Noise analysis in ghost imaging from the perspective of coherent-mode representation*

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First we present a theoretical analysis of classical noise in ghost imaging system based on the coherent-mode representation theory. The classical noise depends crucially on the distribution of the eigenvalues of the coherent-mode representation of the source and the decomposition coefficients of the object imaged. We show that both decreasing the distribution of the decomposition coefficients and increasing the distribution of the eigenvalues can lead to the decrease of classical noise.

Keywords: classical noise, ghost imaging, coherent-mode representation

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1. Introduction

Ghost imaging, a novel technique of obtaining the information about an object or about its diffraction pattern from the correlation measurements, has attracted great interest. The first demonstration of this imaging technique was implemented with twin beams, generated by parametric down-conversion (PDC).[1] The reconstruction of the image should be attributed to the nonlocal quantum correlations between the photon pairs.[2] Then it was discovered, both theoretically and experimentally, that classically correlated fields can be used to realize ghost imaging.[3−5] Especially, a lensless Fourier-transform ghost-imaging system has attracted much interest[6,7] because it has potential applications in X-ray diffraction. Recently, it is shown that the direct ghost-images can be obtained by lensless ghost-imaging systems.[8−10] In addition, a new type of ghost-imaging experiments by measuring the reflected photons were also implemented.[11−13]

To date, many studies focus on the imaging properties in ghost imaging, including the visibility,[14−20] resolution,[21−23] noise,[23−25] and signal-to-noise ratio.[26−29] However, all the results were given under the usually two-dimensional integral representation theory. Recently, a new coherent-mode representation theory has been proposed to analyze ghost imaging with a classical light source, and the results showed that this theory is suitable for evaluating the imaging quality, i.e., resolution.[30] However, to the best of our knowledge, the noise analysis of ghost imaging based on the coherent-mode representation has not been discussed. In this paper, the noise properties of ghost imaging from the perspective of the coherent-mode representation will be studied. Based on the coherent-mode representation of partially coherent fields, the fluctuation in the imaging signal can be changed from the usually two-dimensional integral representation to a new one-dimensional summation representation. The analytical and numerical results show that the noise of ghost imaging can be analyzed by the distribution of the eigenvalues of the coherent-mode representation of the source and the decomposition coefficients of the object imaged.

2. Analytical results

A simplified scheme for ghost imaging is shown in Fig. 1. The pseudo-thermal source is prepared by illuminating a pulsed Nd:YAG laser into a slowly rotating ground glass, then is divided into two beams by a beam splitter (BS), they travel on their respective paths to be detected at spatially separated detection systems. In the test arm, the beam usually passes through a transmission object, and then to the test detector D_1 which is a pointlike detector with no spatial resolution. In the reference arm there is not usually any object, the beam propagates directly to

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a spatially resolving detector D_2. From the previous results,[6,7] the information of the object can be retrieved by measuring the spatial correlation function of the intensities detected by two detectors[14,15]

$$\begin{aligned} G^{(2)}(u_1,u_2) &= \langle E(u_1)E(u_2)E^*(u_2)E^*(u_1)\rangle \\ &= \langle I_1(u_1)\rangle\langle I_2(u_2)\rangle + G(u_1,u_2) \end{aligned} \quad (1)$$

with

$$\begin{aligned} \langle I_i(u_i)\rangle &= \int \Gamma(x_1,x_2)h_i(x_1,u_i)h_i^*(x_2,u_i)\mathrm{d}x_1\mathrm{d}x_2, \\ G(u_1,u_2) &= \langle \Delta I_1(u_1)\Delta I_2(u_2)\rangle \\ &= \Big|\int \Gamma(x_1,x_2)h_1(x_1,u_1) \\ &\quad \times h_2^*(x_2,u_2)\mathrm{d}x_1\mathrm{d}x_2\Big|^2, \end{aligned}$$

where $\langle I_i(u_i)\rangle$ $(i=1,2)$ is the intensity distribution at the i-th detector, $G(u_1,u_2)$ is the intensity fluctuation correlation function, and $\Gamma(x_1,x_2)$ is the second-order correlation function of the source.

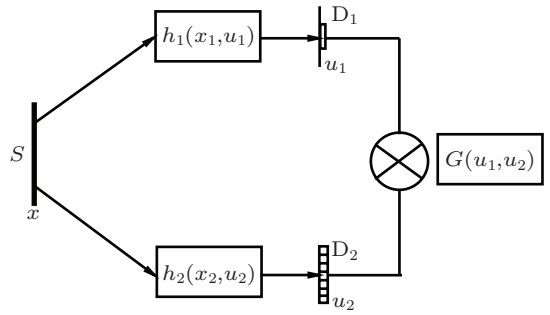


Fig. 1. A simplified scheme for ghost imaging.

To study the noise in ghost imaging, we need to give the fluctuation in imaging signal $G(u_1,u_2)$ which has been defined in Ref. [23]

$$\Delta G(u_1,u_2) = \Big\{\langle[\Delta I_1\Delta I_2]^2\rangle - [G(u_1,u_2)]^2\Big\}^{1/2} \quad (2)$$

with

$$\langle[\Delta I_1\Delta I_2]^2\rangle = [\langle I_1\rangle\langle I_2\rangle + 2G(u_1,u_2)]^2.$$

From Ref. [30], the intensity fluctuation correlation depicted in Eq. (2) can be changed from the usually two-dimensional integral representation to a new one-dimensional summation representation under the second-order coherence theory of optical fields.[31] However, the first term of Eq. (1), i.e., the background term is not included in their results. Here we consider this term, and give the intensity correlation function based on the coherence-mode representation:

$$\begin{aligned} G^{(2)}(u_1,u_2) &= \sum_n \beta_n|f_n(u_1)|^2\sum_n \beta_n|g_n(u_2)|^2 \\ &\quad + \Big|\sum_n \beta_n f_n^*(u_1)g_n(u_2)\Big|^2. \end{aligned} \quad (3)$$

By substituting Eqs. (1), (2), and (4) into Eq. (3), the fluctuation in the imaging signal under the coherence-mode representation can be expressed as:

$$\begin{aligned} \Delta G(u_1,u_2) &= \Big\{\Big(\sum_n \beta_n|f_n(u_1)|^2\sum_n \beta_n|g_n(u_2)|^2\Big)^2 \\ &\quad + 3\Big|\sum_n \beta_n f_n^*(u_1)g_n(u_2)\Big|^4 \\ &\quad + 4\sum_n \beta_n|f_n(u_1)|^2\sum_n \beta_n|g_n(u_2)|^2 \\ &\quad \times\Big|\sum_n \beta_n f_n^*(u_1)g_n(u_2)\Big|^2\Big\}^{1/2}. \end{aligned} \quad (4)$$

For simplicity, we choose a simple $4f$ imaging system[4] to discuss the noise properties. From Ref. [30], the imaging signal, i.e., the conditional intensity fluctuation correlation function under the coherence-mode representation represents a degraded image of the object (see Eq. (7) in Ref. [30]. By considering this equation, the noise in the $4f$ ghost-imaging system can be written as

$$\begin{aligned} \Delta G(0,u_2) &= \Big\{\Big(\sum_n \beta_n|f_n(0)|^2\sum_n \beta_n|\phi_n(-u_2)|^2\Big)^2 \\ &\quad + 3\Big|\sum_n \beta_n f_n^*(0)\phi_n(-u_2)\Big|^4 \\ &\quad + 4\sum_n \beta_n|f_n(0)|^2\sum_n \beta_n|\phi_n(-u_2)|^2 \\ &\quad \times\Big|\sum_n \beta_n f_n^*(0)\phi_n(-u_2)\Big|^2\Big\}^{1/2}, \end{aligned} \quad (5)$$

where β_n and ϕ_n are the corresponding eigenvalue and eigenfunction of the coherent-mode representation of the Gaussian Schell-model source[31]

$$\begin{aligned} \beta_n &= \Big(\frac{\pi}{a+b+c}\Big)^{1/2}\Big(\frac{b}{a+b+c}\Big)^n, \\ \phi_n(x) &= \Big(\frac{2c}{\pi}\Big)^{1/4}\frac{1}{\sqrt{2^n n!}}\,\mathrm{H}_n(x\sqrt{2c})\,\mathrm{e}^{-cx^2}, \end{aligned} \quad (6)$$

where $a=1/4\sigma_l^2$, $b=1/2\sigma_g^2$, and $c=\sqrt{a^2+2ab}$. σ_l is the source's transverse size, σ_g is the transverse coherence width of the source, and $\mathrm{H}_n(x)$ are the Hermite polynomials.

From Eq. (6), it should be noted that the distribution of both $f_n(0)$ and β_n can affect the fluctuation in the imaging signal. So we can analyze the noise properties by the distribution of the eigenvalues of the coherent-mode representation of the source and the decomposition coefficients of the object imaged $f_n(0)$.

3. Numerical results

In this section, we attempt to investigate the relation between the noise properties and the distribution of the decomposition coefficients $f_n(0)$ and the eigenvalues β_n by numerical simulations.

First let us choose a proper object imaged, i.e., a double-slit with the slit width 0.07 mm and the distance between two slits 0.16 mm. Here the parameters are chosen as $\sigma_l = \sqrt{1/4a} = 4.0$ mm, $\sigma_{\rm g} = \sqrt{1/2b} = 300$ μm, and $n = 50$. Figure 2 shows the imaging signal (dashed curve) and its fluctuation (solid curve), and the corresponding distribution of the eigenvalues β_n of the coherent-mode representation of the source and the decomposition coefficients $f_n(0)$ of the object is also presented. From Fig. 2, it is shown that the noise is not very large. At the same time, $f_n(0)$ has a relatively narrow distribution when comparing with the distribution of β_n.

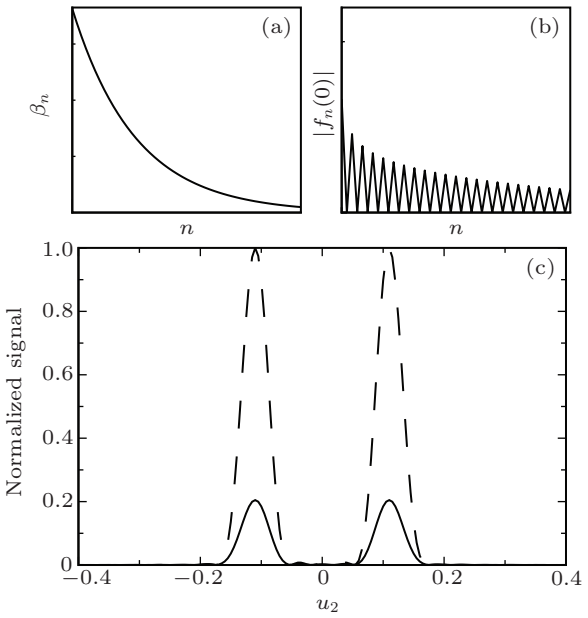


Fig. 2. (a) The first 50 eigenvalues of the coherent-mode representation of the source; (b) the first 50 decomposition coefficients of the object; (c) the normalized imaging signal $G(0, u_2)$ (dashed line) and its fluctuation $\Delta G(0, u_2)$ (solid line) as a function of u_2. The parameters are chosen as $\sigma_l = \sqrt{1/4a} = 4.0$ mm, $\sigma_g = \sqrt{1/2b} = 300$ μm, and $n = 50$.

We next enquire "if the distribution varieties of the decomposition coefficients of the object and the eigenvalues of the coherent-mode representation can reflect the changes of classical noise?" Below we present a series of calculations for various values of σ_l and $\sigma_{\rm g}$. In Fig. 3, We keep $\sigma_{\rm g} = 300$ μm unchanged, but decrease the transverse size from $\sigma_l = 4.0$ mm in Fig. 2 to $\sigma_l = 0.3$ mm. The noise increases as compared with that in Fig. 2. In other words, a decrease of the source's transverse size results in an increase of the noise, which is consistent with the theoretical result[23] from the usually two-dimensional integral representation. It is obvious that only a simple proportional relation could be given from the integral representation. Here, we can understand more clearly the changes of the noise by the distribution varieties of β_n and $f_n(0)$ under the new coherent-mode representation. From Fig. 3, it can be seen that the distribution of β_n has a narrower distribution and $f_n(0)$ becomes wide when the source's transverse size is decreased, the changes result in the increase of classical noise.

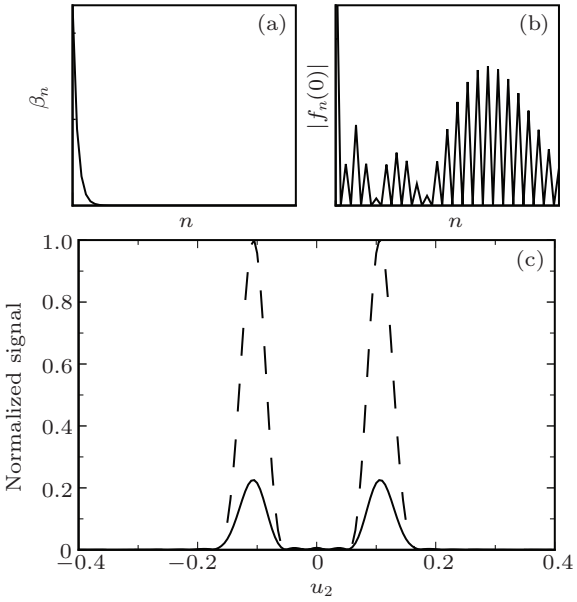


Fig. 3. The same as Fig. 2 except for $\sigma_l = 0.3$ mm.

Clearly, two methods can be used to decrease the classical noise, i.e., increasing the distribution of the eigenvalues of the coherent-mode representation of the source or decreasing the distribution of the decomposition coefficients of the object imaged.

We next explore the influence of the source's transverse coherence width based on the above conclusion. In Fig. 4, we choose a smaller value of the transverse coherence width $\sigma_{\rm g} = 80$ μm, but keep other parameters in Fig. 2 unchanged. It is shown that β_n has a relative wide distribution, while the distribution of $f_n(0)$ becomes narrow when the source's transverse coherence width is decreased, which results in the decrease of classical noise.

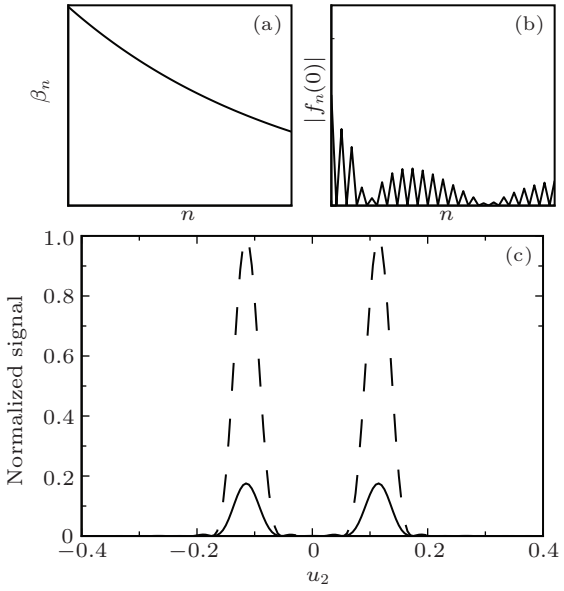


Fig. 4. The same as Fig. 2, but with a decreased transverse coherence width of $\sigma_g = 80$ μm.

4. Conclusions

In conclusion, we have theoretically investigated the noise properties in a ghost imaging system with partially coherent radiation by using the coherent-mode representation theory. The fluctuation in the imaging signal depends crucially on the distribution of the eigenvalues of the coherent-mode representation of the source and the decomposition coefficients of the object imaged. Both increasing the distribution of the eigenvalues and decreasing the distribution of the decomposition coefficients can result in the decrease of classical noise.

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