Geometric phases in qubit-oscillator system beyond conventional rotating-wave approximation*

Wang Yue-Ming(王月明)[a)b)†], Du Guan(杜 冠)[a)], and Liang Jiu-Qing(梁九卿)[a)]

[a)] *Department of Physics, Institute of Theoretical Physics, Shanxi University, Taiyuan* 030006, *China*

[b)] *State Key Laboratory of Quantum Optics and Quantum Optics Devices, Shanxi University, Taiyuan* 030006, *China*

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In this work we investigated the geometric phases of a qubit-oscillator system beyond the conventional rotating-wave approximation. We find that in the limiting of weak coupling the results coincide with that obtained under rotating-wave approximation while there exists an increasing difference with the increase of coupling constant. It was shown that the geometric phase is symmetric with respect to the sign of the detuning of the quantized field from the one-photon resonance under the conventional rotating-wave approximation while a red-blue detuning asymmetry occurs beyond the conventional rotating-wave approximation.

Keywords: geometric phase, rotating-wave approximation, red-blue detuning asymmetry

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1. Introduction

In 1984 Berry[1] discovered that wave function can acquire a geometric phase (GP) factor in addition to the familiar dynamic phase when the Hamiltonian is cyclic and adiabatic. Later, Aharonov and Anandan[2] showed that the phase change experienced by a quantum state taken around a closed path depends only on the path in Hilbert space, regardless of whether or not the transversal is carried out adiabatically. The geometric phases describe remarkable aspects of quantum mechanics and play a fundamental part of our contemporary understanding of quantum physics. To date it has been extensively investigated in many physical fields such as optics, magnetic resonance, molecular and atomic physics, condensed matter physics, and quantum information, and so on.[3−10] The phases have been found in various two-level system (qubit) such as spin-1/2 particle,[11] atom–cavity system in quantum optics,[12−14] solid-state qubits, and so on.[15] As is well known, a two-level system coupled to a quantum harmonic oscillator can usually be reduced to the exactly solvable Jaynes–Cummings model (JCM)[16] describing the interaction of a two-level atom and a single mode quantized electromagnetic field with the rotating-wave approximation (RWA) applied to a Rabi Hamiltonian. The JCM has been widely used as a guide to understand various quantum-optics phenomena[17,18] due to the fact that most of the experiments satisfy the required parameter regime for RWA, that is, near resonance and relatively weak coupling between the two systems.[16,19] However, the theoretical analyses have suggested that the anti-resonant terms may be responsible for some behaviors, qualitatively, of such chaos, lamb shift, quantum Zeno effect (QZE), Bloch–Siegert shift, spontaneous emission spectrum, and entanglement evolution.[20−25] In recent years, the applicable regime and validity of RWA has been discussed, thus various numerical and approximate analytic methods beyond RWA are applied to the Rabi Hamiltonian for deep strong coupling which can be realized in a solid qubit oscillator system.[26−34] The geometric phases in JCM have been investigated in Refs. [12]–[14], which are all based on the RWA; in this paper we shall study the geometric phases in the JCM beyond the conventional RWA.

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†Corresponding author. E-mail: wang_ym@sxu.edu.cn

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2. Model

We consider the Rabi Hamiltonian of a two-level atom coupled to a single-mode quantum field

$$H = \frac{1}{2}\omega_a\sigma_z + \omega a^\dagger a + g(a^\dagger + a)(\sigma_+ + \sigma_-), \quad (1)$$

where ω and ω_a are the field and atomic transition frequencies respectively, and g is the coupling constant (vacuum Rabi frequency) with $\hbar$ set to unity; $a^\dagger$ (a) is the creation (annihilation) operator of the quantized field while $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|, \sigma_+ = |e\rangle\langle g| = \sigma_-^\dagger, \sigma_x = (\sigma_+ + \sigma_-)$ are atomic operators, where $|g\rangle$ and $|e\rangle$ denote the ground and excited atomic states, respectively. We take a rotation around the axis y for the Pauli matrices $\mathrm{e}^{\mathrm{i}\pi\sigma_y/4}\sigma_x\,\mathrm{e}^{-\mathrm{i}\pi\sigma_y/4} = \sigma_z$ and $\mathrm{e}^{\mathrm{i}\pi\sigma_y/4}\sigma_z\,\mathrm{e}^{-\mathrm{i}\pi\sigma_y/4} = -\sigma_x$. It is worthwhile to point out that the exchange transformation of spin operators is done for facilitating the following unitary transformation. Thus the Hamiltonian becomes

$$H' = -\frac{1}{2}\omega_a\sigma_x + \omega a^\dagger a + g(a^\dagger + a)\sigma_z. \quad (2)$$

Perform a unitary transform on H' with the operator

$$S = \frac{g\xi}{\omega}\sigma_z(a^\dagger - a), \quad (3)$$

where ξ is a parameter to be determined. Using the Baker–Hausdorff theorem, we obtain the transformed Hamiltonian $H^S = \exp(S)H'\exp(-S)$,

$$\begin{aligned} H^S &= \omega a^\dagger a - g(\xi-1)(a^\dagger + a)\sigma_z + H_x^S \\ &\quad + H_y^S + \frac{2g^2\xi(\xi-1)}{\omega}, \quad (4) \\ H_x^S &= -\frac{1}{2}\omega_a\sigma_x\Bigg[1 + 2\left(\frac{g\xi}{\omega}\right)^2(a^\dagger - a)^2 \\ &\quad + \frac{2}{3}\left(\frac{g\xi}{\omega}\right)^4(a^\dagger - a)^4 + \cdots\Bigg], \quad (5) \\ H_y^S &= -\frac{1}{2}\mathrm{i}\omega_a\sigma_y\Bigg[2\left(\frac{g\xi}{\omega}\right)(a^\dagger - a) \\ &\quad + \frac{4}{3}\left(\frac{g\xi}{\omega}\right)^3(a^\dagger - a)^3 + \cdots\Bigg]. \quad (6) \end{aligned}$$

We keep the terms of $g\xi/\omega$ up to the quadratic term (higher-order terms are neglected) and only the "energy-conserving" one-excitation (of the oscillator) terms, while the terms involving higher numbers of quanta are discarded,[26] then we have

$$\begin{aligned} &H^S \\ &= \omega a^\dagger a - g(\xi-1)(a^\dagger + a)\sigma_z + \frac{2g^2\xi(\xi-1)}{\omega} \\ &\quad - \mathrm{i}\omega_a\left(\frac{g\xi}{\omega}\right)(a^\dagger - a)\sigma_y + \frac{1}{2}\omega_a\sigma_x\left[\left(\frac{g\xi}{\omega}\right)^2 - 1\right]. \quad (7) \end{aligned}$$

It is worthwhile noting that we can see from Eqs. (4)–(6) it is essential to keep higher-order terms for the multi-quanta process, i.e., in this case the counter-rotating terms should be taken into account. Perform inverse transformations as $\mathrm{e}^{-\mathrm{i}\pi\sigma_y/4}\sigma_x\,\mathrm{e}^{\mathrm{i}\pi\sigma_y/4} = -\sigma_z$ and $\mathrm{e}^{-\mathrm{i}\pi\sigma_y/4}\sigma_z\,\mathrm{e}^{\mathrm{i}\pi\sigma_y/4} = \sigma_x$, and eliminate the counter-rotating-wave terms without approximation by setting $\xi = \omega/2\bar{\omega}$ ($\bar{\omega} \equiv (\omega + \omega_a)/2$). A renormalized JC Hamiltonian in the RWA form can be obtained with the constants omitted

$$\tilde{H}^S = \frac{1}{2}\tilde{\omega}_a\sigma_z + \omega a^\dagger a + \tilde{g}(a^\dagger\sigma_- + a\sigma_+), \quad (8)$$

where

$$\begin{aligned} \tilde{\omega}_a &= \omega_a\left(1 - \frac{g^2}{2\bar{\omega}^2}\right), \\ \tilde{g} &= g\left(1 + \frac{\Delta}{2\bar{\omega}}\right), \\ \tilde{\Delta} &= \Delta - \omega_a\frac{g^2}{2\bar{\omega}^2}. \end{aligned} \quad (9)$$

Here the atom-field detuning is defined as $\Delta = \omega_a - \omega$. From Eqs. (9) we can see in the valid parameter regime of RWA, that is, $g \ll \{\omega, \omega_a\}$ and $\Delta \to 0$, the effective atomic transition frequency $\tilde{\omega}_a = \omega_a$, the effective coupling constant $\tilde{g} = g$; and an effective detuning exists even at zero detuning. We can say the results of Eqs. (8) and (9) include the effect of the counter-rotating-wave terms.

3. Geometric phase

Now we introduce the phase shift operator $R(\phi(t)) = \exp(-\mathrm{i}\phi(t)a^\dagger a)$ which is applied to the Hamiltonian of the system, where $\phi(t) = \omega t$ represents the change of the state of the light field. The geometric phase induced by the light field can be found when $\phi(t)$ is changed slowly from 0 to 2π,[35] which is in fact a geometric effect for the wave function of the system to undergo a cyclic evolution in the parameter space,

$$\begin{aligned} \gamma_\pm &= \mathrm{i}\int_0^{2\pi}\mathrm{d}\phi\,\langle\Psi_n^\pm|\,R^\dagger(\phi)\frac{\mathrm{d}}{\mathrm{d}\phi}R(\phi)\,|\Psi_n^\pm\rangle \\ &= 2\pi\,\langle\Psi_n^\pm|\,a^\dagger a\,|\Psi_n^\pm\rangle, \end{aligned} \quad (10)$$

where $|\Psi_n^\pm\rangle$ are the eigenstates of the Hamiltonian (8),

$$\begin{aligned} |\Psi_n^-\rangle &= \cos\frac{\theta_n}{2}\,|g, n+1\rangle - \sin\frac{\theta_n}{2}\,|e, n\rangle, \\ |\Psi_n^+\rangle &= \sin\frac{\theta_n}{2}\,|g, n+1\rangle + \cos\frac{\theta_n}{2}\,|e, n\rangle, \end{aligned} \quad (11)$$

with the angle θ_n defined as

$$\sin\frac{\theta_n}{2} = \frac{1}{\sqrt{2}}\left(\frac{\tilde{\Omega}_n - \tilde{\Delta}}{\tilde{\Omega}_n}\right)^{1/2},$$
$$\cos\frac{\theta_n}{2} = \frac{1}{\sqrt{2}}\left(\frac{\tilde{\Omega}_n + \tilde{\Delta}}{\tilde{\Omega}_n}\right)^{1/2} \quad (12)$$

with $\tilde{\Omega}_n = [\tilde{\Delta}^2 + 4\tilde{g}^2(n+1)]^{1/2}$. Substituting the expressions of $|\Psi_n^{\pm}\rangle$ into Eq. (10) the geometric phases that the eigenstates $|\Psi_n^{+}\rangle$ and $|\Psi_n^{-}\rangle$ acquire after a cyclic evolution are respectively

$$\gamma_+ = \pi(1-\cos\theta_n) + 2\pi n, \quad (13)$$
$$\gamma_- = -\pi(1-\cos\theta_n) + 2\pi(n+1), \quad (14)$$

where $\cos\theta_n = \tilde{\Delta}/\tilde{\omega}_n$. It can be seen that the geometric phases are dependent on photon number n and are nontrivial even if the field is a vacuum mode, as is the result of quantization of the field. Figure 1 shows how the geometric phase γ_- to be acquired by the ground state $|\Psi_n^{-}\rangle$ varies with the coupling constant g, in which the solid (purple) line represents the results under RWA and the dashed line (orange) those beyond RWA for various detunings.

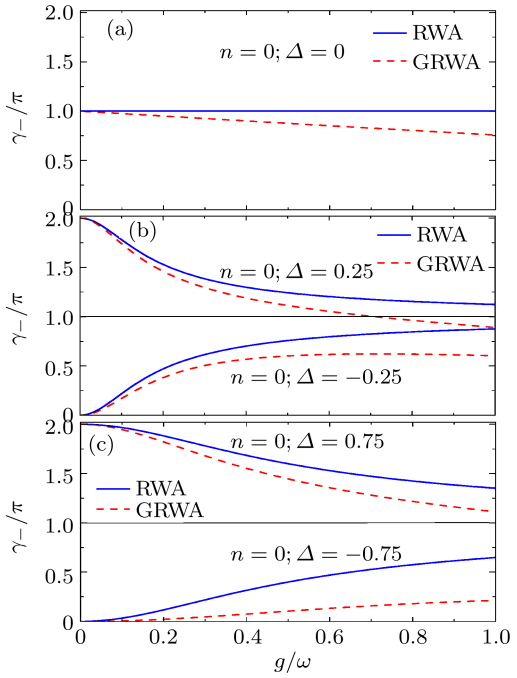


Fig. 1. (color online) The vacuum-induced geometric phase γ_- acquired by the ground state $|\Psi_n^{-}\rangle$ after an adiabatic evolution of the field varies with coupling constant g/ω under various detunings. The solid (purple) line shows the results under RWA and the dashed (orange) line shows the results beyond RWA. (a) $\Delta = 0$; (b) $\Delta = \pm 0.25$; (c) $\Delta = \pm 0.75$. Δ is in units of ω.

This non-trivial geometric phase can be measured by means of an interference procedure between the eigenstate $|\Psi_0^{+}\rangle$ and the ground state $|g,0\rangle$ for which no geometric phase is acquired. For example if the atom-field system is initially in the state $|e,0\rangle = \cos(\theta_0/2)|\Psi_0^{+}\rangle - \sin(\theta_0/2)|\Psi_0^{-}\rangle$ a non-trivial geometric phase can be obtained after the adiabatic operation performed on the field

$$\gamma_0 = \frac{\pi}{2}(1-\cos 2\theta_0) = \frac{\Omega}{4} \quad (15)$$

with the solid angle $\Omega = 2\pi(1-\cos 2\theta_0)$. Equation (15) shows that the initial state $|e\rangle$ coupled to vacuum cavity mode acquires a geometric phase $\Omega/4$ through adiabatic evolution, which is the result of the vacuum photon fluctuation due to the interaction with the qubit. Figure 2 shows how the geometric phase varies with the detuning in the cases of weak coupling ($g/\omega = 0.01$), intermediate strong coupling ($g/\omega = 0.1$), and deep strong coupling ($g/\omega = 1$). It can be seen that from Figs. 1 and 2 in the limit of weak coupling the two results coincide with each other both in the case of RWA and beyond RWA. There exists an increasing difference between the two with the increase of coupling strength. The geometric phase is symmetric with respect to the sign of the detuning of the quantized field from the one-photon resonance

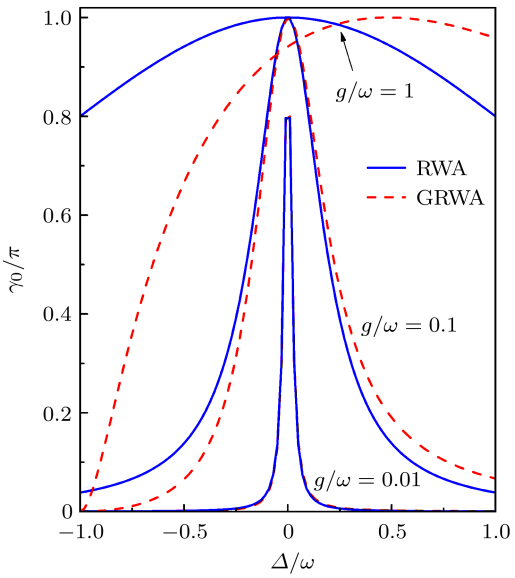


Fig. 2. (color online) The vacuum-induced geometric phase γ_0 acquired by the state $|e\rangle$ coupled to vacuum cavity mode after an adiabatic evolution of the field varies with detunings Δ/ω for various coupling constants. The solid (purple) lines show the results in RWA and the dashed (orange) lines show the results beyond RWA.

in the case of RWA, however the symmetry is broken beyond RWA. A red-blue detuning asymmetry also occurs for the spontaneous emission spectrum of a qubit strongly coupled to a resonant cavity without RWA.[23]

Following Ref. [36], we consider the effect of the decoherence of the field on the geometric phases. With the same procedure the effective Hamiltonian in the no-jump case becomes $\tilde{H}_d^S = \tilde{H}^S - \mathrm{i}\lambda\hat{n}/2$, so the geometric phase γ_0 acquired by the state $|e\rangle$ coupled to vacuum cavity mode can be written as

$$\gamma_0^d = \frac{\pi}{2}\left(1 - \mathrm{Re}\frac{(\tilde{\Delta} - \mathrm{i}\lambda/2)^2 - 4\tilde{g}^2}{(\tilde{\Delta} - \mathrm{i}\lambda/2)^2 + 4\tilde{g}^2}\right), \qquad (16)$$

which can be expanded to the second order in $\lambda/\tilde{\Omega}$, since $\lambda/\tilde{\Omega}$ should be a perturbation quantity with $\tilde{\Omega} = [\tilde{\Delta}^2 + 4\tilde{g}^2]^{1/2}$, then we have

$$\gamma_0^d \approx \gamma_0 + \frac{\pi(1 - \sin^2\theta_0)}{32\sin^2\theta_0(1 + 2\sin^2\theta_0) + 4(1 - \sin^2\theta_0)^2} \times \left(\frac{\lambda}{\tilde{\Omega}}\right)^2, \qquad (17)$$

where $\sin\theta_0 = 2\tilde{g}/\tilde{\Omega}$. The latter part of Eq. (17) can be viewed as a correction for the geometric phases in the case of decoherence with a vanishing first order term in λ/Ω and a remaining quadratic term reflecting the resilience of the geometric phase against the environment, which makes geometric phases more appropriate for a potential fault tolerance when applied to quantum information procession.

Finally we give a discussion on the experimental observation of the geometric phase of the qubit-oscillation system beyond the conventional RWA. The differences of the results between RWA and GRWA are mainly embodied in the ultra-strong coupling regime that is far beyond atom–cavity coupling limit. We know that the current quantum dot cavity QED systems can reach a strong coupling regime in which RWA is not always justified.[37,38] In Ref. [13] a feasible method to observe the vacuum-induced geometric phases within an atom–cavity modes composite system through the usual Ramsey interferometry was proposed, in which two dispersive Raman interactions in a double-Λ configuration are adopted by means of two orthogonally polarized cavity modes and externally injected quasiclassical fields. This scheme can be realized in a quantum dot embedded inside a microdisc structure as illustrated in Ref. [39]. The arbitrary single-bit rotations can be performed by applying external short laser pulses. The Ramsey fringes can also be measured in a quantum dot two-level system within current experimental technologies. We expect the results discussed in this text to be examined in the future experiments.

4. Conclusion

In summary we have calculated the geometric phases of a qubit-oscillator system beyond the conventional rotating-wave approximation, which is different from the results in the case of RWA. In the limiting of weak-coupling the two results coincide while in the strong coupling regime there exists an increasing difference with the increase of coupling constant and a red-blue detuning asymmetry occurs for the geometric phases when beyond the conventional rotating-wave approximation. A possible experimental observability is also briefly discussed.

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