Exponential networked synchronization of master-slave chaotic systems with time-varying communication topologies*

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(Received 11 August 2011; revised manuscript received 19 October 2011)

The networked synchronization problem of a class of master-slave chaotic systems with time-varying communication topologies is investigated in this paper. Based on algebraic graph theory and matrix theory, a simple linear state feedback controller is designed to synchronize the master chaotic system and the slave chaotic systems with a time-varying communication topology connection. The exponential stability of the closed-loop networked synchronization error system is guaranteed by applying Lyapunov stability theory. The derived novel criteria are in the form of linear matrix inequalities (LMIs), which are easy to examine and tremendously reduce the computation burden from the feedback matrices. This paper provides an alternative networked secure communication scheme which can be extended conveniently. An illustrative example is given to demonstrate the effectiveness of the proposed networked synchronization method.

Keywords: exponential networked synchronization, master-slave chaotic systems, algebraic graph theory, communication topology

PACS: 05.45.–a, 05.45.Xt, 05.45.Gg **DOI:** 10.1088/1674-1056/21/4/040503

1. Introduction

In recent years, consensus and synchronization problems have become popular subjects in systems and control, motivated by many applications in physics, biology, and engineering.[1−16] These problems arise in multi-agent systems with the collective objective of reaching agreement about some variables of interest.

In some papers, the emphasis is placed on the communication constraints rather than on the individual dynamics: the agents exchange information according to a communication graph which is not necessarily complete, nor even symmetric or time-invariant. However, in the absence of communication, the agreeing variables usually have no dynamics. It is the exchange of information only that determines the time-evolution of the variables, aiming at asymptotic synchronization to a common value. The convergence of such consensus algorithms has attracted much attention in recent years.[2,6,7] However, in other papers, the emphasis is put on the individual dynamics rather than on the communication limitations: the communication graph is often assumed to be complete, but in the absence of communication, the time-evolution of the systems' variables can be oscillatory or even chaotic. The system's dynamics can be modified through the information exchange, and, as in the consensus problem, the goal of the interconnection is to reach synchronization to a common solution of the individual dynamics.[13−15]

During the past two decades, chaotic synchronization of master-slave systems has been studied extensively in light of its potential applications in secure communication.[17−31] It is worthwhile noting that in the research on master-slave chaotic synchronization, researchers only considered the synchronization prob-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 60904046, 60972164, 60974071, and 60804006), the Special Fund for Basic Scientific Research of Central Colleges, Northeastern University, China (Grant No. 090604005), the Science and Technology Program of Shenyang (Grant No. F11-264-1-70), the Program for Liaoning Excellent Talents in University (Grant No. LJQ2011137), and the Program for Liaoning Innovative Research Team in University (Grant No. LT2011019).

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lem of two chaotic systems, which means that an information signal as a carrier, and the synchronization is necessary to recover the information at the receiver. However, in reality, our environment is an information-rich world and the control involves complex networked communications. Under this circumstance, systems are considered as agents forming a network and information is exchanged between these agents through the network.[1,16] Therefore, the networked secure communication is of great value in applications.

Motivated by the above discussion, in this paper, we consider the master-slave synchronization problem in a network with a master-system and N slave-systems, where the dynamics of the master-system and each slave-system is described by a chaotic system. It is noted that in this scheme of networked secure communication, information can be exchanged among slave-systems, which is utilized in the design of controllers, and the ultimate goal is to realize the master-slave synchronization. Figure 1 illustrates the networked secure communication scheme proposed in this paper. The novelty of the current paper lies in that: we treat a time-varying communication topology and show that an exponential synchronization criterion is derived in the case of a connected graph. It is shown that our analysis relies on several tools from algebraic graph theory, matrix theory, and control theory. The derived criteria are based on linear matrix inequalities, which are easy to check and tremendously reduce the computation burden from the feedback matrix.

The rest of this paper is organized as follows. In Section 2, some preliminaries are presented, and the relevant physical models are described. In Section 3, the master-slave synchronization problem under time-varying communication topology is discussed. Section 4 presents an illustrative example to demonstrate the effectiveness of the analytical method and the proposed networked chaotic synchronization scheme. Finally, some conclusions are drawn and further discussions are carried out in Section 5.

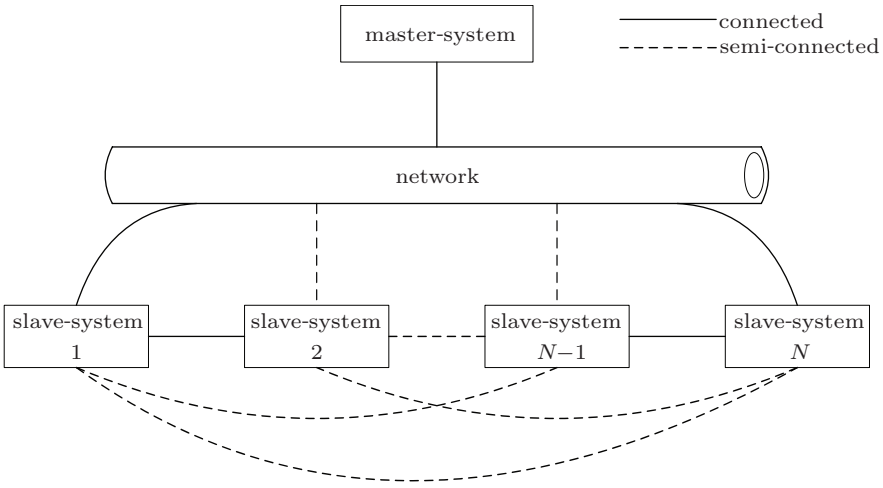


Fig. 1. Networked secure communication scheme.

2. Preliminaries

Throughout this paper, we will use the following notation. $\mathbb{Z}^+$ and $\mathbb{R}^+$ are used to denote the set of all nonnegative integers and the set of all nonnegative real numbers, respectively. $\mathbb{C}$ denotes the complex numbers. W^{T} defines the transpose of a matrix W. We denote by $W > 0$ (≥ 0) that W is positive (semi-positive) definite. Letting $W \geq 0$, $\rho(W)$ denotes the smallest nonzero eigenvalue of W. For a finite set $\mathcal{S}$, $\lfloor \mathcal{S} \rfloor$ denotes the number of its elements. I_m is the identity matrix with dimension m. $|\cdot|$ denotes the absolute value sign. $\|\cdot\|$ refers to Euclidean vector norm or the induced matrix 2-norm. For $\forall x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$, the notation $\|x\| = (\sum_{i=1}^{n} x_i^2)^{1/2}$. The symbol $\otimes$ is the Kronecker product.

2.1. Algebraic graph theory

We first review some elements of algebraic graph theory[26] used in the sequel. For a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with N vertices $\mathcal{V} = \{1, \ldots, N\}$ and edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, the adjacency matrix $\mathcal{A} = \mathcal{A}(\mathcal{G}) = (\mathcal{A}_{ij})$ is an $N \times N$ matrix given by $\mathcal{A}_{ij} = 1$, if $(i, j) \in \mathcal{E}$, and $\mathcal{A}_{ij} = 0$, otherwise. If $(i, j) \in \mathcal{E}$, then i, j are adjacent. The graph is undirected, that is, the edges (i, j) and (j, i) in $\mathcal{E}$. The set of neighbours of vertex i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}, i \neq j\}$. A path is a sequence of connected edges in a graph. A graph is connected if there is a path between every pair of vertices. A component of graph $\mathcal{G}$ is a connected subgraph that is maximal. The degree d_i of vertex i is given by $d_i = \sum_j \mathcal{A}_{ij}$. Let $\Delta = \mathrm{diag}(d_1, \ldots, d_N)$. The Laplacian of $\mathcal{G}$ is a symmetric positive semi-definite matrix $L = [l_{ij}] \in \mathbb{R}^{N \times N}$ such that $L = \Delta - \mathcal{A}$. For a connected graph $\mathcal{G}$, L has a single zero eigenvalue with the corresponding eigenvector $\mathbf{1} = [1, \ldots, 1]^{\mathrm{T}}$.

2.2. Model description

Suppose that a networked chaotic system consists of a master-system and N slave-systems. N slave-systems interact over a network whose topology is given by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with adjacency matrix $\mathcal{A}$. Slave-system j can send information to slave-system i only if $(j, i) \in \mathcal{E}$. The dynamics of slave-system i is given by

$$\dot{x}_i(t) = Cx_i(t) + Af_i(x_i(t)) + u_i(t), \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the slave-system i's state, $C = (c_{ij})_{n \times n} \in \mathbb{R}^{n \times n}$ and $A = (a_{ij})_{n \times n} \in \mathbb{R}^{n \times n}$ are system parameter matrices; $f_i(x_i(t))$ represents a nonlinear function, and $u_i(t) \in \mathbb{R}^n$ is the slave-system i's input, which can only use local information from its neighbours. The master-system, labeled as $i = 0$, has nonlinear dynamics as

$$\dot{x}_0(t) = Cx_0(t) + Af_0(x_0(t)), \tag{2}$$

where $x_0(t) \in \mathbb{R}^n$ denotes the master-system's state. It is worthwhile noting that the master-system's dynamics is independent of others. Here, we set the slave-systems' parameters and those of the master-system to be the same because this case has a wide range of practical backgrounds such as school of fishes, group of birds, and so on.

Before starting the main results, we present some necessary assumption, definitions, and lemmas.

Assumption 1 (A1) The nonlinear function $f_i(\cdot)$ satisfies the following condition:

$$(\xi - \zeta)^{\mathrm{T}}(f_i(\xi) - f_i(\zeta)) \leq (\xi - \zeta)^{\mathrm{T}} \Gamma (\xi - \zeta), \tag{3}$$

for arbitrary $\xi, \zeta \in \mathbb{R}^n$ and $\xi \neq \zeta$, and $\Gamma \in \mathbb{R}^{n \times n}$ is a positive diagonal matrix, $i = 0, 1, \ldots, N$.

Definition 1 (D1) The master-slave synchronization in a network of chaotic systems (1) and (2) is said to be achieved if, for each slave-system $i \in \{1, \ldots, N\}$, there is a local state feedback $u_i(t)$ of $\{x_j(t) : j \in \mathcal{N}_i\}$ such that the closed-loop system satisfies

$$\lim_{t \to \infty} \|x_i(t) - x_0(t)\| = 0, \ i = 1, \ldots, N, \tag{4}$$

for any initial condition $x_i(0), i = 0, 1, \ldots, N$.

Lemma 1 (L1)[32] Let $\mathcal{B} = [b_{ij}] \in \mathbb{R}^{N \times N}$ be any matrix, and let

$$R_i'(\mathcal{B}) \equiv \sum_{j=1, j \neq i}^{n} |b_{ij}|, \ 1 \leq i \leq n,$$

denote the deleted absolute row sums of $\mathcal{B}$. Then all the eigenvalues of $\mathcal{B}$ are located in the union of n discs

$$G(\mathcal{B}) \equiv \bigcup_{i=1}^{N} \{z \in \mathbb{C} : |z - b_{ii}| \leq R_i'(\mathcal{B})\}.$$

Lemma 2 (L2)[32] Let any matrix $\mathcal{B} = [b_{ij}] \in \mathbb{R}^{N \times N}$ have all diagonal entries nonzero and be diagonally dominant with $|b_{ii}| > R_i'(\mathcal{B})$ for all but one value of $i = 1, \ldots, N$. Then $\mathcal{B}$ is invertible.

Remark 1 Note that **A1** is very mild. For example, all linear and piecewise linear functions satisfy this condition. In addition, if $\partial f_i / \partial \xi_j$ $(i, j = 0, 1, \ldots, N)$ are bounded, the above condition is satisfied. Therefore, many well-known systems satisfy the assumption A1, such as the Lorenz system,[33] Chen system,[34] Lü system,[30] chaotic neural networks,[23] Chua's circuit, etc.

3. The criterion for exponential networked synchronization with time-varying communication topologies

Here, we need to account for all possible graphs $\{\bar{\mathcal{G}}_p : p \in \mathcal{P}\}$, where $\mathcal{P}$ is an index set for all graphs defined on vertices on $\{0, 1, \ldots, N\}$. The dependence of the graphs upon time can be characterized by a switching signal $\sigma : [0, \infty) \to \mathcal{P}$, that is, at each time t, the graph is $\bar{\mathcal{G}}_{\sigma(t)}$. It is assumed in this paper that σ

switches finite times in any bounded time interval. In order to realize the networked chaotic synchronization, a simple linear state feedback controller consisting of the local exchange information among slave-systems with a time-varying communication topology is designed as follows:

$$u_i(t) = \mathcal{K}\sum_{j\in\mathcal{N}_i(t)} l_{ij}(t)(x_j(t)-x_i(t)) + \mathcal{K}d_i(t)(x_0(t)-x_i(t)), \quad i=1,\ldots,N. \tag{5}$$

It should be noted that the neighbours $\mathcal{N}_i(t)$ of each slave-system and the index number $d_i(t)$ describing the neighbours of the master-system vary with time. We use $L_{\sigma(t)}$ and $D_{\sigma(t)}$ to describe the time-dependence of graph topology.

The dynamics of master-slave error system $e_i(t) = x_i(t) - x_0(t)$ can be written as

$$\begin{aligned}\dot{e}_i(t) &= Ce_i(t) + Af_i(e_i(t)) + u_i(t)\\ &= Ce_i(t) + Af_i(e_i(t))\\ &\quad + \mathcal{K}\sum_{j\in\mathcal{N}_i(t)} l_{ij}(t)(e_j(t)-e_i(t))\\ &\quad - \mathcal{K}d_i(t)e_i(t), \quad i=1,\ldots,N,\end{aligned} \tag{6}$$

where $f_i(e_i(t)) = f_i(x_i(t)) - f_0(x_0(t))$.

If the above equation (6) is written in a compact form, then we can obtain

$$\begin{aligned}\dot{e}(t) &= (I_N\otimes C)e(t) + (I_N\otimes A)f(e(t))\\ &\quad - (L_{\sigma(t)}\otimes I_n)(I_N\otimes\mathcal{K})e(t)\\ &\quad - (D_{\sigma(t)}\otimes I_n)(I_N\otimes\mathcal{K})e(t)\\ &= (I_N\otimes C - (L_{\sigma(t)}+D_{\sigma(t)})\otimes\mathcal{K})e(t)\\ &\quad + (I_N\otimes A)f(e(t))\\ &= (I_N\otimes C - H_{\sigma(t)}\otimes\mathcal{K})e(t)\\ &\quad + (I_N\otimes A)f(e(t)),\end{aligned} \tag{7}$$

where $e(t) = (e_1^{\mathrm{T}}(t),\ e_2^{\mathrm{T}}(t),\ \ldots,\ e_N^{\mathrm{T}}(t))^{\mathrm{T}}$, $f(e(t)) = (f_1^{\mathrm{T}}(e_1(t)),\ f_2^{\mathrm{T}}(e_2(t)),\ \ldots,\ f_N^{\mathrm{T}}(e_N(t)))^{\mathrm{T}}$, $D = \mathrm{diag}(d_1(t),\ d_2(t),\ \ldots,\ d_N(t))$, $H_{\sigma(t)} = L_{\sigma(t)} + D_{\sigma(t)}$.

To guarantee the stability of the above error system, consider the following switching sequence

$$S = (t_0, p_0), (t_1, p_1), \ldots, \tag{8}$$

where $t_k \in \mathbb{R}^+$ and $p_k \in \mathcal{P}$. The sequence may or may not be infinite. In the finite case, all further results hold. Thus, to ease notation, the infinite case is presented in what follows. Throughout, it is assumed that the switching sequence S is minimal in the sense that $p_k \neq p_{k+1}$ where $i \in \mathbb{Z}^+$. Let S^p denote the set of switching instants that the graph $\bar{\mathcal{G}}_p$ is active, where S_j^p denotes the j-th element of S^p.

Take projections of the switching sequence S onto its first and second coordinates, yielding the sequence of indices

$$\pi_t(S) = t_0, t_1, t_2, \ldots, \pi_p(S) = p_0, p_1, p_2, \ldots.$$

The interval completion $I(T)$ of a strictly increasing sequence of times $T = t_0, t_1, \ldots$ is the set

$$I(T) = \bigcup_{k\in\mathbb{Z}^+} [t_k, t_{k+1}). \tag{9}$$

For each $p \in \mathcal{P}$, denote $\delta_p := \rho(H_p)$. Since $\mathcal{P}$ is finite, the set $\{\delta_p : p \in \mathcal{P}\}$ is finite. Define

$$\delta_{\min} := \min\{\delta_p : p \in \mathcal{P}\}, \tag{10}$$

which is positive and independent of time.

Theorem 1 Suppose that

(i) the graphs $\bar{\mathcal{G}}_{\sigma(s)}$ are connected across each interval $s \in [t_k, t_{k+1}), k = 0, 1, \ldots$;

(ii) there exist a symmetric positive definite P and a control matrix $\mathcal{K}$, such that

1) $\bar{\mathcal{K}} + \bar{\mathcal{K}}^{\mathrm{T}} > 0$,

2)

$$\begin{pmatrix} C^{\mathrm{T}}P + PC + \Gamma^{\mathrm{T}}\Gamma - \delta_{\min}(\bar{\mathcal{K}} + \bar{\mathcal{K}}^{\mathrm{T}}) + \delta_{\min}I_n & PA \\ A^{\mathrm{T}}P & -I \end{pmatrix} < 0,$$

where $\bar{\mathcal{K}} = P\mathcal{K}$;

3) $\forall p \in \mathcal{P},\ j \in \{1, 2, \ldots, \lfloor S^p \rfloor\},\ V_p(S_j^p) - V_p(S_{j+1}^p) \geq \varepsilon$, where ε is a positive constant.

Then, under the designed control law (5), the dynamics of all the slave-systems exponentially asymptotically synchronize to the homogeneous trajectory of the master-system under time-varying communication topologies.

Proof Choose the following Lyapunov function candidate

$$V_p(e(t)) = e^{\mathrm{T}}(t)(I_N\otimes P)e(t), \tag{11}$$

where $e(t)$ is any solution of master-slave error sys-

tem. Obviously, $V_p(t)$ is continuously differentiable at any time except for switching instants.

(I) The first step is to show that at any non-switching instants, $\dot{V}_p(e(t)) \leq 0$. Noting that for every $p \in \mathcal{P}$, H_p is symmetric and positive-definite, and its N eigenvalues are labeled as $\{\lambda_p^1, \ldots, \lambda_p^N\}$, we can find an orthogonal matrix T_p such that

$$T_p H_p T_p^{\mathrm{T}} = \Lambda_p := \mathrm{diag}(\lambda_p^1, \ldots, \lambda_p^N) > 0. \quad (12)$$

Assuming communication topology $\bar{\mathcal{G}}_p$ is active at time t, the time derivative of $V_p(e(t))$ is

$$\begin{aligned}
&\dot{V}_p(e(t)) \\
&= e^{\mathrm{T}}(t)((I_N \otimes C^{\mathrm{T}} - H_p \otimes \mathcal{K}^{\mathrm{T}})(I_N \otimes P) \\
&\quad + (I_N \otimes P)(I_N \otimes C - H_p \otimes \mathcal{K})) \\
&\quad + 2e^{\mathrm{T}}(t)(I_N \otimes P)(I_N \otimes A)f(e(t)) \\
&= e^{\mathrm{T}}(t)(I_N \otimes (C^{\mathrm{T}}P + PC) \\
&\quad - H_p \otimes (\mathcal{K}^{\mathrm{T}}P + P\mathcal{K}))e(t) \\
&\quad + 2e^{\mathrm{T}}(t)(I_N \otimes PA)f(e(t)) \\
&\leq e^{\mathrm{T}}(t)(I_N \otimes (C^{\mathrm{T}}P + PC + PAA^{\mathrm{T}}P + \Gamma^{\mathrm{T}}\Gamma) \\
&\quad - H_p \otimes (\bar{\mathcal{K}} + \bar{\mathcal{K}}^{\mathrm{T}}))e(t), \quad (13)
\end{aligned}$$

which can be rewritten, by letting $\tilde{e}(t) = (T_p \otimes I_n)e(t)$, as

$$\begin{aligned}
&\dot{V}_p(e(t)) \\
&\leq \tilde{e}^{\mathrm{T}}(t)(I_N \otimes (C^{\mathrm{T}}P + PC + PAA^{\mathrm{T}}P + \Gamma^{\mathrm{T}}\Gamma) \\
&\quad - \Lambda_p \otimes (\bar{\mathcal{K}} + \bar{\mathcal{K}}^{\mathrm{T}}))\tilde{e}(t) \\
&= \sum_{i=1}^{N} \tilde{e}_i^{\mathrm{T}}(t)(C^{\mathrm{T}}P + PC + PAA^{\mathrm{T}}P \\
&\quad + \Gamma^{\mathrm{T}}\Gamma - \lambda_p^i(\bar{\mathcal{K}} + \bar{\mathcal{K}}^{\mathrm{T}}))\tilde{e}_i(t) \\
&\leq \sum_{i=1}^{N} \tilde{e}_i^{\mathrm{T}}(t)(C^{\mathrm{T}}P + PC + PAA^{\mathrm{T}}P \\
&\quad + \Gamma^{\mathrm{T}}\Gamma - \delta_p(\bar{\mathcal{K}} + \bar{\mathcal{K}}^{\mathrm{T}}))\tilde{e}_i(t) \\
&\leq -\delta_{\min} \sum_{i=1}^{N} \tilde{e}_i^{\mathrm{T}}(t)\tilde{e}_i(t) \\
&= -\delta_{\min}\tilde{e}^{\mathrm{T}}(t)(I_N \otimes I_n)\tilde{e}(t) \\
&= -\delta_{\min}e^{\mathrm{T}}(t)(I_N \otimes I_n)e(t) \\
&\quad \leq -\frac{\delta_{\min}}{\|P\|_2}e^{\mathrm{T}}(t)(I_N \otimes P)e(t) \\
&= -\frac{\delta_{\min}}{\|P\|_2}V_p(e(t)). \quad (14)
\end{aligned}$$

Then we can derive

$$V_p(e(t)) \leq \exp\left[-\left(\frac{\delta_{\min}}{\|P\|_2}t\right)\right]V_p(e(0)), \quad (15)$$

which yields $\dot{V}_p(e(t)) < 0$ for all $p \in \mathcal{P}$.

(II) We proceed to show $\lim_{k\to\infty} V_{p_k}(t_k) = 0$. Divide the graphs into two sets: a) the graphs $\bar{\mathcal{G}}_f$, with the corresponding index set $\mathcal{P}_f$, which appears within a finite time horizon, i.e., there is a t_f such that $\forall \bar{\mathcal{G}} \in \bar{\mathcal{G}}_f$, $\bar{\mathcal{G}}$ is not active for $t \geq t_f$; b) the graphs $\bar{\mathcal{G}}_f$, with the corresponding index set $\mathcal{P}_f$, which are active within infinite time horizon. Thus for $t \geq t_f$, one can regroup the switching sequence as follows:

$$S = T_0, T_1, \ldots, \quad (16)$$

where

$$\begin{aligned}
&T_0 = (t_f, p_f), (t_{f+1}, p_{f+1}), \ldots, (t_{n_0}, p_{n_0}), \\
&T_1 = (t_{n_0+1}, p_{n_0+1}), (t_{n_0+2}, p_{n_0+2}), \ldots, (t_{n_1}, p_{n_1}), \\
&\vdots = \vdots
\end{aligned}$$

such that $\forall k \in \mathbb{Z}^+$, $\{\pi_p(T_k)\} = \mathcal{P}_f$.

Define $V_k^{\max} = \max_{t\in I(\pi_t(T_k))} V_{\sigma(t)}(t)$ with $V_{\sigma(t)}(t)$ defined by Eq. (11). Suppose $V_k^{\max} = V_{p_k}(t_k')$, $p_k \in \pi_p(T_k)$, $t_k' \in I(\pi_t(T_k))$, one has

$$\begin{aligned}
V_{k+1}^{\max} &= V_{p_{k+1}}(t_{k+1}') < V_{p_{k+1}}(t_k'') \leq V_{p_k}(t_k') \\
&= V_k^{\max}, \quad (17)
\end{aligned}$$

where $t_k'' \in \pi_t(T_k)$ is a starting time when $\bar{\mathcal{G}}_{p_{k+1}}$ is active. Hence, $\lim_{k\to\infty} V_k^{\max} = 0$.

4. An illustrative example

In this section, an illustrative example is given to verify the exponentially networked chaotic synchronization criteria established above. Consider a communication network consisting of a master-system and six slave-systems, of which each subsystem represents a Lorenz chaotic system. The system's parameters are given as follows:

$$C = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -8/3 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Here, the nonlinear function

$$f_i(x_i) = [\,0 \;\; -x_i^1 x_i^3 \;\; x_i^1 x_i^2\,]^{\mathrm{T}}, \quad i = 0, \ldots, 6.$$

Four possible communication topologies, which are referred to as $\{\bar{\mathcal{G}}_a, \bar{\mathcal{G}}_b, \bar{\mathcal{G}}_c, \bar{\mathcal{G}}_d\}$, are shown in Fig 2. In Fig. 3, a finite switching automation is illustrated where the set of states represents the discrete states of a network with time-varying communication topologies. Communication network topology starts at $\bar{\mathcal{G}}_a$ and switches to the next one for 1/4 s according to

the switching machine as shown in Fig. 2. Then, we can choose $t_k = k, t_{k+1} = k + 1/4$ with $k = 0, 1, \ldots$.

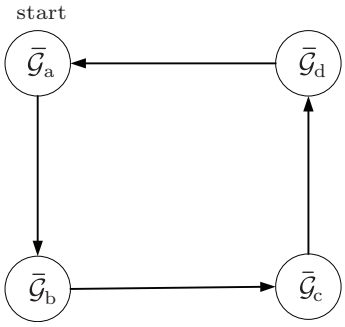


Fig. 2. The finite switching automation with four different communication topologies.

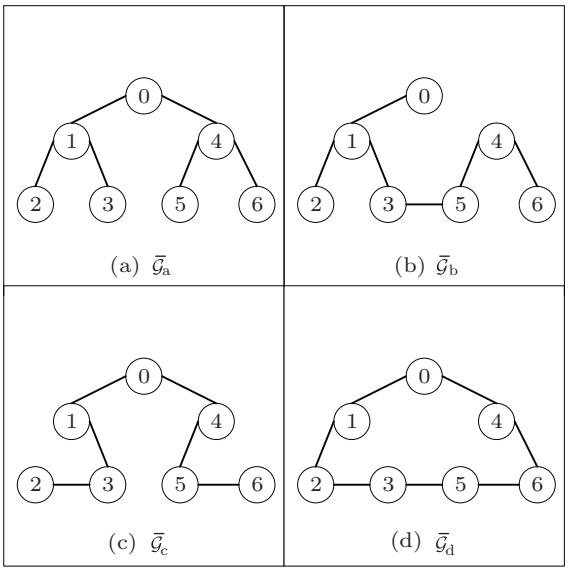


Fig. 3. The illustration of different communication graphs.

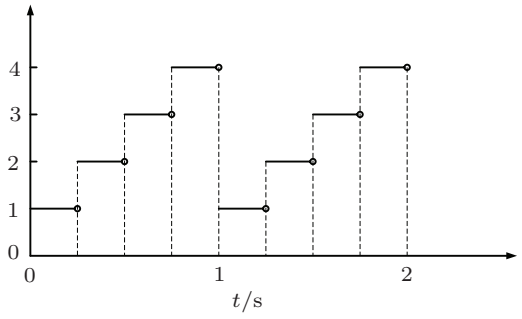


Fig. 4. A switching signal describing time-varying communication topologies.

Based on the above graphs, the matrices H_a, H_b, H_c, H_d and D_a, D_b, D_c, D_d can be easily determined. A simple calculation yields that $\delta_{\min} = 0.0784$.

On the other hand, from the numerical results, it is found that there exist some constants $M_1 = 20$, $M_2 = 26$, and $M_3 = 47$, such that the chaotic attractor (x_0^1, x_0^2, x_0^3) of Lorenz system satisfies $|x_0^1| \leq M_1$, $|x_0^2| \leq M_2$, $|x_0^3| \leq M_3$. According to Assumption 1, one has

$$\begin{aligned}
&(x_i - x_0)^{\rm T}(f_i(x_i) - f_0(x_0)) \\
&= e_i^{\rm T}\left(0, -e_i^1 e_i^3 - x_0^1 e_i^3 - x_0^3 e_i^1, e_i^1 e_i^2 + x_0^1 e_i^2 + x_0^2 e_i^1\right)^{\rm T} \\
&\leq \frac{M_2 + M_3}{2}(e_i^1)^2 + \frac{M_3}{2}(e_i^2)^2 + \frac{M_2}{2}(e_i^3)^2 \\
&= \left(e_i^1, e_i^2, e_i^3\right)\begin{pmatrix} (M_2+M_3)/2 & 0 & 0 \\ 0 & M_3/2 & 0 \\ 0 & 0 & M_2/2 \end{pmatrix}\begin{pmatrix} e_i^1 \\ e_i^2 \\ e_i^3 \end{pmatrix}.
\end{aligned}$$

Therefore, we have $\Gamma = \begin{pmatrix} 73/2 & 0 & 0 \\ 0 & 47/2 & 0 \\ 0 & 0 & 13 \end{pmatrix}$. By using Matlab LMI toolbox, we obtain the feasible solutions as

$$P = \begin{pmatrix} 43.4050 & -9.4319 & 0 \\ -9.4319 & 35.3312 & 0 \\ 0 & 0 & 61.0709 \end{pmatrix},$$

$$\mathcal{K} = \begin{pmatrix} 742.8254 & 244.6700 & 0 \\ 266.6415 & 869.9610 & 0 \\ 0 & 0 & 544.4363 \end{pmatrix}.$$

In this simulation, we choose the initial conditions as $x_0 = (1, 2, 3)^{\rm T}$, $x_1 = (-8, -0.5, 1.5)^{\rm T}$, $x_2 = (-2, -2.5, 2.5)^{\rm T}$, $x_3 = (3, 3.6, -3)^{\rm T}$, $x_4 = (2, 2.5, -2)^{\rm T}$, $x_5 = (-3, -1, 3)^{\rm T}, x_6 = (5, 3, -4.5)^{\rm T}$. Figures 5 and 6 describe the dynamics of the master-system x_0 and the synchronization dynamics of a chosen slave-system x_3, respectively. Figure 7 gives the state curves of master-system x_0 and the state curves of slave-system x_3. Under the function of the synchronization controller (5), the master-slave synchronization error trajectories of $e_i = x_i - x_0, i = 1, \ldots, 6$ are represented in Fig. 8. It can be seen that as $t \to \infty$, the evolution of $e_i = x_i - x_0, i = 1, \ldots, 6$ tends to zero, that is, the states of slave-systems synchronize to that of master-system with time-varying communication topologies.

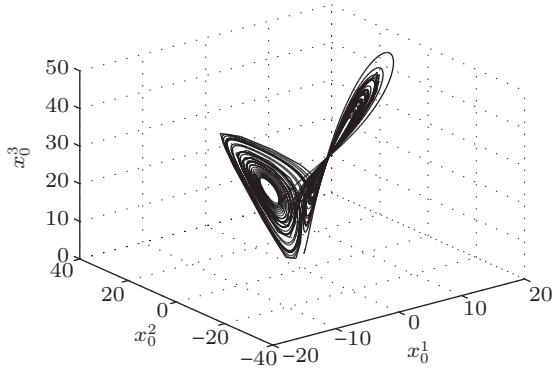


Fig. 5. The dynamics of master-system x_0 representing a Lorenz system.

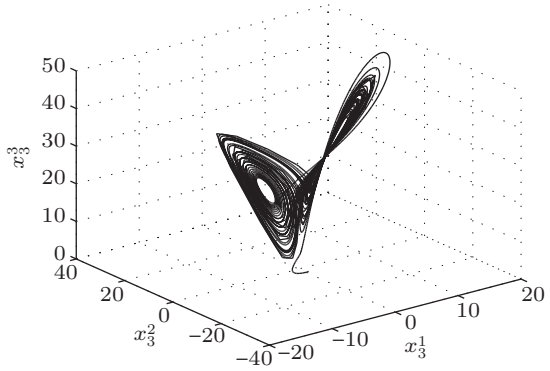


Fig. 6. The dynamics of controlled slave-system x_3.

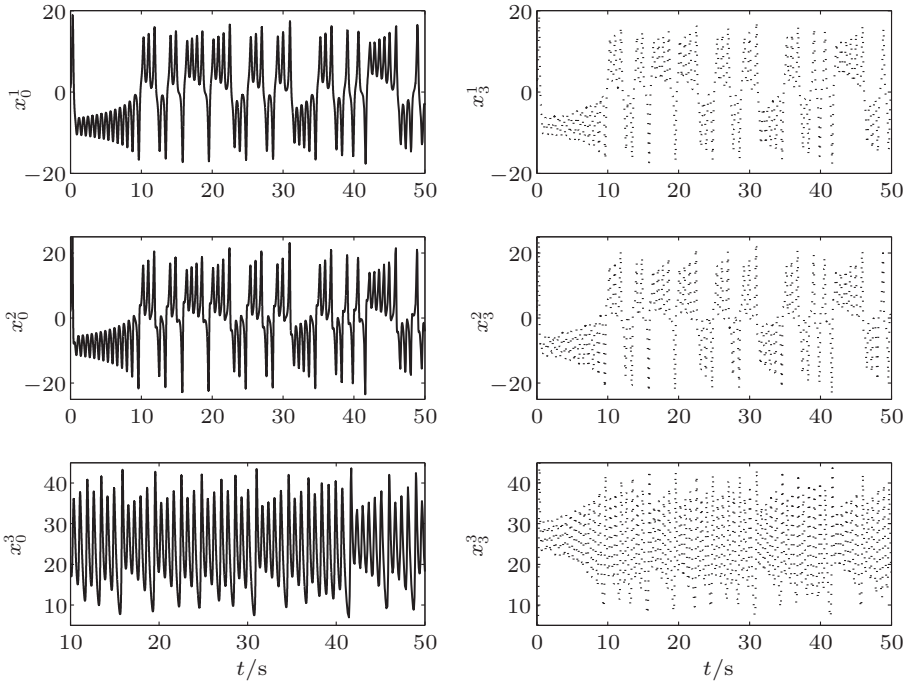


Fig. 7. The state evolution curves of master-system x_0 and the state response curves of slave-system x_3.

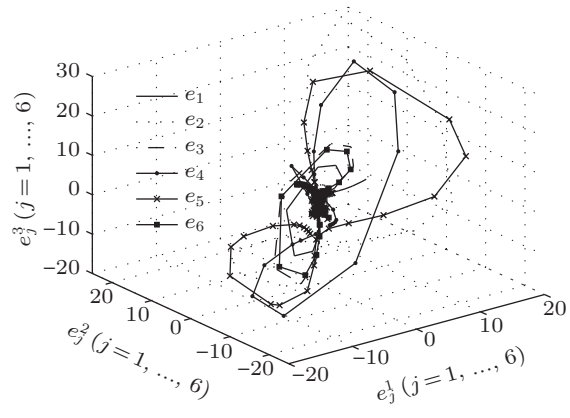


Fig. 8. The state trajectories of master-slave synchronization error systems.

5. Conclusion

The master-slave synchronization in a network of a class of chaotic systems with time-varying communication topology is investigated in this paper. The analysis course relies on several tools from algebraic graph theory, matrix theory, and Lyapunov stability theory, etc. Moreover, a sufficient criterion to realize the exponential networked master-slave chaotic synchronization with switching communication topology. It is noted that we give a networked secure communication scheme without considering chaotic signal transmission with communication channel bandwidth constraints or information quantization, which

deserves more efforts for further extensive study.

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