Simplified triangular ground plane cloak by oblique multilayer dielectrics*

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Based on the effective medium theory, the triangular ground plane cloak can be realized by thin layered systems. Two solutions of parameter setting of the layered cloak are suggested to demonstrate the invisibility performance of a hybrid incoming wave. The hybrid parameters are derived from the equivalent of both anisotropies of permittivity and permeability to the alternating layers. The performance of the designed layered cloak is validated by both TM and TE wave simulations with near-field distributions and average scattering power outflows on an observation semicircle. From the simulation results, the layered cloak with both hybrid parameters and improved hybrid parameters can reflect the incoming TM/TE waves in a specular direction, and the latter behaves with a better overall invisibility performance.

Keywords: transformation media, ground plane cloak, effective medium theory

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1. Introduction

Based on transformation optics, invisibility cloaks[1,2] have attracted intensive attention on both theoretical analysis and practical implementation.[3] There are two kinds of invisibility cloak: the free-space cloak,[1−5] which is commonly deformed from an empty space into a shell region, can ensure the electromagnetic (EM) waves are bent round the inside hidden region, leaving the object inside the region without interacting with the EM waves. However, the full constitutive parameters for an ideal free-space invisibility cloak would be singular on the inner boundaries, which is hard to practically realize and is usually working at a narrow frequency. Then some simplified cloaks have been proposed to avoid the singularity;[6,7] however, these simplified structures would lead to unexpected scattering, which degrades the invisibility performance. Besides the free-space cloak, the so-called carpet or ground plane cloak has also attracted lots of attention, mimicking a flat perfect electric conducting (PEC) ground plane[8] and having been experimentally verified in microwave frequency.[9] Such a ground plane cloak could avoid both material and geometric singularities and can work in a broadband frequency region. The material for the ground plane cloak is commonly still inhomogeneous and mostly designed using a quasi-conformal method, which allows minimal anisotropy and facilitates fabrication.[10,11] An alternative approach is proposed to design a square cloak[12] and triangular cloaks[13−17] by rotating coordinates and utilizing alternating layers of isotropic dielectrics. The parameters of the triangular cloak are homogeneous, leading to easy realization; and it has recently been introduced in macroscopic scale based on a natural birefringent crystal and experimentally demonstrated at visible frequencies.[14,15] The hidden object is relatively very small compared with the cloak device for the low anisotropy of natural material. To obtain high-anisotropy material, a uniform SiO_2 grating structure with proper filling factor has been fabricated on a bulk Si substrate and it was verified experimentally for the cloak.[17] However, the concealed area is still a few times smaller than the cloak itself and it should be fabricated on a host medium (Si substrate) for the rigorous limitations. Furthermore, all the cloaks realized by multilayer structure are designed for TM polarization electromagnetic wave with permeability being constant for simplicity.

In this paper, we suggest that oblique alternating layers are set with hybrid parameters and improved hybrid parameters to realize the triangular ground

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plane cloak. The layered cloak with both parameter settings has good invisibility performances for TE and TM polarization incoming waves. When the background medium is air, two kinds of dielectrics can be realized by electric plasma and normal material, respectively, without a limitation of ratio of hidden area to the cloak device.

2. Scheme design

Coordinate transformation is usually chosen to design the ground plane cloak by space squeezing. A schematic diagram illustrating a triangular cloak is shown in Fig. 1. The region within the dashed triangle in the virtual space [Fig. 1(a)] is squeezed into two triangles in the physical space [Fig. 1(b)] with inner perfect electric conducting (PEC) boundaries, resulting in a triangular region blind to external illumination. It can be seen that the deformation is only along y direction in a uniformly linear manner. The transformation equations are given as

$$x = x', \quad y = \frac{\delta d}{a}x' + (1-\delta)y' + \delta d, \quad z = z' \quad \text{(for region I)};$$
$$x = x', \quad y = -\frac{\delta d}{a}x' + (1-\delta)y' + \delta d, \quad z = z' \quad \text{(for region II)}. \tag{1}$$

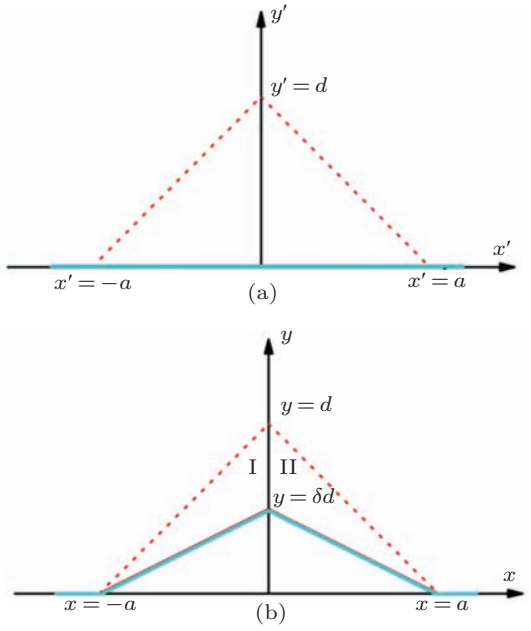


Fig. 1. (colour online) The scheme of coordinate transformation from the virtual space (a) to physical space (b).

Then we can obtain the parameters in regions I and II from the standard procedure of transformation optics[1]

$$\overleftrightarrow{\varepsilon} = \overleftrightarrow{\mu} = \begin{bmatrix} 1/P & \pm K/P & 0 \\ \pm K/P & \left(K^2+P^2\right)/P & 0 \\ 0 & 0 & 1/P \end{bmatrix}, \tag{2}$$

where $K = \delta d/a$, $P = 1-\delta$. The positive sign in the tensor is for region I and negative sign for region II. Elements of the permittivity or permeability tensor are constants, which represents a spatially invariant anisotropic dielectric. It also can be seen that the tensor is symmetric, which can be expressed as a diagonal constant tensor $\overleftrightarrow{\varepsilon} = \mathrm{diag}\,[\varepsilon_u, \varepsilon_v, \varepsilon_w]$ in a rotated $Ouvw$ coordinate system as two-dimensionally depicted in Fig. 2. The elements of the diagonal tensor can be written as follows:

$$\begin{aligned} \varepsilon_v &= (P + 1/P + K^2/P)/2 \\ &\quad + \sqrt{(P + 1/P + K^2/P)^2 - 4}/2, \\ \varepsilon_u &= (P + 1/P + K^2/P)/2 \\ &\quad - \sqrt{(P + 1/P + K^2/P)^2 - 4}/2, \\ \varepsilon_w &= \varepsilon_z. \end{aligned}$$

and the rotating angles of the original $Oxyz$ coordinate system are obtained as $\sin\alpha = -\sqrt{(1/P - \varepsilon_u)/(\varepsilon_v - \varepsilon_u)}$ for region I, and $\sin\alpha = \sqrt{(1/P - \varepsilon_u)/(\varepsilon_v - \varepsilon_u)}$ for region II.

The derivation of the permeability tensor is analogous. For simplicity, a two-dimensional cloak is commonly verified. For the TM wave, the parameters are simplified as ε_u, ε_v, and μ_w, while for the TE wave, they are μ_u, μ_v, and ε_w. The anisotropies of permeability and permittivity can be verified separately through an alternating layered system with two kinds of isotropic dielectrics.[18] When the layer thickness is much less than the wavelength, the following effective permittivity can be determined by

$$\varepsilon_u = 2\varepsilon_1\varepsilon_2/(\varepsilon_1 + \varepsilon_2), \quad \varepsilon_v = \varepsilon_w = (\varepsilon_1 + \varepsilon_2)/2. \tag{3}$$

Here we consider that the layers are parallel to Ovw plane and the layer thickness is kept identical.

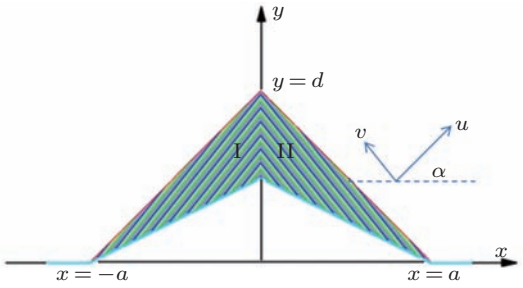


Fig. 2. (colour online) Rotated coordinate system and layered structure for modeling the triangular ground cloak.

3. Simulation and results

To verity the above formulation and the cloak performance, we make full-wave simulations based on the finite element method (FEM). Considering a representative example, we let $a = d = 1$ m and $\delta = 0.5$, and obtain results as $K = P = 0.5$. The working frequency is set to be 2.0 GHz. Assuming the zero-angle is along positive x axis, and the incident wave is at an azimuth angle 45°. For TM wave illumination, figure 3(a) demonstrates the magnetic field distribution of flat PEC ground plane corresponding to Fig. 1(a).

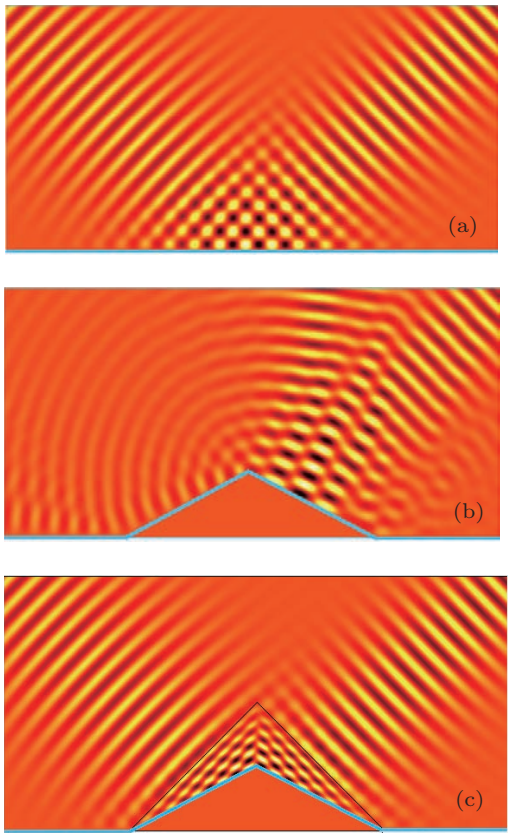


Fig. 3. (colour online) Magnetic field distribution for a Gaussian beam incident at an azimuth angle 45°. (a) PEC ground, (b) bare bump, (c) ideal coating (transformation media).

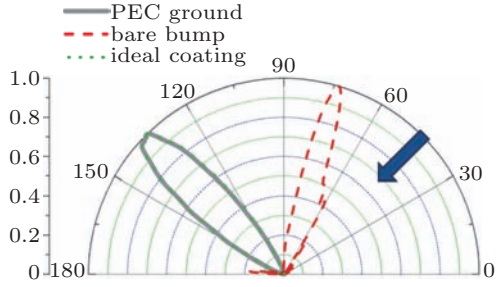


Fig. 4. (colour online) Comparison of the normalized average scattering power outflows in Fig. 3.

Figure 3(c) depicts the magnetic field distribution of the triangular ground cloak covered with ideal coating. Figure 3(b) shows the magnetic field distribution when the incident wave strikes on the bare bump without cloak. As indicated in the results, the magnetic field scattered by the bare PEC bump [Fig. 3(b)] is quite irregular, while in the cases that the bump is covered with the triangular ground plane cloak [Fig. 3(c)], the magnetic scattering field is confined highly in a specular direction, mimicking that in the case of a flat reflective PEC ground plane [Fig. 3(a)]. We also make the comparison of average scattering power outflows on the observation semicircle which centres at the coordinate origin with radius of 1.95 m. Figure 4 shows the normalized average scattering power outflows corresponding to the three plots in Fig. 3, and it indicates both the PEC bump covered with ideal coating and the flat PEC ground can reflect the incident wave in a specular direction, which is the direct consequence of the invariance of Maxwell's equations in different coordinates. Based on this coordinate transformation, the designed triangular cloak could shield any object underneath the triangular PEC bump.

In the following we demonstrate the implementation of the layered cloak with simplified parameters for the TM wave, considering that only the anisotropy of permittivity is realized by alternating layers with $\varepsilon_1 = 5.0376$ and $\varepsilon_2 = 0.1985$, and permeability is constant with $\mu = 2$. Magnetic field distribution for the TM wave illumination and electrical field distribution for the TE wave incidence are depicted in Figs. 5(a) and 5(b), respectively. We find that the TM wave has a perfect performance of cloaking, but the TE wave is

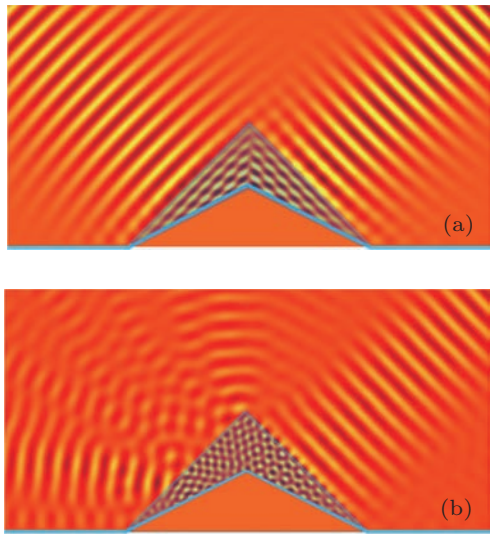


Fig. 5. (colour online) Magnetic field distribution of the TM wave (a) and electromagnetic field distribution of the TE wave (b) for a Gaussian beam incident at an azimuth angle 45°.

blocked inside the cloak and has an irregular scattering. The corresponding normalized average scattering power outflows are also depicted in Fig. 6. In the following text, all the near-field distributions of the layered cloak are marked with shadows and the ratio of the layer thickness to working wavelength is set to be 5.26% in FEM simulations.

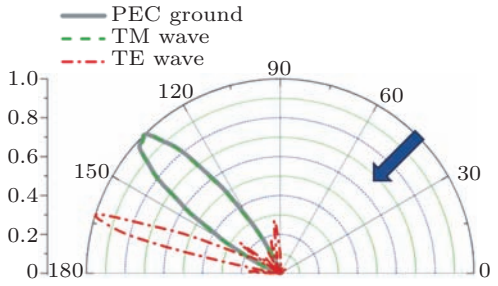


Fig. 6. (colour online) Comparison of the normalized average scattering power outflows in Fig. 5.

For a hybrid incoming electromagnetic wave, which always can be separated into TM and TE wave components, we consider that both the anisotropies of permittivity and permeability are modulated with step index profile by layered structure. So we obtain the hybrid parameters for both TM and TE wave cloaking performances as follows:

$$\begin{aligned} \varepsilon_1 &= \mu_1 = \varepsilon_v + \sqrt{\varepsilon_v^2 - 1}, \\ \varepsilon_2 &= \mu_2 = \varepsilon_v - \sqrt{\varepsilon_v^2 - 1}. \end{aligned} \tag{4}$$

Based on the index matching, improved hybrid parameters can be given as[19]

$$\varepsilon_1 = \mu_2 = \varepsilon_v + \sqrt{\varepsilon_v^2 - 1},$$

$$\varepsilon_2 = \mu_1 = \varepsilon_v - \sqrt{\varepsilon_v^2 - 1}. \tag{5}$$

Figures 7(a) and 7(b) illustrate the magnetic field distribution with a Gaussian beam striking on the layered cloak with hybrid parameters and index matching hybrid parameters, respectively. We have seen similar field distributions as shown in Figs. 3(c) and 5(a). Next, we also set the incoming wave as the TE polarization wave, and different from the case in Fig. 5(b), the cloak with both settings of hybrid parameters can reflect the incoming TE wave specularly. The electrical field distributions are demonstrated in Figs. 7(c) and 7(d). From Fig. 7, we can find that there is little difference in the field distributions of TM and TE waves, and this attributes to the duality of Maxwell equations for TM and TE waves. The layered cloak with index matching hybrid parameters has a better overall invisibility performance, which attributes to the smooth profile of the reflection index. The comparison of normalized scattering power outflows is made in Fig. 8, and it can be seen that both TM and TE waves can be reflected specularly by the layered cloak with two kinds of hybrid parameters, indicating that it is a reasonable and feasible solution for both TM and TE wave performance. From the expression of ε_v, we can obtain the variations $\varepsilon_1 = \mu_1 = \varepsilon_v + \sqrt{\varepsilon_v^2 - 1} \subset [1, \infty)$ and $\varepsilon_2 = \mu_2 = \varepsilon_v - \sqrt{\varepsilon_v^2 - 1} \subset (0, 1]$, so the layers can be realized by normal dielectric and electric plasmas, respectively.

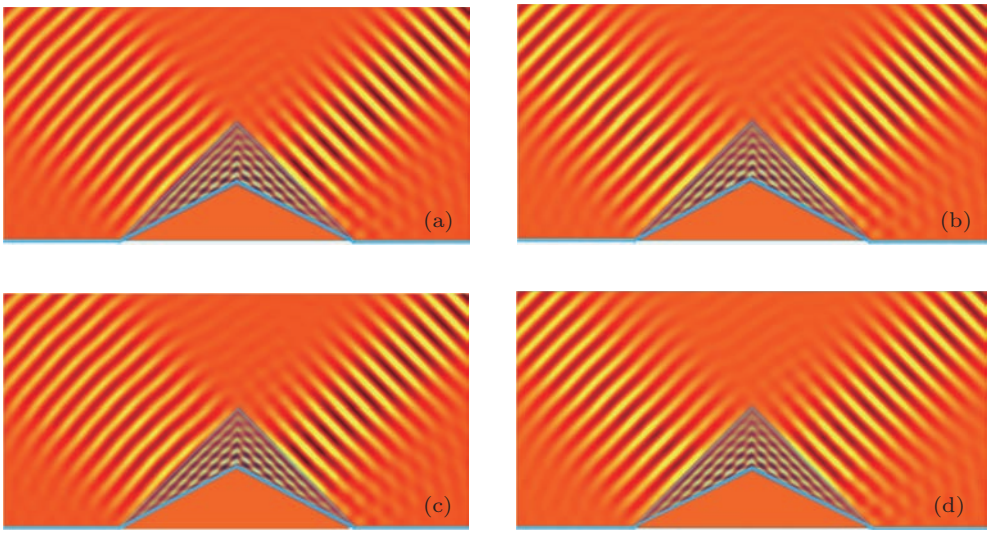


Fig. 7. (colour online) Magnetic field distribution of the layered cloak with hybrid parameters (a) and index matching hybrid parameters (b) for TM wave incident at an azimuth angle 45°; Electrical field distribution of the layered cloak with hybrid parameters (c), and index matching hybrid parameters (d) for TE wave incident at an azimuth angle 45°.

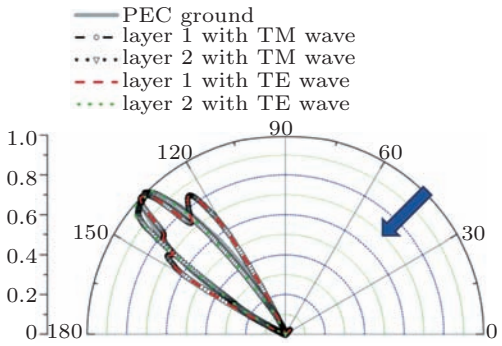


Fig. 8. (colour online) Comparison of the normalized average scattering power flows in Fig. 7 [Layer 1 is for hybrid parameters and layer 2 for index matching hybrid parameters].

4. Conclusions

Based on transformation media and effective medium theory, we have designed layered systems to realize a triangular ground plane cloak, which has spatially invariant anisotropic parameters. We suggested two solutions of parameter setting of the layered cloak for both TM and TE wave invisibility performances. From the FEM simulation results, a conclusion can be drawn that the layered cloak with both anisotropies of permittivity and permeability modulated by alternating layers can behave with a good invisibility performance for a hybrid incoming wave. The layered cloak of improved hybrid parameters with index matching has a better overall performance of invisibility in the practical implementations.

References

[1] Pendry J B, Schuring D and Smith D R 2006 *Science* **312** 1780

[2] Leonhardt U 2006 *Science* **312** 1777

[3] Schurig D, Mock J J, Justice B J, Cummer S A, Pendry J B, Starr A F and Smith D R 2006 *Science* **314** 977

[4] Wang X H, Qu S B, Xia S, Wang B K, Xu Z, Ma H, Wang J F, Gu C, Wu X, Lu L and Zhou H 2010 *Chin. Phys.* B **19** 064101

[5] Ma H, Qu S B, Xu Z, Zhang J Q and Wang J F 2009 *Chin. Phys.* B **18** 179

[6] Jiang W X, Cui T J, Yang X M, Cheng Q, Liu R and Smith D R 2009 *Appl. Phys. Lett.* **93** 194102

[7] Ma H, Qu S B, Xu Z, Zhang J Q and Wang J F 2009 *Chin. Phys.* B **18** 1850

[8] Li J and Pendry J B 2008 *Phys. Rev. Lett.* **101** 203901

[9] Liu R, Ji C, Mock J J, Chin J Y, Cui T J and Smith D R 2009 *Science* **323** 366

[10] Kallos E, Argyropoulos C and Yang H 2009 *Phys. Rev.* A **79** 063825

[11] Valentine J, Li J, Zentgraf T, Bartal G and Zhang X 2009 *Nat. Mater.* **8** 568

[12] Xu X, Feng Y, Yu Z, Jiang T and Zhao J 2010 *Opt. Express* **18** 24477

[13] Xi S, Chen H, Wu B I and Kong J A 2009 *IEEE Microw. Wireless Compon. Lett.* **19** 131

[14] Zhang B, Luo Y, Liu X and Barbastathis G 2011 *Phys. Rev. Lett.* **106** 033901

[15] Chen X, Luo Y, Zhang J, Jiang K, Pendry J B and Zhang S 2011 *Nat. Commun.* **2** 176

[16] Xu X, Feng Y, Hao Y, Zhao J and Jiang T 2009 *Appl. Phys. Lett.* **95** 184102

[17] Zhang J, Liu L, Luo Y, Zhang S and Mortensen N A 2011 *Opt. Express* **19** 8625

[18] Wood B, Pendry J B and Tsai D P 2006 *Phys. Rev.* B **74** 115116

[19] Qiu C W, Hu L and Zouhdi S 2010 *Opt. Express* **18** 14950