Characteristics of synchronized traffic in mixed traffic flow*

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In this paper, the characteristics of synchronized traffic in mixed traffic flow are investigated based on the braking light model. By introducing the energy dissipation and the distribution of slowdown vehicles, the effects of the maximum velocity, the mixing ratio, and the length of vehicles on the synchronized flow are discussed. It is found that the maximum velocity plays a great role in the synchronized flow in mixed traffic. The energy dissipation and the distribution of slowdown vehicles in the synchronized flow region are greatly different from those in free flow and a traffic jamming region. When all of vehicles have the same maximum velocity with $V_{\max} > 15$, the mixed traffic significantly displays synchronized flow, which has been demonstrated by the relation between flow rate and occupancy and estimation of the cross-correlation function. Moreover, the energy dissipation in the synchronized flow region does not increase with occupancy. The distribution of slowdown vehicles shows a changeless platform in the synchronized flow region. This is an interesting phenomenon. It helps to deeply understand the synchronized flow and greatly reduce the energy dissipation of traffic flow.

Keywords: cellular automata, synchronized flow, mixed traffic flow, energy dissipation

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1. Introduction

Recently, traffic problems have attracted the attention of the physics community because of the observed non-equilibrium phase transitions and various nonlinear dynamical phenomena.[1] Moreover, financial damage due to environmental pollution and energy dissipation from traffic is huge every year. About 20 percent more fuel consumption and air pollution is caused by impeded traffic and stop-and-go traffic.[2] Vehicle emissions of SO_2, NO_X, CO, CO_2, dust particles, smog, and noise have reached or even exceeded levels comparable to those from industrial production or private households, and are harmful to the environment and human health. Furthermore, research has found that doubling fuel consumption will cause a double or quadruple increase in the environmental load, while vehicle velocity drops off from 40 km per hour to 10 km per hour.[2] It is extremely urgent to reduce vehicle emissions and ease traffic jams in order to enhance people's life quality. Many traffic models, such as fluid-dynamical models, kinetic models, car-following models, cellular automata (CA) models, etc., have been presented to describe the complicated traffic phenomenon.[1−5] CA traffic models have been extensively applied and investigated because of their simplicity, high computational efficiency and the flexibility in reproducing the complicated features observed in real traffic.[1,3,4] The NaSch model is a typical CA traffic model used to simulate stop-and-start waves in real traffic.[5] Based on the NaSch model, many CA models have been extended to the investigation of the influences of real factors in traffic system such as highway junctions, two-level crossings, tollbooths on highway traffic, etc.[6−11] However, the empirical observation of highway traffic has shown the existence of very complex spatiotemporal structures. There are three traffic phases: free flow; synchronized traffic; and wide moving jams. The characteristics of free flow and wide moving jams are intuitively clear. Synchronized traffic is characterized by a large variance in flow and density measurements and the velocity is significantly lower than in free-flow traffic. Moreover, it has a strong correlation between the density on different

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lanes caused by lane changing. In more recent years, many three-phase traffic models, such as the KKW-1, 2, 3 model, the braking light model (BL model), the ATD-model, the SA model, the MCD model, and so on, were successively proposed to explain synchronized traffic and corresponding phase transitions of traffic, i.e., the F $\Leftrightarrow$ S $\Leftrightarrow$J transitions are found in the empirical observation.[12−17]

Recently, problems of energy dissipation in traffic systems have been extensively investigated in the framework of the car-following model, CA traffic models and the city traffic model.[18−24] The additional energy consumption of traffic flow under the action of an external disturbance was estimated by using the optimal velocity model.[18] The relation between the stability and the energy consumption by analysing and comparing energy consumptions for several typical car-following models was studied.[19] The effects of speed limits, stochastic noise, boundary conditions, and "go and stop" traffic on energy dissipation were discussed in the NaSch model.[21,22] The influences of the maximum velocity and the length of vehicle in mixed traffic have been investigated in the NaSch model and the FI model.[23,24] However, to our knowledge, the relations of energy dissipation and vehicle distribution for synchronized flow in mixed traffic flow to the speed limit, length of vehicles, and the mixed ratio C_n of vehicles have not been explored so far. These factors will affect the vehicle safety conditions, vehicle acceleration, and deceleration behaviours, etc., which are very important elements of a microscopic three-phase traffic theory.

In this paper, we attempt to study the characteristics of synchronized traffic in mixed traffic flow including the computation of the fundamental diagram, the cross-correlation function to identify the synchronized traffic, and a discussion of the energy dissipations from their corresponding distributions of vehicles via the BL model. The influences of the maximum velocity $V_{\max}$, length of vehicle, and the ratio C_n of the mixed vehicles on synchronized traffic will be discussed respectively. The results of this paper may lead to a deeper understanding of some features of synchronized traffic or may provide schemes for reducing the energy dissipation of existing traffic systems.

The rest of the present paper is organized as follows. Section 2 is devoted to the description of the BL model, the definition of energy dissipation and the statistical distribution of slowdown vehicles in energy dissipation. In Section 3, the simulation and the analysis are carried out. Finally, some conclusions are given in Section 4.

2. Model and energy dissipation

The model is defined on a single lane road consisting of L cells of equal size numbered by $n = 1, 2, \ldots, L$ and the time is discrete. Each site can be either empty or occupied by a car with the speeds of 0, 1, 2, ..., $V_{\max}$, where $V_{\max}$ is the speed limit (the maximum velocity). Let $X_n(t)$ and $V_n(t)$ denote the position and the velocity of the n-th vehicle at time t, respectively. In the BL model,[15,16] the driving strategies have been considered. When vehicles can have larger intervals between themselves, these vehicles move with their desired velocity $V_{\max}$. At intermediate distances, drivers react to the velocity changes of the next vehicle via observing the brake lights in front. When vehicles have small intervals, the drivers adjust their velocities in order to drive in safety. Moreover, the effect of slow-to-start on traffic is considered. The accelerations of standing vehicles and directly after braking events are delayed. These rules are realized by a set of evolutive steps. The evolution rules of BL model are as follows.

(i) Determination of the randomization parameter:

$$\begin{cases} p = p(v_n(t), b_{n+1}(t), t_h, t_s), \\ b_n(t+1) = 1. \end{cases} \tag{1}$$

(ii) Acceleration:

$$\begin{aligned} &\text{if} \quad (b_{n+1}(t) = 0 \text{ and } \ b_n(t) = 0 \text{ or } t_h \geq t_s), \quad \text{then :} \\ &\qquad v_n(t+1) = \min(v_n(t) + 1, v_{\max}), \\ &\text{else} \\ &\qquad v_n(t+1) = v_n(t). \end{aligned}$$

(iii) NaSch braking rule:

$$V_n(t+1) = \min(d_n^{\text{eff}}, V_n(t+1)), \tag{2}$$

$$\text{if } (V_n(t+1) < V_n(t)), \text{ then } b_n(t+1) = 1. \tag{3}$$

(iv) Randomization:

$$\text{if } (\text{rand}() < p) \text{ then : } \begin{cases} v_n(t+1) = \max(v_n(t+1) - 1, 0), \\ \text{if } (p = p_b) \text{ then } b_n(t+1) = 1. \end{cases} \tag{4}$$

(v) Vehicle motion

$$X_n(t+1) = X_n(t) + V_n(t+1), \tag{5}$$

where $d_n = X_{n+1} - X_n - l$ is the gap between consecutive vehicles (where l is the length of the n-th vehicle) and b_n is the status of the brake light (on (off)$\rightarrow b_n = 1(0)$). The randomization parameter p for the n-th vehicle can take on three different values p_0, p_d, and p_b, i.e.,

$$p(v_n(t), b_{n+1}(t), t_h, t_s) = \begin{cases} p_b, & \text{if } b_{n+1} = 1 \text{ and } t_h < t_s, \\ p_0, & \text{if } v_n = 0, \\ p_d, & \text{in all other cases,} \end{cases} \tag{6}$$

$$d_n^{\text{eff}} = d_n + \max(v_{\text{anti}} - \text{gap}_{\text{safety}}, 0), \tag{7}$$

where the braking parameters p_0, p_d, and p_b depend on its current velocity V_n(t), the status b_{n+1} of the brake light of the preceding vehicle $n+1$ and the two times of interaction t_h and t_s. The d_n^{eff} is the effective headway via considering the safe gap and the expected velocity of the leading vehicle. The another parameters of the model are the maximal velocity $v_{\max}$, the cut-off time of interaction h and the minimal safety $\text{gap}_{\text{safety}}$. The two times of interaction $t_h = d_n/V_n(t)$ and $t_s = \min(V_n(t),\ h)$, where h determines the range of interaction with the brake light, are introduced to compare the time t_h needed to reach the position of the leading vehicle with a velocity-dependent interaction horizon t_s. Finally, $d_n^{\text{eff}} = d_n + \max(v_{\text{anti}} - \text{gap}_{\text{safety}},\ 0)$ denotes the effective gap, where $v_{\text{anti}} = \min(d_{n+1},\ V_{n+1})$ is the expected velocity of the leading vehicle in the next time step. The effectiveness of the anticipation is controlled by the parameter $\text{gap}_{\text{safety}}$. Accidents are avoided only if the constraint $\text{gap}_{\text{safety}} \geq 1$ is fulfilled.

In mixed traffic, many factors affect the formation of synchronized traffic due to the mixture of long and short vehicles or fast and slow vehicles. Generally, the cross-correlation function ($CC_{\rho,q}(\tau)$) is used to identify the synchronized flow. The cross-correlation function is the covariance ($CC_{\rho,q}(\tau)$) between local density and flow on road. It is denoted as

$$CC_{\rho,q}(\tau) = \frac{\langle \rho(t) q(t+\tau) \rangle - \langle \rho(t) \rangle \langle q(t+\tau) \rangle}{\sqrt{\langle \rho^2(t) \rangle - \langle \rho(t) \rangle^2} \sqrt{\langle q^2(t+\tau) \rangle - \langle q(t+\tau) \rangle^2}}, \tag{8}$$

where ρ is the local density. Traffic states with $(CC_{\rho,q}(\tau)) \approx 0$ are identified as synchronized flow and it indicates the non-correlation of density and flow. Otherwise, the traffic state is a free-flow regime or a wide moving jam.[15−17]

The kinetic energy of a vehicle moving with velocity is $mv^2/2$, where m is the mass of the vehicle. In mixed traffic flow, the mass of the vehicle is proportional to its length. For simplicity, the frictional resistance, air drag and other dissipation factors are neglected. When the vehicle decelerates, we define its reduced kinetic energy as the dissipated energy. The dissipated energy of n-th vehicle from time $t-1$ to t is

$$e(i,t) = \begin{cases} m/2[v^2(i,t-1) - v^2(i,t)], & \text{for } v(i,t) < v(i,t-1), \\ 0, & \text{for } v(i,t) \geq v(i,t-1). \end{cases} \tag{9}$$

Thus, the energy dissipation is

$$E_d = \frac{1}{T}\frac{1}{N}\sum_{t=t_0+1}^{t_0+T}\sum_{i=1}^{N} e(i,t), \tag{10}$$

where N is the number of vehicles in the system, t_0 is the relaxation time, and m is the mass of the vehicle. The energy dissipation is caused by the deterministic deceleration and the randomization step. Then E_d is the total average energy dissipation.

In light of Eq. (10), if more vehicles slow down, more vehicle kinetic energy is dissipated. In order to analyse the characteristics of the energy dissipation further, we introduce the distribution of vehicles with the energy dissipation in the process of the deterministic and stochastic decelerations as follows:

$$\text{distribution} = \frac{1}{T}\frac{1}{N}\sum_{t=t_0+1}^{t_0+T} n_{s(d)}, \tag{11}$$

where $n_{s(d)}$ is the amount of energy dissipation in the process of the deterministic and the stochastic decelerations within time T. Obviously, the corresponding number of vehicles in the energy dissipation takes on different values for different occupancies (densities).

3. Simulation and analysis

In the following, simulations are carried out on a loop road of 10^4 cells. One time step corresponds to 1 s. Each cell corresponds to a length of 1.5 m on a real road. The mixed traffic is composed of long and short vehicles on a single-lane. The lengths of the long and short vehicles are denoted as $L_{\rm l}$ and $L_{\rm s}$, respectively. Their corresponding numbers are $N_{\rm l}$ and $N_{\rm s}$. In order to describe the mixing traffic, we introduce an occupation ratio to reflect the number of long vehicles. The occupation ratio $C_{\rm l}$ is expressed as the percentage of long vehicles amounting to all vehicles, i.e., $C_{\rm l} = (N_{\rm l} \times L_{\rm l}/L)$. The mixing ratio $C_n = C_{\rm l}/C$ represents the percentage of occupation ratio $C_{\rm l}$ of long vehicles amounting to the total occupation ratio C. The total occupation ratio C is defined as $C = N_{\rm l} \times L_{\rm l}/L + N_{\rm s} \times L_{\rm s}/L$. The maximum velocities of long and short vehicles are assumed to be $V_{\max 1}$ and $V_{\max 2}$, respectively.

In the process of the simulations, the parameters are chosen as $p_0 = 0.5$, $p_d = 0.1$, $p_b = 0.94$, $h = 6$, $\text{gap}_{\text{safety}} = 7$, and 2×10^4 time steps T for simulation. After $t_0 = 2 \times 10^4$ time steps to remove the transient state, the fundamental diagrams, the energy dissipation, and vehicle distributions of the energy dissipation are obtained by the statistical average over 50 simulation runs. In our simulation, the unit of length is in units of cells using the dimensionless calculation. The mass of a single vehicle occupying a cell is 1 and the mass of the length l of a vehicle occupying l cells is l.

First, we study the effects of different maxima of velocity $V_{\max}$ on synchronized traffic with the identical vehicles on the road ($C_n = 1.0$). We choose $V_{\max} = 5$, 10, 15, 30, 50 and the length of vehicle to be 5. The flow rate is $Q = \rho\langle v\rangle$, where $\langle v\rangle$ is the average velocity of the vehicles.

Figure 1(a) shows the plots of traffic flow rate versus occupancy C for different maxima of velocity $V_{\max}$. It can be found that when $V_{\max} = 5$ and 10, the flow rate Q increases with occupancy C increasing until it reaches a maximum value and then gradually reduces to a minimum value. When $V_{\max} = 30$ and 50, there exist three regions: free flow, synchronized flow, and traffic jamming in the fundamental diagram. Based on the study of Ref. [15], we take $V_{\max} = 5$, 15, and 50 for example to identify the characteristics in the case of $C_1 < C < C_2$ by the cross-correlation function ($CC_{p,q}(\tau)$). Figure 1(b) shows that when the occupancy $C = 0.18$ and $V_{\max} = 15$ and 50, the cross-correlation function approaches zero with the increase of time while the cross-correlation function with $V_{\max} = 5$ approaches a large value. It indicates that when the maximum velocity of vehicles exceeds $V_{\max} = 15$, the traffic flow takes on the characteristics of synchronized traffic. When the maximum velocity $V_{\max}$ equals 30, the correlation function has different values in the occupancy $C = 0.18$ and 0.55. Figure 1(c) shows the approximate zero-valued correlation functions in the occupancy $C = 0.18$ while the correlation function has a large value in the occupancy $C = 0.55$. From the above discussion, it is found that the maximum velocity has a great effect on the generation of synchronized traffic.

Figures 2(a) and 2(b) exhibit the plots of the energy dissipation and the deceleration vehicles distribution versus occupancy C for different maxima of velocity $V_{\max}$, which correspond to the flow rate-occupancy curves in Fig. 1(a). The energy dissipation of three-phase traffic is obviously different from that of two-phase traffic. The energy dissipation E_d of three-phase traffic increases with the increase of density in a free-flow regime, but decays in the synchronized traffic region. When

it reduces to a minimum value, it rises to the next maximum value and then gradually decreases to zero, which corresponds to the wide moving jamming region. In contrast, the energy dissipation E_d of two-phase traffic ascends to a maximum value in a free-flow regime and then gradually descends to zero. Obviously, it is a new characteristic of synchronized traffic. Figure 2(c) shows the corresponding distribution of slowdown vehicles and in the synchronized traffic region it presents a changeless platform. It implies that the slowdown vehicles remain nearly constant. For simplicity, the mixed traffic flow is composed of long vehicles and short vehicles with the same maximum velocity and occupancy $C_n = 0.3$. The parameters of the mixed traffic are chosen as $V_{\max} = 10$, 15, 20, 50 and length of vehicle $L_{\rm l} = 5$, $L_{\rm s} = 10$. By simulation, the flow rates versus occupancy C for the different maxima of velocity $V_{\max}$ are given in Fig. 3(a). Figure 3(b) shows their cross-correlation function between local density and flux with occupancy $C = 0.2$. The cross-correlation function in Fig. 3(c) shows the existence of the synchronized flow and the traffic jamming in the different density regions for $V_{\max} = 20$. Figures 4(a) and 4(b) display the energy dissipations and the distributions of decelerating vehicle versus occupancy C corresponding to Fig. 3(a), respectively. In a way similar to that in the case of homogeneous traffic, we can deduce that when the maximum velocity of traffic flow is approximately greater than 15, there exists synchronized traffic with a larger flow rate, lower velocity, and lower energy dissipation.

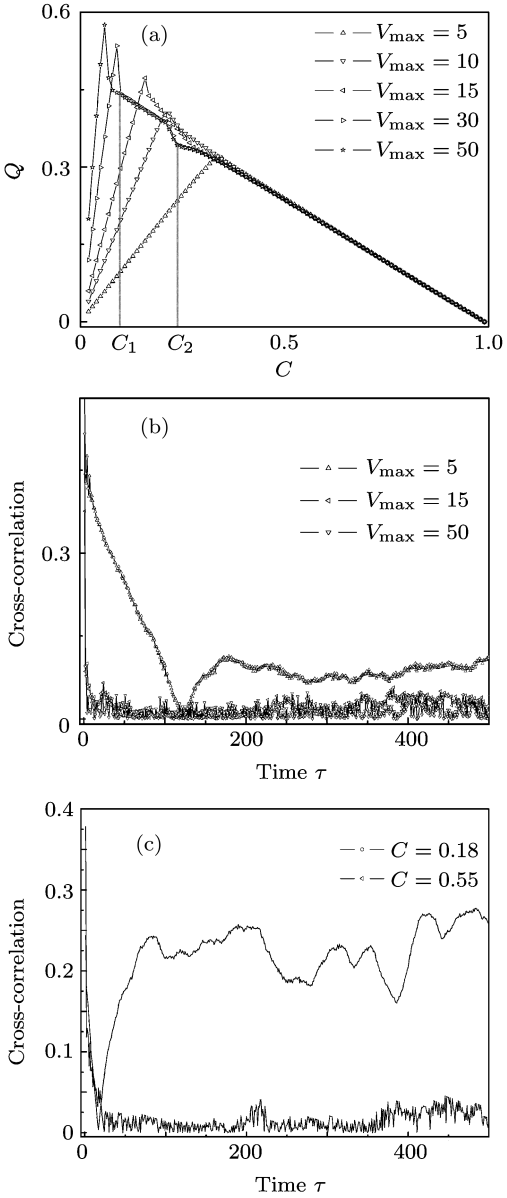


Fig. 1. (a) Fundamental diagram for different maximum values of $V_{\max}$. (b) Cross-correlation function (the occupancy C=0.18). (c) Cross-correlation function in the synchronized flow and the traffic jamming region for $V_{\max} = 30$.

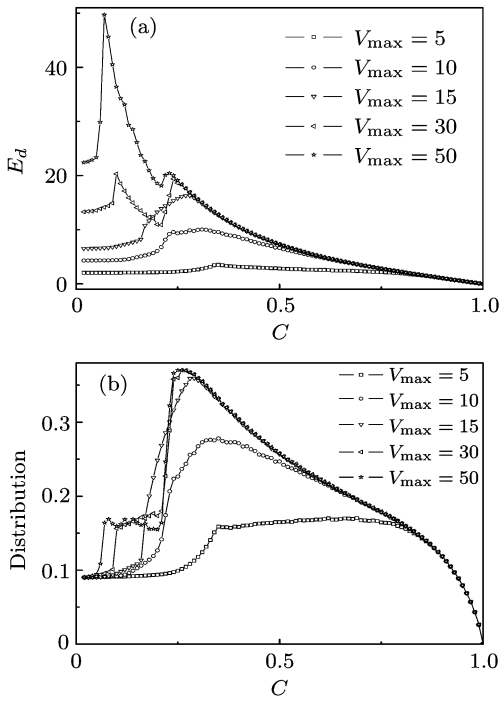


Fig. 2. (a) Energy dissipations for different maxima of $V_{\max}$. (b) The corresponding distribution of deceleration vehicles.

Generally, we should study the effects of the different maxima of velocity on the mixed traffic ($C_n = 0.3$). In mixed traffic flow, long and short vehicles have the different maxima of velocity. The maxima of velocity and the length of short vehicles are chosen as $V_{\max 2} = 20$, 30, 80, 100, and $L_{\rm s} = 10$,

respectively. The maximum velocity and the length of

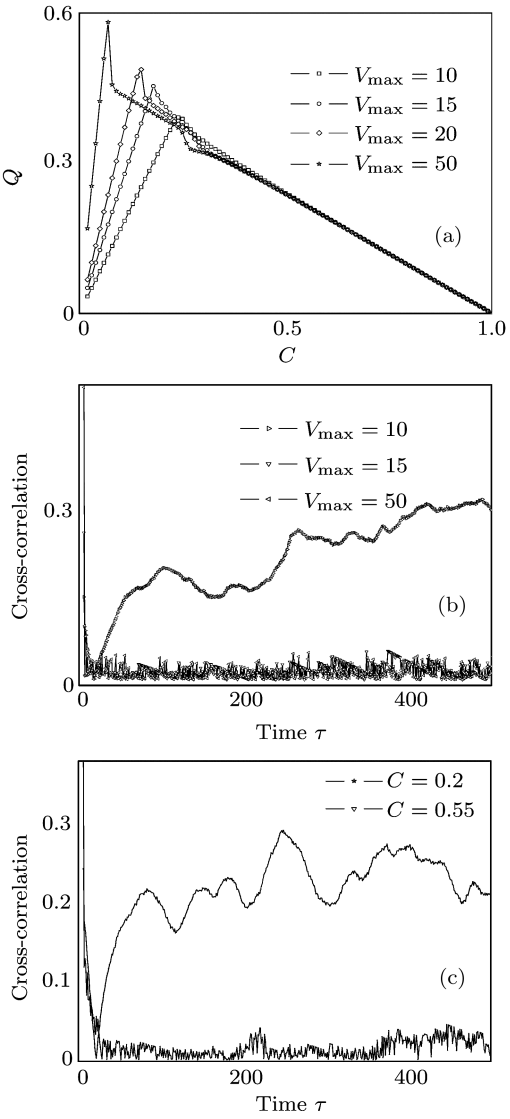


Fig. 3. (a) Fundamental diagram for different maximum values of $V_{\max}$. (b) Cross-correlation functions (the occupancy $C = 0.20$). (c) Cross-correlation functions in the synchronized flow and traffic jamming regions ($V_{\max} = 20$).

long vehicle are $V_{\max 1} = 20$ and $L_{\rm l} = 20$. The traffic flow rates versus occupancy C (i.e. the fundamental diagram) for different maxima of velocity $V_{\max 2}$ are shown in Fig. 5(a). We can see that when the short and the long vehicles have the same maximum velocity ($V_{\max 1} = V_{\max 2} = 20$), the fundamental diagram displays three regions: free flow ($0 < C < C_1$), synchronized flow ($C_1 < C < C_2$), and traffic jamming ($C > C_2$). However, when the value of $V_{\max 2}$ increases and the $V_{\max 1}$ keeps constant, all curves of the flow rate are superimposed together and the obvious characteristics of synchronized traffic are not seen. Figures 5(b) and 5(c) show the corresponding energy dissipations and distributions of slow-down vehicles versus occupancy C for different maxima of velocity

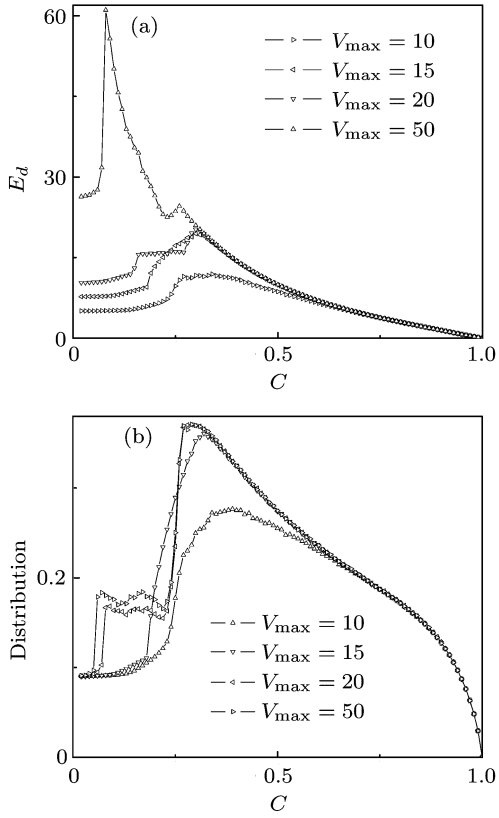


Fig. 4. (a) Plots of energy dissipation E_d versus C for different maxima of $V_{\max}$. (b) The corresponding distributions of the deceleration vehicle versus C.

$V_{\max 2}$, respectively. It is found that when short and long vehicles have the same maximum velocity ($V_{\max 1} = V_{\max 2} = 20$), the energy dissipation corresponding to the flow rate takes on the different characteristics in three regions. The energy dissipation in the synchronized traffic exhibits a platform. When the value of $V_{\max 2}$ increases and the $V_{\max 1}$ keeps constant, all curves of the energy dissipation are almost superimposed together. Figure 5(c) demonstrates that few vehicles are decelerated in synchronized traffic to maintain the properties of the larger flow rate, lower velocity and lower energy dissipation. Next, we study the effects of different values of mixing ratio C_n on synchronized traffic in mixed traffic flow. Assuming that short and long vehicles have the same maximum velocity $V_{\max} = 30$, the mixing ratios and the lengths of vehicles are chosen as $C_n = 0$, 0.1, 0.5, 0.9, 1.0, and $L_{\rm s} = 5$ and $L_{\rm l} = 20$, respectively. The traffic flow rates, the energy dissipations and the distributions of slowdown vehicles for the different mixing

ratios are shown in Figs. 6(a), 6(b), and 6(c), respectively. Obviously, there exist three regions showing

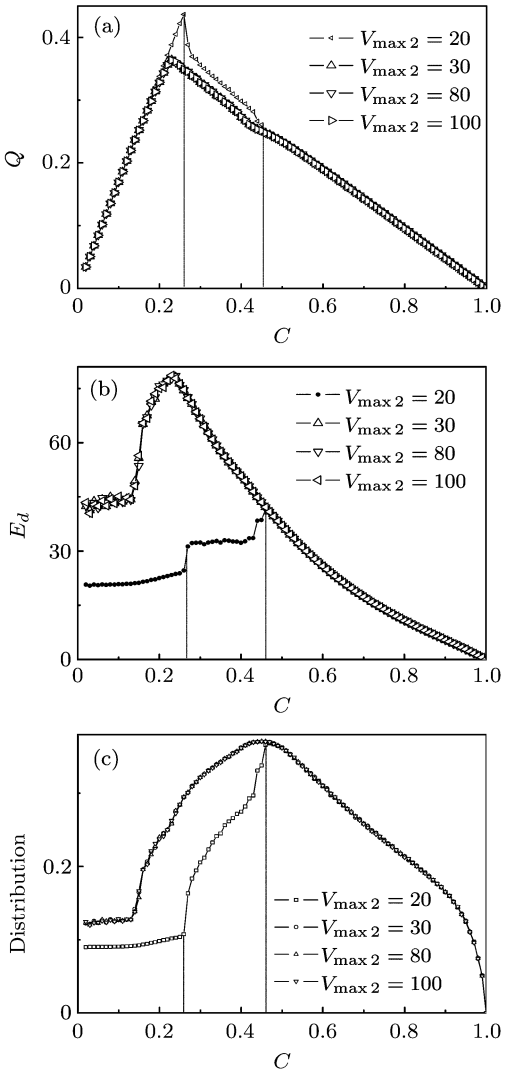


Fig. 5. (a) Fundamental diagram, (b) energy dissipation versus C, and (c) distributions of the decelerated vehicles versus C, for different maxima of $V_{\max 2}$.

free flow, synchronized flow and traffic jamming. The energy dissipation in synchronized traffic increases with the increase of mixing ratio, but at the same mixing ratio, the energy dissipation in synchronized traffic is reduced with the increase of density or occupancy. It indicates that the more long vehicles there are, the more the energy dissipation is. The distribution of slowdown vehicles in Fig. 6(c) tends to shift toward the right with the increase of long vehicles, but the curve in synchronized traffic remains constant and shows a platform. Finally, we study the influences of different values of vehicle length $L_{\rm l}$ on mixed traffic flow. It is well known that the length of vehicle is an important factor in the mixed traffic flow. Assume that the maximum velocity of vehicles on road lane is $V_{\max} = 30$, the long vehicle lengths $L_{\rm l}$ are 10, 30, 50, 80, and 100, the short vehicle length $L_{\rm s} = 5$ and the mixing ratio $C_n = 0.5$. Figures 7(a), 7(b), and 7(c) show the traffic flow rates, the energy dissipations, and the distributions of slowdown vehicles for the different long vehicle values $L_{\rm l}$, respectively. Figure 7(a) exhibits three regions: free flow, synchronized flow, and traffic jamming. Similarly, the energy dissipation in synchronized flow increases with the increase of long vehicles and reduces with the increase of density or occupancy, but the curves are almost superimposed together when the length of a long vehicle exceeds 30. Moreover, it is also found that the distribution curves of slowdown vehicles in Fig. 7(c) each show a changeless platform and the curves are almost superimposed together with the increase of long vehicles.

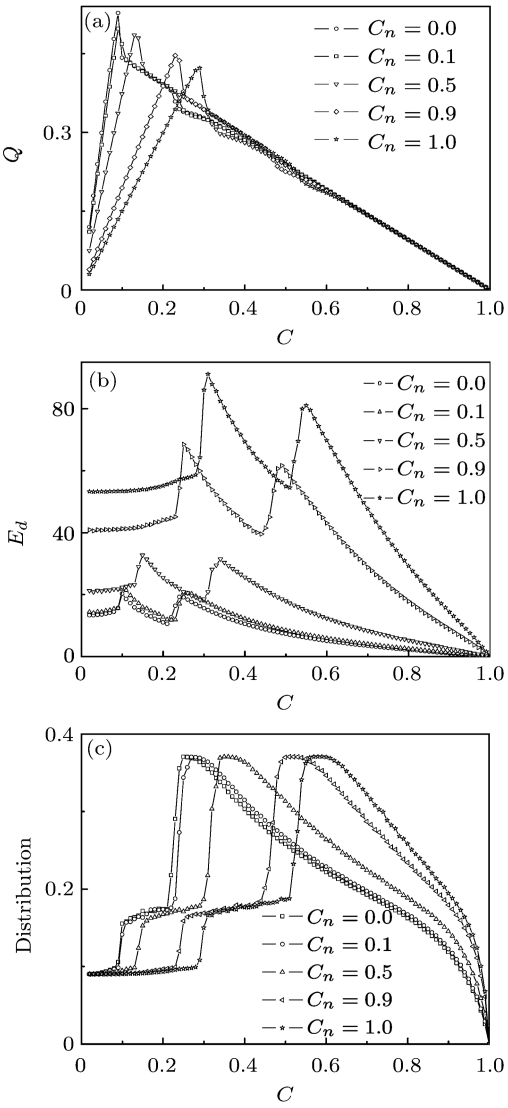


Fig. 6. (a) Fundamental diagram, (b) energy dissipation versus C, and (c) distributions of decelerated vehicle versus C, for different mixing ratios C_n.

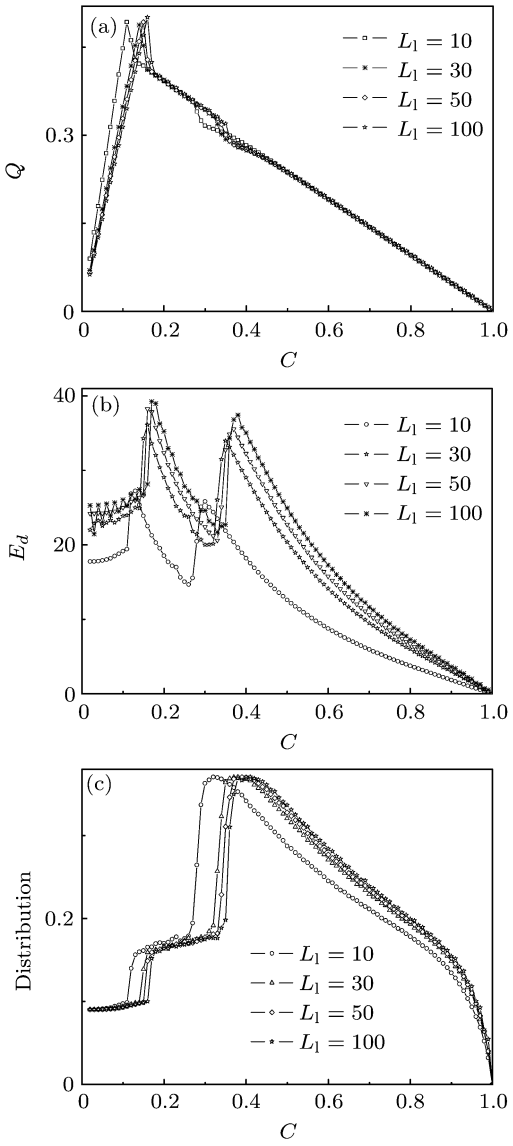


Fig. 7. (a) Fundamental diagrams, (b) energy dissipation versus C, and (c) distributions of decelerated vehicle versus C for different length of L_1.

4. Conclusion

In this paper, we studied the characteristics of synchronized traffic in mixed traffic flow by using the BL model. Our simulation results show the important features of synchronized traffic. The maximum velocity plays a great role in forming the synchronized flow in mixed traffic. When all of the vehicles have the same maximum velocity ($V_{\max} > 15$), the mixed traffic significantly displays the synchronized flow. The result is confirmed by the fundamental diagram and the estimation of the cross-correlation function.

Next, the energy dissipations in free flow and a traffic jamming regime increase with the increases of the maximum velocity, the mixing ratio and the length of vehicles, but the energy dissipation in synchronized flow decreases with density increasing or remains constant. These are new findings about the characteristics of synchronized traffic. It is good to reasonably control the congested traffic to a synchronized flow and reduce the energy dissipation of traffic flow to a great extent. Finally, the distribution of slowdown vehicles corresponding to the synchronized flow shows a changeless platform.

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