC coupled-cavity waveguide will have potential applications in optoelectronic devices, such as delay devices, dispersion compensating components, optical modulators, and the low-threshold photonic crystal laser. Moreover, the above figures and analysis show that the application of this PC waveguide can be flexible by adjusting the radius R. By changing R from 0.17a to 0.50a, band A and band B cover the communication wavelength of about 7 nm. In the A band, in a large range of radius R from 0.22a to 0.35a, the group refractive index is larger than 1800. Through optimizing the radius R of the PC waveguide, the largest group index of 1856 can be obtained in the B band (Fig. 6(b) at radius R 0.22a), which is more than 5 times larger than the largest one that reported before.[12] If we consider both the large group refractive index and high transmittance, we can choose the values of radius R to be 0.17a, 0.245a–0.30a, 0.47a–0.50a, etc., when the group refractive indices are larger than 200, while the transmission amplitude higher than 8%, and the highest transmission amplitude reaches 16% in the B band. For example, for the radius R of 0.275a, the group refractive index reaches the largest value in the A band, while the transmittance is only 0.78%, and the transmission spectrum is shown in Fig. 7(a). However, the group refractive index is 202, and the transmittance reaches 16% in B band (Fig. 7(b)). It can be found that the group index is sensitive to the value of radius R of the hole separating the long cavity. This phenomenon also demonstrates that the slow light in the PC waveguide is determined by the structure of the coupled-cavity waveguide. Changing the radius of the hole separating each cavity will affect the coupling-strength between the cavities, and then the group velocity is changed. Usually, this process for approaching a small group velocity will strongly reduce both the transmission and the bandwidth of the slow-light region. However, in our case, as mentioned above, the slow-light transmittance can achieve around 0.10, and

the bandwidth of slow light is more than 6 nm.

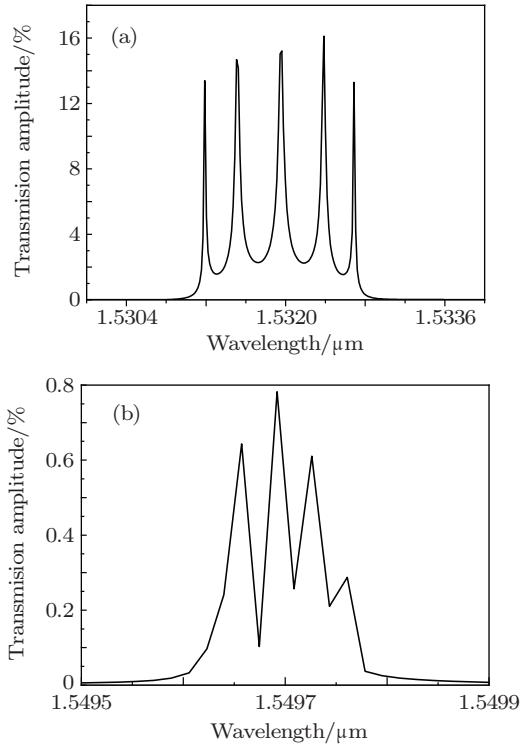


Fig. 7. For $R = 0.275a$, (a) transmission spectrum in the B band; (b) transmission spectrum in the A band.

In this paper, a small group velocity in a coupled-cavity waveguide is calculated by using the 2D FDTD method for computation ease. If the three-dimensional (3D) FDTD method is used to simulate a similar structure we used for the case of finite thickness (i.e., a PC slab), a very large computer memory will be required. Similar to the case of a ridge waveguide studied in Ref. [15], although the third dimension is neglected in the calculation, for the width of PC is much larger than the height, the 2D FDTD method is sufficiently valid for first-order prediction of practical performance.[16] Moreover, the 2D FDTD method is confirmed by experiments in calculating transmission spectrum and group velocity in a PC waveguide.[17−19] It can be found that the group velocity calculated by 2D FDTD method fits well to that measured experimentally.[19] Therefore, the results of group velocities obtained in this paper are valid and convincing. If the resolution of the spectrometer in the experiment is high enough, we believe that the high-resolution transmission spectrum we simulated is measurable, and the lowest group velocity in this paper is reachable experimentally.

3. Photonic band structure of the PC waveguide

The band structure of PC in Fig. 8 with radius $r = 0.275a$ is calculated using the PWE method. It can be found that the photonic band gap is located in the range of $0.20314a/\lambda$–$0.25288\ a/\lambda$, which corresponds to the wavelength range 1.280 μm–1.593 μm with lattice constant a of 323.71 nm. This band gap corresponds to the photonic band gap range of 1.30 μm–1.585 μm from the transmission spectrum in Fig. 1, calculated by the FDTD.

In the PWE calculation, a proper supercell of $9\times9(9a\times4.5\sqrt{3}a)$ shown as the dashed-line in Fig. 1(b) is selected for calculating the band structure of a coupled-cavity waveguide with TE polarization. Figure 8(b) shows the band structure for the PC waveguide with the values of radius r being $0.275a$ and R $0.230a$. The photonic band gap in Fig. 8(b) corresponding to that in Fig. 8(a) is shown by the vertical arrow line. Around the wavelength range $0.209437\ a/\lambda$–$0.228\ a/\lambda$ (1.41741 μm–1.543042 μm), there are several PC waveguide modes. The C mode is around $0.216687\ a/\lambda$–$0.218753\ a/\lambda$ (1.47733 μm–1.49141 μm), corresponding to C band in Fig. 2 (wavelength range of 1.48650 μm–1.50126 μm). It can be found that C mode is not so flat in Fig. 8(b), therefore, the group velocity in C band is not so small. The normalized frequency of the B mode is in the range of $0.211605\ a/\lambda$–$0.211742\ a/\lambda$ (1.526244 μm–1.527232 μm), corresponding to the guide mode of the B band in Figs. 2 and 4(c). Another mode is around $0.209437\ a/\lambda$–$0.209453\ a/\lambda$ (1.54292 μm–1.543042 μm, named A mode), corresponding to the guide mode in the A band in Fig. 2 and Fig. 4(a). It can be found that these two waveguide modes, A and B, in the photonic band structure are very flat. Therefore, the corresponding group velocity is very small. For the B mode around Γ point, the group index can be evaluated to be 17254, while that of the A mode around the Γ point reaches 72186. The large group refractive index calculated from the transmission spectrum corresponds to the large group index near the Γ point in the band structure. It was reported in some literature that the electric field of the waveguide mode could be well localized in the area of the PC waveguide.[5,7,12] In the range of slow group velocity, the electric field of the localized light can leak to the adjacent holes, which thereby induces the loss of guide

mode. The field distributions of two guide modes, A and B, in Fig. 8(b) are shown in Figs. 8(c) and 8(d), respectively. It can be found that the TE mode is confined in the coupled-cavity waveguide. Therefore, the slow light is due mainly to the light localization in the cavity and the coupling between cavities. Based on the above results, the PC parameters may be optimized to obtain a smaller group velocity.

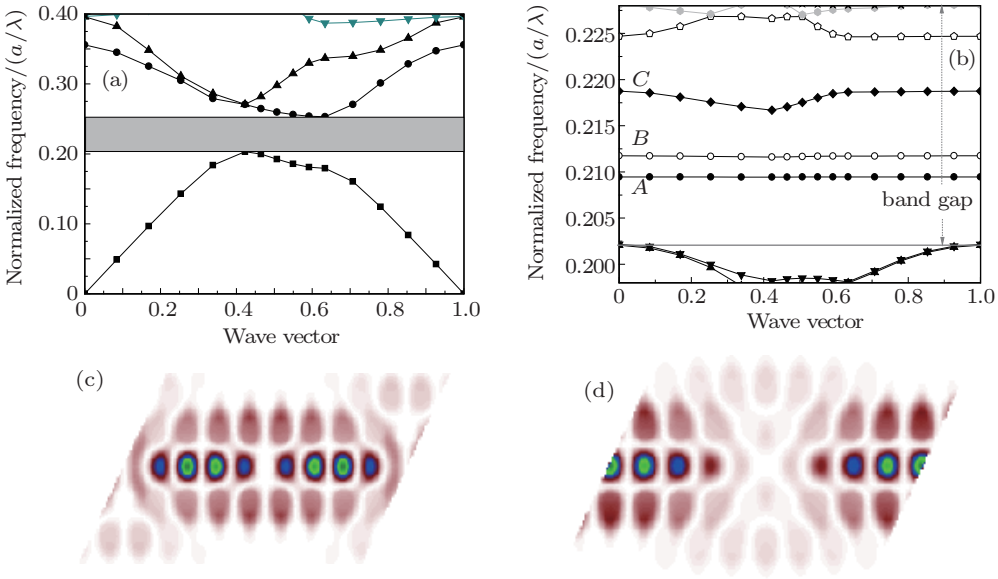


Fig. 8. (colour online) (a) Photonic band structure of PC with radius r of $0.275a$; (b) photonic band structure of coupled-cavity waveguide; (c) electric field of PC waveguide for mode A in (b); (d) electric field of PC waveguide for mode B in (b). The gray area represents the photonic band gap.

4. Conclusion

Through optimizing the parameters of photonic crystal, a photonic crystal waveguide constructed by using long coupled cavities is designed for slow light. The largest group index 1856 in the photonic crystal coupled-cavity waveguide is obtained from the transmission spectrum by using the FDTD method. For applications of this photonic crystal waveguide, the radius of the hole separating each cavity can be flexibly adjusted to obtain a large group index with a large variation range of radius. A considerable group refractive index with high transmittance can also be obtained, showing that a group refractive index larger than 200 with transmittance higher than 16% is reachable. To explain the mechanism of slow light, the band structure is calculated by using the PWE method. The result shows that the slow light of waveguide mode, calculated from PWE, is very flat, which means that the group velocity is very small. This result is consistent with the small group velocity calculated from the transmission spectrum using the FDTD method.

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