The two-mode quantum Fresnel operator and the multiplication rule of 2D Collins diffraction formula*

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(Received 27 September 2011; revised manuscript received 10 November 2011)

By using the two-mode Fresnel operator we derive a multiplication rule of two-dimensional (2D) Collins diffraction formula, the inverse of 2D Collins diffraction integration can also be conveniently derived in this way in the context of quantum optics theory.

Keywords: 2D Collins diffraction formula, two-mode Fresnel operator

PACS: 42.25.Fx, 42.50.–p **DOI:** 10.1088/1674-1056/21/4/044203

1. Introduction

Usually the optical transformation occurs in the three-dimensional (3D) space and the input and output optical signals are distributed two-dimensionally. After an input of two-dimensional (2D) optical signal denoted by $f(x_1, x_2)$ undergoing an optical imaging process (or passing an optical imaging instrument) which is characterized by a real matrix $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ in which the parameters obeying $AD - BC = 1$,[1] the output amplitude of the optical signal becomes $g(x_1', x_2')$, the whole process can be described by the Collins diffraction formula[2]

$$g(x_1', x_2') = \iint_{-\infty}^{\infty} \prod_{j=1,2} \frac{1}{\sqrt{2\pi \,\mathrm{i}\, B}} \times \exp\left[\frac{\mathrm{i}}{2B}(Ax_j^2 - 2x_j x_j' + Dx_j'^2)\right] \times f(x_1, x_2)\,\mathrm{d}x_1\,\mathrm{d}x_2, \quad (1)$$

which is very useful in wave propagation in a near-axis optical lens and imaging of classical optics. If the optical field $g(x_1', x_2')$ undergoes another optical imaging process characteristic of another matrix $\begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$ described by the Collins integration

$$h(x_1'', x_2'') = \iint_{-\infty}^{\infty} \prod_{j=1,2} \frac{1}{\sqrt{2\pi \,\mathrm{i}\, B'}} \times \exp\left[\frac{\mathrm{i}}{2B'}(A'x_j'^2 - 2x_j' x_j'' + D'x_j''^2)\right] \times g(x_1', x_2')\,\mathrm{d}x_1'\,\mathrm{d}x_2', \quad (2)$$

then can these two successive transformations' result, be equivalent to a single integration transformation related to $f(x_1, x_2)$ and $h(x_1'', x_2'')$, i.e., does a multiplication rule exist? The answer is affirmative. In the following we will discuss the question by virtue of the two-mode Fresnel operator in the context of quantum optics and will derive a new theorem about the multiplication rule of 2D Collins diffraction formula and its inverse. In Section 2, we give the 2D Collins formula in entangled representation and its inverse derived by virtue of the two-mode Fresnel operator, and in Section 3 we will give another form of the 2D Collins formula in the two-mode coordinate representation. In Section 4, we will derive the multiplication rule of the 2D Collins diffraction formula obtained by virtue of the two-mode Fresnel operator. Finally, we give some discussions.

*Project supported by the Doctoral Scientific Research Startup Fund of Anhui University, China (Grant No. 33190059), the National Natural Science Foundation of China (Grant No. 10874174), the Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20113401120004), and the Open Funds from National Laboratory for Infrared Physics, Chinese Academy of Sciences (Grant No. 201117).

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2. 2D Collins diffraction formula in entangled representation and its inverse derived by virtue of the two-mode Fresnel operator

In Refs. [3] and [4] the quantum two-mode Fresnel operator is defined in the two-mode coherent state representation by the following form

$$F_2(r,s) = s\int \frac{1}{\pi^2}\mathrm{d}^2 z_1 \mathrm{d}^2 z_2 \left|sz_1 + rz_2^*, rz_1^* + sz_2\right\rangle \times \left\langle z_1, z_2\right|, \tag{3}$$

where

$$\left|z_1, z_2\right\rangle = \exp\left(-\frac{1}{2}\left|z_1\right|^2 - \frac{1}{2}\left|z_2\right|^2 + z_1 a_1^\dagger + z_2 a_2^\dagger\right)\left|00\right\rangle$$

is the canonical two-mode coherent state,[5,6] F_2 is unitary, it transfoms (z_1, z_2) to $(sz_1 + rz_2^*, rz_1^* + sz_2)$ in quantum phase space, $|s|^2 - |r|^2 = 1$. By using the integration within an ordered product (IWOP)[7−10] technique to perform the integration (3) we obtain

$$F_2(r,s) = \frac{1}{s^*}\exp\left(\frac{r}{s^*}a_1^\dagger a_2^\dagger\right) \times :\exp\left[\left(\frac{1}{s^*} - 1\right)(a_1^\dagger a_1 + a_2^\dagger a_2)\right]: \times \exp\left(-\frac{r^*}{s^*}a_1 a_2\right). \tag{4}$$

Noting the fact that relations about s, r are related to A, B, C, and D by

$$s = \frac{1}{2}\left[A + D - \mathrm{i}(B - C)\right], \quad r = \frac{-1}{2}\left[A - D + \mathrm{i}(B + C)\right], \tag{5}$$

we can further obtain

$$\begin{aligned} &F_2(A,B,C) \\ &= \frac{2}{A + D + \mathrm{i}(B - C)} \\ &\times :\exp\left\{-\frac{A - D + \mathrm{i}(B + C)}{A + D + \mathrm{i}(B - C)}a_1^\dagger a_2^\dagger \right. \\ &+ \left(\frac{2}{A + D + \mathrm{i}(B - C)} - 1\right)(a_1^\dagger a_1 + a_2^\dagger a_2) \\ &\left. + \frac{A - D - \mathrm{i}(B + C)}{A + D + \mathrm{i}(B - C)}a_1 a_2\right\}: . \end{aligned} \tag{6}$$

Then using the completeness relation of the two-mode coherent state

$$\iint \frac{\mathrm{d}^2 z_1 \mathrm{d}^2 z_2}{\pi^2}\left|z_1, z_2\right\rangle\left\langle z_1, z_2\right| = 1, \tag{7}$$

and the overlap between the entangled representation $|\eta\rangle$[11−14] ($\eta = \eta_1 + \mathrm{i}\,\eta_2$ is a complex number) and the two-mode coherent state $|z_1, z_2\rangle$,

$$\left\langle \eta\right|\left.z_1, z_2\right\rangle = \exp\left(-\frac{\left|\eta\right|^2}{2} - \frac{\left|z_1\right|^2 + \left|z_2\right|^2}{2} + \eta z_1 - \eta^* z_2 + z_1 z_2\right), \tag{8}$$

we can obtain

$$\begin{aligned} \kappa_2^M(\eta,\eta') &\equiv \left\langle\eta\right| F_2(A,B,C)\left|\eta'\right\rangle \\ &= \frac{1}{2\,\mathrm{i}\,B}\exp\left\{\frac{\mathrm{i}}{2B}[A\left|\eta'\right|^2 + D\left|\eta\right|^2 \right. \\ &\left. - (\eta'\eta^* + \eta'^*\eta)]\right\}, \end{aligned} \tag{9}$$

where the superscript M of $\kappa_2^M(\eta,\eta')$ is the parameter matrix $[A,B,C,D]$, the subscript 2 representing the transformation of Eq. (9) is two-dimensional. We can call Eq. (9) as integration kernel of the 2D Collins formula in the entangled representation, for letting $|g\rangle = F_2(A,B,C)\,|f\rangle$ and projecting $|g\rangle$ to the entangled representation $|\eta'\rangle$, then using the completeness relation of the entangled representation we can have

$$\begin{aligned} g(\eta') &\equiv \left\langle\eta'\right.\left|g\right\rangle = \left\langle\eta'\right| F_2(A,B,C)\left|f\right\rangle \\ &= \int\frac{\mathrm{d}^2\eta}{\pi}\left\langle\eta'\right| F_2(A,B,C)\left|\eta\right\rangle\left\langle\eta\right|\left.f\right\rangle \\ &= \int\frac{\mathrm{d}^2\eta}{\pi}\kappa_2^M(\eta',\eta)f(\eta) \\ &= \frac{1}{2\,\mathrm{i}\,B}\int\frac{\mathrm{d}^2\eta}{\pi}\exp\left\{\frac{\mathrm{i}}{2B}[A\left|\eta\right|^2 + D\left|\eta'\right|^2 \right. \\ &\left. - (\eta\eta'^* + \eta^*\eta')]\right\}f(\eta). \end{aligned} \tag{10}$$

For the inverse of matrix $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is matrix $M^{-1} = \begin{pmatrix} D & -B \\ -C & A \end{pmatrix}$, so accordingly the inverse of the 2D Collins formula is

$$\begin{aligned} f(\eta) &= \left\langle\eta\right|\left.f\right\rangle = \left\langle\eta\right| F_2(A,B,C)^{-1}\left|g\right\rangle \\ &= \int\frac{\mathrm{d}^2\eta'}{\pi}\left\langle\eta\right| F_2\left(\begin{pmatrix} D & -B \\ -C & A \end{pmatrix}\right)\left|\eta'\right\rangle\left\langle\eta'\right|\left.g\right\rangle \\ &= \int\frac{\mathrm{d}^2\eta'}{\pi}\kappa_2^{M^{-1}}(\eta,\eta')g(\eta'), \end{aligned} \tag{11}$$

where

$$\kappa_2^{M^{-1}}(\eta,\eta') = \frac{1}{-2\,\mathrm{i}\,B}\exp\left\{\frac{\mathrm{i}}{-2B}[D\left|\eta'\right|^2 + A\left|\eta\right|^2\right.$$

$$-(\eta'\eta^* + \eta'^*\eta)]\Big\}. \tag{12}$$

So far, we have derived the 2D Collins formula and its inverse by virtue of the two-mode Fresnel operator in the context of quantum optics. The reason why we can call Eq. (10) as the 2D Collins formula is that if letting $\eta_1 = x_1, \eta_2 = x_2$ and simultaneously $\eta_1' = x_1', \eta_2' = x_2'$, we can obtain

$$\begin{aligned}\kappa_2^M(\eta', \eta) &= \kappa_2^M(x_1', x_2'; x_1, x_2)\\ &= \kappa_1^M(x_1, x_1') \otimes \kappa_1^M(x_2, x_2'),\end{aligned} \tag{13}$$

where

$$\begin{aligned}&\kappa_1^M(x, x')\\ &= \frac{1}{\sqrt{2\pi\,\mathrm{i}\,B}} \exp\left[\frac{\mathrm{i}}{2B}(Ax^2 - 2x'x + Dx'^2)\right]\end{aligned} \tag{14}$$

is the integration kernel of 1D Collins formula. Compared with Eq. (1), we can see Eq. (13) representing that the $F_2(A, B, C)$ is truly a two-mode Fresnel transformation.

3. 2D Collins formula in the two-mode coordinate representation

Further using the overlap between the entangled representation $|\eta\rangle$ and the two-mode coordinate representation $|x_1, x_2\rangle$

$$\langle \eta = \eta_1 + \mathrm{i}\,\eta_2 \,|x_1, x_2\rangle$$

$$\begin{aligned}&= \exp(\mathrm{i}\,\eta_1\eta_2)\delta(\sqrt{2}\eta_1 + x_2 - x_1)\\ &\quad \times \exp(-\mathrm{i}\sqrt{2}\eta_2 x_1),\end{aligned} \tag{15}$$

we can obtain the matrix element of the two-mode Fresnel operator in the two-mode coordinate representation, which can be called the integration kernel of the 2D Collins formula in coordinate representation

$$\begin{aligned}&\langle x_1', x_2'|\, F_2(A, B, C)\, |x_1, x_2\rangle\\ &= \int \frac{\mathrm{d}^2\eta'\,\mathrm{d}^2\eta}{\pi^2}\, \langle x_1', x_2'|\, \eta'\rangle\\ &\quad \times \langle \eta'|\, F_2(A, B, C)\, |\eta\rangle\, \langle \eta|\, x_1, x_2\rangle\\ &= \frac{1}{\sqrt{2\pi\,\mathrm{i}\,B}} \exp\left[\frac{\mathrm{i}}{2B}(A\lambda^2 - 2\lambda\lambda' + D\lambda'^2)\right] \frac{1}{\sqrt{2\pi\,\mathrm{i}(-C)}}\\ &\quad \times \exp\left[\frac{\mathrm{i}}{2(-C)}(D\mu^2 - 2\mu\mu' + A\mu'^2)\right],\end{aligned} \tag{16}$$

where

$$\begin{aligned}\lambda &= \frac{x_1 - x_2}{\sqrt{2}}, \quad \lambda' = \frac{x_1' - x_2'}{\sqrt{2}},\\ \mu &= \frac{x_1 + x_2}{\sqrt{2}}, \quad \mu' = \frac{x_1' + x_2'}{\sqrt{2}}.\end{aligned} \tag{17}$$

Projecting the input optical signal $f(x_1, x_2)$ and output optical signal $g(x_1', x_2')$ on the coordinate representation, i.e., letting $f(x_1, x_2) = \langle x_1, x_2|\, f\rangle$, $g(x_1', x_2') = \langle x_1', x_2'|\, g\rangle$, then using Eq. (16) we have another 2D Collins diffraction formula in two-mode coordinate representation,

$$\begin{aligned}g(x_1', x_2') &= \iint_{-\infty}^{\infty} \mathrm{d}x_1\,\mathrm{d}x_2\, \langle x_1', x_2'|\, F_2(A, B, C)\, |x_1, x_2\rangle\, \langle x_1, x_2|\, f\rangle\\ &= \iint_{-\infty}^{\infty} \mathrm{d}x_1\,\mathrm{d}x_2 \frac{1}{\sqrt{2\pi\,\mathrm{i}\,B}} \exp\left[\frac{\mathrm{i}}{2B}(A\lambda^2 - 2\lambda\lambda' + D\lambda'^2)\right]\\ &\quad \times \frac{1}{\sqrt{2\pi\,\mathrm{i}(-C)}} \exp\left[\frac{\mathrm{i}}{2(-C)}(D\mu^2 - 2\mu\mu' + A\mu'^2)\right] f(x_1, x_2).\end{aligned} \tag{18}$$

Accordingly, the inverse of the above 2D Collins formula (18) is

$$\begin{aligned}f(x_1, x_2) &= \iint_{-\infty}^{\infty} \mathrm{d}x_1'\,\mathrm{d}x_2'\, \langle x_1, x_2|\, F_2\begin{pmatrix} D & -B\\ -C & A\end{pmatrix} |x_1', x_2'\rangle\, \langle x_1', x_2'|\, g\rangle\\ &= \iint_{-\infty}^{\infty} \mathrm{d}x_1'\,\mathrm{d}x_2' \frac{1}{\sqrt{-2\pi\,\mathrm{i}\,B}} \exp\left[\frac{\mathrm{i}}{-2B}(D\lambda'^2 - 2\lambda'\lambda + A\lambda^2)\right]\\ &\quad \times \frac{1}{\sqrt{2\pi\,\mathrm{i}\,C}} \exp\left[\frac{\mathrm{i}}{2C}(A\mu'^2 - 2\mu'\mu + D\mu^2)\right].\end{aligned} \tag{19}$$

So far, we have derived the 2D Collins formula and its inverse in two forms (one in entangled state representation, another in the coordinate representation) by virtue of the two-mode Fresnel operator in the context of quantum optics.

4. Multiplication rule of the Collins diffraction formula obtained by virtue of the two-mode Fresnel operator

Using Eq. (3) and the overlap of coherent state we can obtain

$$\begin{aligned}
&F_2(r',s')F_2(r,s)\\
&= ss'\int \frac{\mathrm{d}^2z_1'\,\mathrm{d}^2z_2'\mathrm{d}^2z_1\mathrm{d}^2z_2}{\pi^4}\,|sz_1'+rz_2'^*, rz_1'^*+sz_2'\rangle\\
&\quad\times\langle z_1', z_2'|\, sz_1+rz_2^*, rz_1^*+sz_2\rangle\,\langle z_1,z_2|\\
&= \frac{1}{s'^*s^*+r'^*r}\exp\left(\frac{s'r+r's^*}{s'^*s^*+r'^*r}a_1^\dagger a_2^\dagger\right)\\
&\quad:\exp\left[\left(\frac{1}{s'^*s^*+r'^*r}-1\right)(a_1^\dagger a_1+a_2^\dagger a_2)\right]:\\
&\quad\times\exp\left[-\frac{s'^*r^*+r'^*s}{s'^*s^*+r'^*r}a_1a_2\right];
\end{aligned}\tag{20}$$

by letting $s''=s's+r'r^*, r''=s'r+r's^*$, equation (20) becomes

$$\begin{aligned}
&F_2(r',s')F_2(r,s)\\
&= \frac{1}{s''^*}\exp\left(\frac{r''}{s''^*}a_1^\dagger a_2^\dagger\right)\\
&\quad:\exp\left[\left(\frac{1}{s''^*}-1\right)(a_1^\dagger a_1+a_2^\dagger a_2)\right]:\exp\left(-\frac{r''^*}{s''^*}\right)\\
&= \exp\left(\frac{r''}{s''^*}a_1^\dagger a_2^\dagger\right)\\
&\quad\times\exp\left[(a_1^\dagger a_1+a_2^\dagger a_2+1)\ln(s''^*)^{-1}\right]\\
&\quad\times\exp\left(-\frac{r''^*}{s''^*}\right).
\end{aligned}\tag{21}$$

Note that it just happens

$$\begin{pmatrix} s'' & -r''\\ -r''^* & s''^*\end{pmatrix}=\begin{pmatrix} s' & -r'\\ -r'^* & s'^*\end{pmatrix}\begin{pmatrix} s & -r\\ -r^* & s^*\end{pmatrix},\tag{22}$$

or

$$\begin{aligned}
\begin{pmatrix} A'' & B''\\ C'' & D''\end{pmatrix}&=\begin{pmatrix} A' & B'\\ C' & D'\end{pmatrix}\begin{pmatrix} A & B\\ C & D\end{pmatrix}\\
&=\begin{pmatrix} A'A+B'C & A'B+B'D\\ C'A+D'C & C'B+D'D\end{pmatrix},
\end{aligned}\tag{23}$$

$$A''D''-B''C''=1,\tag{24}$$

hence we have

$$\begin{aligned}
&F_2(r',s')F_2(r,s)\\
&= s\int\frac{1}{\pi^2}\mathrm{d}^2z_1\,\mathrm{d}^2z_2\,|s''z_1+r''z_2^*, r''z_1^*+s''z_2\rangle\,\langle z_1,z_2|\\
&= F_2(A'',B'',C''),
\end{aligned}\tag{25}$$

so two successive Fresnel transformations are still a Fresnel transformation, and still we have $\det\begin{pmatrix} A'' & B''\\ C'' & D''\end{pmatrix}=1$. Then from Eqs. (10) and (18) we can obtain the multiplication rule of two 2D Collins formulas in two forms. One is in the entangled representation

$$\begin{aligned}
h(\eta'') &= \frac{1}{2\,\mathrm{i}(A'B+B'D)}\\
&\quad\times\int\frac{\mathrm{d}^2\eta}{\pi}\exp\left\{\frac{\mathrm{i}}{2(A'B+B'D)}\right.\\
&\quad\times\left[(A'A+B'C)\,|\eta|^2+(C'B+D'D)\,|\eta''|^2\right.\\
&\quad\left.\left.-(\eta\eta''^*+\eta^*\eta'')\right]\right\}f(\eta);
\end{aligned}\tag{26}$$

and another is in the two-mode coordinate representation

$$\begin{aligned}
&h(x_1'',x_2'')\\
&= \iint_{-\infty}^{\infty}\mathrm{d}x_1\,\mathrm{d}x_2\frac{1}{\sqrt{2\pi\,\mathrm{i}(A'B+B'D)}}\\
&\quad\times\exp\left\{\frac{\mathrm{i}}{2(A'B+B'D)}[(A'A+B'C)\lambda^2\right.\\
&\quad\left.-2\lambda\lambda''+(C'B+D'D)\lambda''^2]\right\}\\
&\quad\times\frac{1}{\sqrt{2\pi\,\mathrm{i}(-(C'A+D'C))}}\\
&\quad\times\exp\left\{\frac{\mathrm{i}}{2(-(C'A+D'C))}[(C'B+D'D)\mu^2\right.\\
&\quad\left.-2\mu\mu''+(A'A+B'C)\mu''^2]\right\}f(x_1,x_2),
\end{aligned}\tag{27}$$

where

$$\begin{aligned}
\lambda&=\frac{x_1-x_2}{\sqrt{2}},\quad \lambda''=\frac{x_1''-x_2''}{\sqrt{2}},\\
\mu&=\frac{x_1+x_2}{\sqrt{2}},\quad \mu''=\frac{x_1''+x_2''}{\sqrt{2}}.
\end{aligned}\tag{28}$$

Equations (26) and (27) are new theorems about the multiplication rule of the 2D Collins formula. The inverse transformations of Eqs. (26) and (27) are

$$f(\eta)=\frac{1}{-2\,\mathrm{i}(A'B+B'D)}$$

$$\times \int \frac{\mathrm{d}^2\eta''}{\pi} \exp\left\{\frac{\mathrm{i}}{-2(A'B+B'D)} \times \left[(A'A+B'C)\left|\eta''\right|^2 + (C'B+D'D)\left|\eta\right|^2 - (\eta''\eta^* + \eta''^*\eta)\right]\right\} h(\eta'') \tag{29}$$

and

$$\begin{aligned} & f(x_1, x_2) \\ = & \iint_{-\infty}^{\infty} \mathrm{d}x_1'' \mathrm{d}x_2'' \frac{1}{\sqrt{-2\pi\,\mathrm{i}(A'B+B'D)}} \\ & \times \exp\left\{\frac{\mathrm{i}}{-2(A'B+B'D)}[(C'B+D'D)\lambda''^2 \right. \\ & \left. - 2\lambda''\lambda + (A'A+B'C)\lambda^2]\right\} \\ & \times \frac{1}{\sqrt{2\pi\,\mathrm{i}(C'A+D'C)}} \\ & \times \exp\left\{\frac{\mathrm{i}}{2(C'A+D'C)}(A'A+B'C)\mu'^2 - 2\mu''\mu \right. \\ & \left. + (C'B+D'D)\mu^2\right\} h(x_1'', x_2''), \end{aligned} \tag{30}$$

as

$$\begin{pmatrix} A'A+B'C & A'B+B'D \\ C'A+D'C & C'B+D'D \end{pmatrix}^{-1} = \begin{pmatrix} C'B+D'D & -(A'B+B'D) \\ -(C'A+D'C) & A'A+B'C \end{pmatrix}. \tag{31}$$

In summary, we have derived two forms of new theorems about the multiplication rule of the 2D Collins formula obtained by virtue of the two-mode Fresnel operator in quantum optics theory. The obtained multiplication rule of the 2D Collins formula will bring some convenience to the study of the 2D Collins diffraction.

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