Scattering and absorption of particles by a black hole involving a global monopole*

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Under the conditions that the wavelength of a particle is much larger than its radius of central mass, and the Schwarzschild field is weak, the scattering of a particle has been studied by many researchers. They obtained that scalar and vector particles abide by Rutherford's angle distribution by using the low level perturbation method and the scattered field's approximation in a weak field. The scattering cross section of a photon coincides with the section in Newton's field of point mass. We can obtain the photon's polarization effect by calculating the second-order perturbation in the linear Schwarzschild field. This article discusses the scattering and absorption of a particle by a black hole involving a global monopole by using the aforesaid method.

Keywords: scattering, absorption, black hole, wave function

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1. Introduction

Scattering caused by gravitational sources has been an important test for general relativity (GR) since its inception. An early success for Einstein's theory came with the verification of the prediction that starlight is deflected when it passes close to the Sun. The experiment conducted by Eddington and his team during the solar eclipse of 1919 was important in establishing GR as a credible theory of gravitation. More recently, gravitational lensing observations provide strong tests of the theory, as well as information about lensing mass distributions and distant astronomical sources.

Gravitational scattering from astrophysical objects may be understood entirely in terms of geometric optics through the analysis of classical geodesics. Nevertheless, to obtain a deeper theoretical understanding of extreme objects such as black holes many authors have also considered the scattering of coherent waves. If the wavelength of the incident wave is comparable to the size of the event horizon of a black hole, then the wave will be diffracted by the black hole. Diffraction effects are responsible for many interesting phenomena in nature, such as glories, rainbows, etc., so wave scattering from black holes is an interesting field in its own right, even if observations may not be realizable in practice. In this paper, we study how quantum waves are scattered and absorbed by a black hole involving a global monopole. The quantum waves scattered by a Schwarzschild black hole has been considered in numerous papers[1−16] and books.[17,18] Most authors studied the massless scalar wave (with spin $s = 0$) for its mathematical simplicity, or massless electromagnetic ($s = 1$) and gravitational ($s = 2$) waves for physical relevance. In this paper, we focus instead on fermion scattering ($s = 1/2$), which has been less frequently discussed.[18] We investigate the effect of particle mass on scattering cross section and polarization. The interaction between an incident plane wave and a black hole may be understood in terms of three quantities: the total absorption cross section $\sigma_{\rm A}$, the differential scattering cross section $\mathrm{d}\sigma/\mathrm{d}\Omega$, and the polarization P that is induced in an initially unpolarized beam (note that $\mathrm{d}\sigma/\mathrm{d}\Omega$ and P are functions of the scattering angle θ). In this paper we calculate these quantities using a variety of analyses. When the gravitational source collapses into a black hole, it is not enough to only consider linear terms because the post-Newtonian term in the interatomic potential has a modified effect on the scattering image. The spin of the particle scattered by a spherically symmetric

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black hole can also be caused by polarization effects. A black hole's charge and angular momentum have a strong effect on the absorption of particles, this may be related to the extreme wave radiation of bosons. Fermions are not extreme radiation effects, as it is different from bosons in Kerr–Newman field. The absorption cross section of a weakly relativistic electron absorbed by a small Schwarzschild black hole is one-eighth of scalar particles. This article discusses the scattering and absorption of particles by a black hole involving a global monopole.

2. The wave function of a scalar particle

Assuming that the mass of a scalar particle is μ, the metric of a black hole involving a global monopole is given by[19]

$$\begin{aligned} \mathrm{d}s^2 = & \left(1-8\pi\eta^2-\frac{2m}{r}\right)\mathrm{d}t^2 \\ & - \left(1-8\pi\eta^2-\frac{2m}{r}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \end{aligned} \quad (1)$$

where m is the mass of the black hole involving a global monopole, and η is the symmetry-breaking factor. The Lagrangian is given by

$$L = \partial_\mu \Phi g^{\mu\nu}\partial_\nu \Phi^* - \mu^2 \Phi\Phi^*, \quad (2)$$

and Klein–Gordon equation has the form as

$$(-g)^{-1/2}\partial_\nu[(-g)^{-1/2}g^{\mu\nu}\partial_\mu\Phi] + \mu^2\Phi = 0, \quad (3)$$

$$(-g)^{-1/2}\partial_\nu[(-g)^{-1/2}g^{\mu\nu}\partial_\mu\Phi^*] + \mu^2\Phi^* = 0. \quad (4)$$

Now we study the equation of the wave function Φ. The wave function has the form

$$\Phi_{\omega lm} = \frac{1}{r}R_{\omega l}(r)\mathrm{Y}_l{}^m(\theta,\varphi)\,\mathrm{e}^{\,\mathrm{i}\omega l}, \quad (5)$$

where ω and l are the energy and angular momentum of the system, respectively, m is the projection of angular momentum on the z axis. The radial wave function at infinity is the sum of the incident wave function and output wave function

$$\begin{aligned} R_{\omega l} = & \exp(-\mathrm{i}\omega\upsilon r - \mathrm{i}\varepsilon\ln 2\omega\upsilon r) \\ & + A_l(\omega)\exp(-\mathrm{i}\omega\upsilon r + \mathrm{i}\varepsilon\ln 2\omega\upsilon r), \end{aligned} \quad (6)$$

where

$$\upsilon = (1-\mu^2/\omega^2)^{1/2},$$

$$\varepsilon = [\omega M(1+\upsilon^2)]/\upsilon.$$

The boundary condition at the horizon surface is

$$R_{\omega l} = \exp[-\mathrm{i}\omega r'], \quad (7)$$

where r' is determined by

$$\mathrm{d}r'/\mathrm{d}r = \left(1-8\pi\eta^2-\frac{2m}{r}\right)-1.$$

The oscillation amplitude $A_l(\omega)$ can be obtained from the boundary condition (7).

3. Scattering and absorption of scalar particles

The scattering wave function's asymptotic behaviour of particles flowing along the z axis has the form as follows:

$$\begin{aligned} \Psi = & \exp(-\mathrm{i}\omega t)\{\exp[\mathrm{i}\omega\upsilon z - \mathrm{i}\varepsilon\ln\omega\upsilon(r-z)] \\ & + \left[\frac{f(\theta)}{r}\right]\exp(\mathrm{i}\omega\upsilon r + \mathrm{i}\varepsilon\ln 2\omega\upsilon r)\}. \end{aligned} \quad (8)$$

Expanding the elastic scattering amplitude according to eigenstates, we obtain

$$f(\theta) = \frac{\mathrm{i}}{2\omega\upsilon}\sum_{l=0}^{\infty}(2l+1)(1-S_l)\mathrm{P}_l(\cos\theta), \quad (9)$$

where

$$S_l = (-1)^{l+1}A_l.$$

The total cross-sectional area of elastic scattering is

$$\sigma_{\mathrm{e}} = \frac{\pi}{\omega^2\upsilon^2}\sum_{l=0}^{\infty}(2l+1)|1-S_l|^2, \quad (10)$$

and the total absorption cross-sectional area is

$$\begin{aligned} \sigma_{\mathrm{A}} = & \frac{\pi}{\omega^2\upsilon^2}\sum_{l=0}^{\infty}(2l+1)(1-|S_l|^2) \\ = & \frac{\pi}{\omega^2\upsilon^2}\sum_{l=0}^{\infty}(2l+1)T_l. \end{aligned} \quad (11)$$

T_l is the local absorption coefficient.

4. Scattering and absorption of scalar particles with lower speed

Under approximate conditions, considering $\upsilon^2 \ll 1$ and $\eta \ll m$, then $r_+ \approx 2m$, using the radial connection method we can obtain

$$A_l(\omega) = (-1)^{l+1}\frac{\Gamma(1+l-\mathrm{i}\varepsilon)}{\Gamma(1+l+\mathrm{i}\varepsilon)}\left(1-\frac{1}{2}T_l\right)$$

$$= (-1)^{(l+1)} S_l^C \left(1 - \frac{1}{2} T_l\right), \tag{12}$$

where S_l^C is the matrix element of scattering operator. The local absorption coefficient has the form

$$T_l(\omega) = 2\Gamma(2\omega v r_+)^{2l+1} \frac{2\pi\varepsilon}{1 - \mathrm{e}^{-2\pi\varepsilon}} \frac{\langle l! \rangle^6}{\langle 2l \rangle!^2 \langle 2l+1 \rangle!^2} \times \prod_{k=1}^{l} \left(1 + 4\frac{\Gamma^2}{k^2}\right)\left(1 + \frac{\varepsilon^2}{k^2}\right), \tag{13}$$

where $\Gamma \equiv \langle \omega r_+ \rangle r_+ / \langle r_+ \rangle \approx 2\omega m$. We can obtain the total absorption cross section with the form

$$\sigma_{\mathrm{A}} = \frac{16\pi m^2}{v^2} 2\pi\omega m \frac{1 + v^2}{1 - \exp[-2\pi\omega m / v(1+v^2)]}. \tag{14}$$

Elastic scattering amplitude has the form as follows:

$$f(\theta) = \frac{\mathrm{i}}{4\omega v} \sum_{l=0}^{\infty} (2l+1) T_l S_l^C(\varepsilon) \mathrm{P}_l(\cos\theta). \tag{15}$$

If $(1+v^2)\omega m \neq 0$, then we can obtain the revised Rutherford scattering angular distribution as

$$\frac{\mathrm{d}\sigma_{\mathrm{e}}}{\mathrm{d}\Omega} = m^2 \frac{(1+v^2)^2}{4v^4 \sin^4(\theta/2)} \left[1 + 32\pi \frac{\omega^2 m^2 v}{1 - \exp(-2\pi\varepsilon)} \times \sin^2\frac{\theta}{2} \sin\left(2\varepsilon \ln\sin\frac{\theta}{2}\right)\right]. \tag{16}$$

If η is small, the elastic scattering is very weak and dependent on the angle, then we can obtain

$$\frac{\mathrm{d}\sigma_{\mathrm{e}}}{\mathrm{d}\Omega} \approx 64\omega^2 m^4 \left[1 + \frac{1}{3}(\omega v m)^2 \cos\theta\right] \tag{17}$$

from Eq. (16).

5. Electron scattering

In this section we study the scattering and absorption of particles by the black hole involving a global monopole. We can know from the quantum electrodynamics that the scattering amplitude is determined by the coefficients of A and B, which have the form

$$A = \frac{1}{2\mathrm{i}\omega v} \sum_{l=0}^{\infty} [(l+1)(S_l^- - 1) + l(S_l^+ - 1)] \mathrm{P}_l(\cos\theta),$$
$$B = \frac{1}{2\omega v} \sum_{l=0}^{\infty} [S_l^- - S_l^+] \mathrm{P}_l^1(\cos\theta), \tag{18}$$

where $\pm$ correspond to the states of $k > 0$ and $k < 0$. Given a non-polarized incident particle, then the differential scattering cross section is

$$\frac{\mathrm{d}\sigma_{\mathrm{e}}}{\mathrm{d}\Omega} = |A|^2 + |B|^2, \tag{19}$$

and the total absorption cross-sectional area is

$$\sigma_{\mathrm{A}} = \frac{\pi}{\omega^2 v^2} \sum_{l=0}^{\infty} [(l+1)(1 - |S_l^-|^2) + l(1 - |S_l^2|^2)] = \frac{\pi}{\omega^2 v^2} \sum_{l=0}^{\infty} [(l+1)T_l^- + lT_l^+] = \frac{\pi}{\omega^2 v^2} \sum_k T_k. \tag{20}$$

Using the radial connection method similar to Eq. (12), we can obtain the outgoing wave amplitude in the form

$$A_l^{\pm}(\omega) \approx (-1)^{l+1} S_l^C(\varepsilon)(1 - T_l^{\pm}/2). \tag{21}$$

Substituting Eq. (21) into Eq. (18), the coefficients of A and B are

$$A = \frac{\mathrm{i}}{4\omega v} \sum_{l=0}^{\infty} [(l+1)T_l^- + lT_l^+] S_l^C(\varepsilon) \mathrm{P}_l(\cos\theta), \tag{22}$$

$$B = \frac{1}{4\omega v} \sum_{l=0}^{\infty} (T_l^+ - T_l^-) S_l^C(\varepsilon) \mathrm{P}_l^1(\cos\theta). \tag{23}$$

Absorption cross-section area is determined by the S-wave absorption as follows:

$$\sigma_{\mathrm{A}} = \frac{\pi m^2}{v^2} \left(1 + \frac{\mu}{\omega}\right) 2\pi\omega m \times \frac{1 + v^2}{1 - \exp\left[(-2\pi\omega m / v)(1+v^2)\right]}. \tag{24}$$

Under approximate conditions, the coefficient B is not equal to zero because of absorption, so the scattered beam is polarized. Polarization vector is perpendicular to the scattering plane. If only $P_{1/2}$-wave contribution, can we obtain the scattering particle polarization vector

$$P = -4\pi \frac{(1 - \mu/\omega)(\omega m)^2 v}{1 - \exp -2\pi\varepsilon} \sin^3\frac{\theta}{2} \cos\frac{\theta}{2} \times \left[(\varepsilon^2 - 1)\cos\left(2\varepsilon \ln\sin\frac{\theta}{2}\right) + 2\varepsilon \sin\left(2\varepsilon \ln\sin\frac{\theta}{2}\right)\right] \boldsymbol{n} \times \boldsymbol{n}'. \tag{25}$$

Equation (25) shows that the degree of polarization $\Delta(\theta)$ is the oscillation function of the angle, when $\theta = 0$ or $\theta = \pi$, $\Delta = 0$. But if $\varepsilon \gg 1$ or $1 \gg \omega m \gg v$, then $v \to 0$, the polarization vector of the model is very small, and when $v \gg \omega m$, the polarization vector of the model is basically proportional to ωm. So the horizon of the black hole involving a global monopole can be viewed as a star polarizer. In addition, we can

obtain that the differential scattering cross section of electrons with the absorption effect is

$$\frac{\mathrm{d}\sigma_{\mathrm{e}}}{\mathrm{d}\Omega} = m^2 \frac{1+v^2}{4v^4 \sin^4(\theta/2)} \left[1 + 2\pi(1+\mu/\omega) \times \frac{\omega^2 m^2 v}{1-\exp -2\pi\varepsilon} \sin^2 \frac{\theta}{2} \sin\left(2\varepsilon \ln \sin \frac{\theta}{2}\right)\right]. \quad (26)$$

Relativistic effects are not considered in Eq. (26).

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