Deterministic implementations of fermionic quantum SWAP and Fredkin gates for spin qubits based on charge detection*

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We propose a scheme to implement fermionic quantum SWAP and Fredkin gates for spin qubits with the aid of charge detection. The scheme is deterministic without the need of qubit–qubit interaction, and the proposed setups consist of simple polarizing beam splitters, single-spin rotations, and charge detectors. Compared with linear optics quantum computation, this charge-measurement-based qubit scheme greatly enhances the success probability for implementing quantum SWAP and Fredkin gates and greatly simplifies the experimental realization of scalable quantum computers with noninteracting electrons.

Keywords: SWAP gate, Fredkin gate, charge detection

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The perspective of a high computational power generates intense efforts to build quantum computers. The two-qubit and three-qubit entangling unitary gates, which are the essential building blocks of a quantum computer and lie at the heart of quantum-information processing that requires precise control and manipulation of the states of quantum systems, have attracted a lot of attention. Linear optics quantum computation (LOQC) has undergone a rapid development during the last decade.[1] Many quantum gates, such as the controlled-NOT (CNOT) gate, the controlled phase (CP) gate, the SWAP gate, the Fredkin gate, and so on, can be implemented between photons with certain probability (usually, the success probability is very low) using auxiliary single photons, passive linear optical interferometers, single photon detectors, and feedforward.[2−18] Despite the fact that the success probability of the above gates are only probabilistic and has some finite probability of failure, a universal quantum computer can also be built with linear optics technology.[15] Free-electron quantum computation (FEQC), which can be achieved by using conduction electrons in a solid state, provides another essential tool in quantum-information processing. In comparison with linear-optics-based quantum computation: (I) it is unnecessary to convert material qubits to radiation by using free electrons acting as the flying qubits; (II) the implementation of quantum gate operations can be greatly enhanced by the charge detection.[19−22] In 2002, Bose and Home[21] proposed a scheme of arbitrarily high efficiency for entangling the spins of two independent uncorrelated identical particles by using beam splitters and which way detection, providing a strong motivation for developing two-particle interferometry for various systems in tandem with absorptionless which way detectors. In 2004, Beenakker *et al.*[22] constructed a deterministic CNOT gate between flying electronic qubits by means of polarizing beam splitters, local single-spin rotations, assisted by charge detectors, and they showed that quantum-information processing with such electrons could be simpler but more computationally powerful than by using photons.

The realization of a generic two-qubit gate may require a sequence of up to three CNOT gates combined with single-qubit rotations. In the case of three-qubit gates, the situation is even worse. For example, the quantum SWAP gate requires three CNOT gates, or can be decomposed into six Hadamard (H)

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gates and three CP gates, while a Fredkin gate requires seven two-qubit gates, as shown in Fig. 1. It is thus desirable to seek alternative simpler schemes for quantum SWAP and Fredkin gates. In this paper, motivated by the work of Beenakker *et al.*,[22] we propose a charge-detection-based scheme to implement fermionic quantum SWAP and Fredkin gates for spin qubits deterministically. For the implementation of fermionic quantum SWAP gate, it is direct to let the spin qubits pass through a series of setups, such as polarizing beam splitters, single-spin rotations, and charge detectors, any ancillary single spin qubit or entangled qubit sources are not needed. While for the Fredkin gate, it needs to use four ancillary single spin qubits, in the state $(|\uparrow\rangle+|\downarrow\rangle)/\sqrt{2}$, for applying deterministic CNOT gates proposed in Ref. [22] to control and target qubits. The experimental realization of the scheme may open up the possibility for quantum computation in solid-state systems without the need of qubit–qubit interaction.

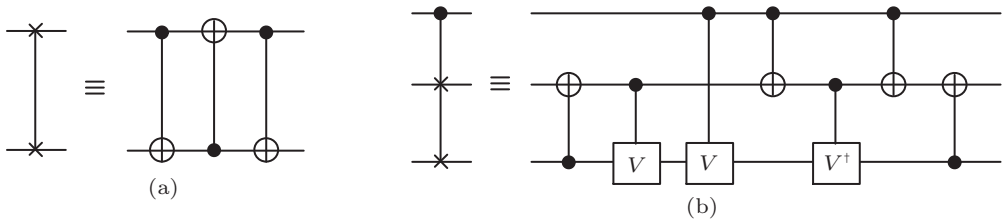


Fig. 1. (a) Decomposition of the SWAP gate from CNOT gates. (b) Seven-gate implementation of a Fredkin gate, where $V=\sqrt{\sigma_x}$, is a 2×2 matrix.

The schematic diagram for the implementation of a deterministic quantum SWAP gate is shown in Fig. 2, where the polarizing beam splitters represented by the double dashed lines transmit spin up and reflect spin down. The σ_x is the Pauli operator representing local spin-flip operation, and p_j $(j=1,2,3,4)$ are charge detectors, which can serve as parity detectors recording electronic bunching ($p_j=0$) or antibunching ($p_j=1$) because they can work in a nondestructive way and can distinguish occupation number unity from occupation number 0 or 2, but cannot distinguish between 0 and 2.[22,23] Here nondestructive means that the charge detectors are able to determine the charge of the electrons without disturbing the spin (spin and charge commute, so a measurement of the charge leaves the spin qubit unaffected). H_1 and H_2 are two types of Hadamard gate transformations, with $H_1=(\sigma_x+\sigma_z)/\sqrt{2}$ and $H_2=(\sigma_x-\sigma_z)/\sqrt{2}$, whose actions are given by

$$\begin{aligned} H_1 &\begin{cases} |\uparrow\rangle \to \frac{1}{\sqrt{2}}(|\uparrow\rangle+|\downarrow\rangle), \\ |\downarrow\rangle \to \frac{1}{\sqrt{2}}(|\uparrow\rangle-|\downarrow\rangle), \end{cases} \\ H_2 &\begin{cases} |\uparrow\rangle \to \frac{1}{\sqrt{2}}(|\downarrow\rangle-|\uparrow\rangle), \\ |\downarrow\rangle \to \frac{1}{\sqrt{2}}(|\uparrow\rangle+|\downarrow\rangle). \end{cases} \end{aligned} \tag{1}$$

We consider two noninteracting electrons in Fig. 2 prepared initially in an arbitrary spin input state written as

$$\begin{aligned} |\phi\rangle =\ & \alpha_1|\uparrow\rangle_{a_{1\rm in}}|\uparrow\rangle_{a_{2\rm in}}+\alpha_2|\uparrow\rangle_{a_{1\rm in}}|\downarrow\rangle_{a_{2\rm in}} \\ & +\alpha_3|\downarrow\rangle_{a_{1\rm in}}|\uparrow\rangle_{a_{2\rm in}}+\alpha_4|\downarrow\rangle_{a_{1\rm in}}|\downarrow\rangle_{a_{2\rm in}}, \end{aligned} \tag{2}$$

where $\sum_{i=1}^{4}|\alpha_i|^2=1$. After a series of setups shown in Fig. 2, the spin state of electrons from the output ports b_i $(i=1,2,3,4)$ is given by

$$|\phi\rangle'=\frac{1}{2}\big(|\phi\rangle_1+|\phi\rangle_2+|\phi\rangle_3+|\phi\rangle_4\big), \tag{3}$$

with

$$\begin{aligned} |\phi\rangle_1 =\ & \alpha_1|\uparrow\rangle_{b_2}|\uparrow\rangle_{b_3}+\alpha_2|\downarrow\rangle_{b_2}|\uparrow\rangle_{b_3} \\ & +\alpha_3|\uparrow\rangle_{b_2}|\downarrow\rangle_{b_3}+\alpha_4|\downarrow\rangle_{b_2}|\downarrow\rangle_{b_3}, \\ |\phi\rangle_2 =\ & \alpha_1|\downarrow\rangle_{b_1}|\downarrow\rangle_{b_4}+\alpha_2|\uparrow\rangle_{b_1}|\downarrow\rangle_{b_4} \\ & +\alpha_3|\downarrow\rangle_{b_1}|\uparrow\rangle_{b_4}+\alpha_4|\uparrow\rangle_{b_1}|\uparrow\rangle_{b_4}, \\ |\phi\rangle_3 =\ & -\alpha_1|\downarrow\rangle_{b_1}|\uparrow\rangle_{b_3}-\alpha_2|\uparrow\rangle_{b_1}|\uparrow\rangle_{b_3} \\ & -\alpha_3|\downarrow\rangle_{b_1}|\downarrow\rangle_{b_3}-\alpha_4|\uparrow\rangle_{b_1}|\downarrow\rangle_{b_3}, \\ |\phi\rangle_4 =\ & -\alpha_1|\uparrow\rangle_{b_2}|\downarrow\rangle_{b_4}-\alpha_2|\downarrow\rangle_{b_2}|\downarrow\rangle_{b_4} \\ & -\alpha_3|\uparrow\rangle_{b_2}|\uparrow\rangle_{b_4}-\alpha_4|\downarrow\rangle_{b_2}|\uparrow\rangle_{b_4}. \end{aligned} \tag{4}$$

Then the charge detectors p_j $(j=1,2,3,4)$ record the electronic bunching ($p_j=0$) or antibunching ($p_j=1$), and pass the electrons to the output ports $b_{j\rm out}$. After that, one can easily see from Eq. (4) that a two-qubit quantum SWAP gate can be obtained in a deterministic fashion, by applying local single-qubit spin rotation operations to the qubits appropriately according

to the measurement results of the charge detectors. For example, if we have $p_2 = p_3 = 1$ and $p_1 = p_4 = 0$, the state of electrons $|\phi\rangle_1$ is obtained, which shows that a typical two-qubit SWAP gate from the output ports $b_{2\text{out}}$ and $b_{3\text{out}}$ is implemented, and in this case it does not need to perform any operation on the spin qubits. For other cases of the measurement results, please see Table 1 for details.

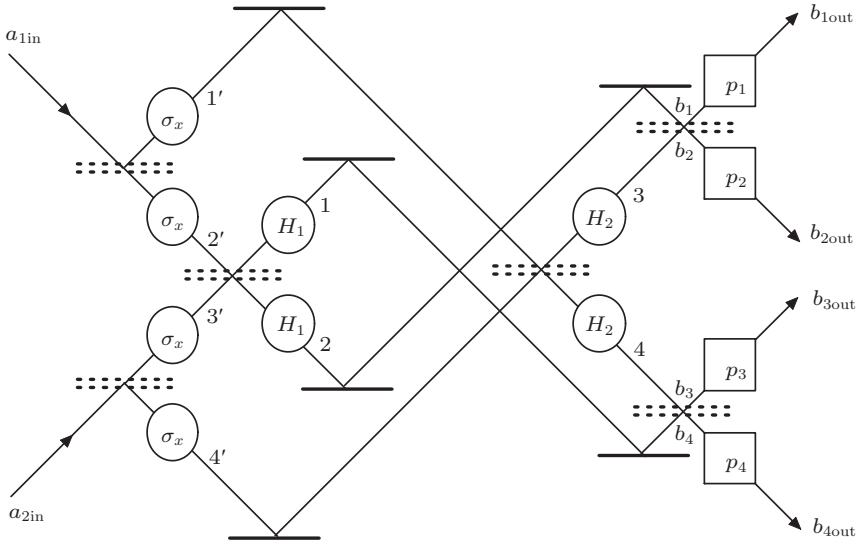


Fig. 2. Schematic diagram of implementing a deterministic SWAP gate for noninteracting electrons. The double dashed lines denote polarizing beam splitters and the solid horizonal lines denote mirrors.

Table 1. The obtained output states from the spin qubit output ports and the single-qubit local rotation operations required for achieving the quantum SWAP gate corresponding to different measurement values of charge detectors p_1, p_2, p_3, and p_4, respectively. Here $|\phi\rangle_i$ $(i = 1, 2, 3, 4)$ are denoted by Eq. (4), σ_x^{α} are Pauli operators on the spin qubit output from the ports α, and I is the identity operator.

Values of charge detection				Output states	Operations
p_1	p_2	p_3	p_4		
0	1	1	0	$\|\phi\rangle_1$	$I^{b_2} \otimes I^{b_3}$
1	0	0	1	$\|\phi\rangle_2$	$\sigma_x^{b_1} \otimes \sigma_x^{b_4}$
1	0	1	0	$\|\phi\rangle_3$	$\sigma_x^{b_1} \otimes I^{b_3}$
0	1	0	1	$\|\phi\rangle_4$	$I^{b_2} \otimes \sigma_x^{b_4}$

In the following we show how to construct a deterministic fermionic quantum Fredkin gate, as shown in Fig. 3. For three noninteracting electrons prepared initially in an arbitrary spin input state written as

$$\begin{aligned}|\psi\rangle \;=\; & \beta_1|\uparrow\rangle_{c_{\text{in}}}|\uparrow\rangle_{t_{1\text{in}}}|\uparrow\rangle_{t_{2\text{in}}} + \beta_2|\uparrow\rangle_{c_{\text{in}}}|\uparrow\rangle_{t_{1\text{in}}}|\downarrow\rangle_{t_{2\text{in}}} \\ & + \beta_3|\uparrow\rangle_{c_{\text{in}}}|\downarrow\rangle_{t_{1\text{in}}}|\uparrow\rangle_{t_{2\text{in}}} \\ & + \beta_4|\uparrow\rangle_{c_{\text{in}}}|\downarrow\rangle_{t_{1\text{in}}}|\downarrow\rangle_{t_{2\text{in}}} + \beta_5|\downarrow\rangle_{c_{\text{in}}}|\uparrow\rangle_{t_{1\text{in}}}|\uparrow\rangle_{t_{2\text{in}}} \\ & + \beta_6|\downarrow\rangle_{c_{\text{in}}}|\uparrow\rangle_{t_{1\text{in}}}|\downarrow\rangle_{t_{2\text{in}}} \\ & + \beta_7|\downarrow\rangle_{c_{\text{in}}}|\downarrow\rangle_{t_{1\text{in}}}|\uparrow\rangle_{t_{2\text{in}}} \\ & + \beta_8|\downarrow\rangle_{c_{\text{in}}}|\downarrow\rangle_{t_{1\text{in}}}|\downarrow\rangle_{t_{2\text{in}}}, \end{aligned} \tag{5}$$

where $\sum_{k=1}^{8} |\beta_k|^2 = 1$, after a series of setups shown in Fig. 3, we can obtain the spin state of electrons from the output ports c, 1, 2, 3, and 4 as

$$|\psi\rangle' = \frac{1}{2}\big(|\psi\rangle_1 + |\psi\rangle_2 + |\psi\rangle_3 + |\psi\rangle_4\big), \tag{6}$$

with

$$\begin{aligned}|\psi\rangle_1 \;=\; & \beta_1|\uparrow\rangle_c|\uparrow\rangle_2|\uparrow\rangle_3 + \beta_2|\uparrow\rangle_c|\uparrow\rangle_2|\downarrow\rangle_3 \\ & + \beta_3|\uparrow\rangle_c|\downarrow\rangle_2|\uparrow\rangle_3 + \beta_4|\uparrow\rangle_c|\downarrow\rangle_2|\downarrow\rangle_3 \\ & + \beta_5|\downarrow\rangle_c|\uparrow\rangle_2|\uparrow\rangle_3\beta_6|\downarrow\rangle_c|\downarrow\rangle_2|\uparrow\rangle_3 \\ & + \beta_7|\downarrow\rangle_c|\uparrow\rangle_2|\downarrow\rangle_3 + \beta_8|\downarrow\rangle_c|\downarrow\rangle_2|\downarrow\rangle_3, \\ |\psi\rangle_2 \;=\; & \beta_1|\uparrow\rangle_c|\downarrow\rangle_1|\downarrow\rangle_4 + \beta_2|\uparrow\rangle_c|\downarrow\rangle_1|\uparrow\rangle_4 \\ & + \beta_3|\uparrow\rangle_c|\uparrow\rangle_1|\downarrow\rangle_4 + \beta_4|\uparrow\rangle_c|\uparrow\rangle_1|\uparrow\rangle_4 \\ & + \beta_5|\downarrow\rangle_c|\downarrow\rangle_1|\downarrow\rangle_4 + \beta_6|\downarrow\rangle_c|\uparrow\rangle_1|\downarrow\rangle_4 \\ & + \beta_7|\downarrow\rangle_c|\downarrow\rangle_1|\uparrow\rangle_4 + \beta_8|\downarrow\rangle_c|\uparrow\rangle_1|\uparrow\rangle_4, \\ |\psi\rangle_3 \;=\; & \beta_1|\uparrow\rangle_c|\downarrow\rangle_1|\uparrow\rangle_3 + \beta_2|\uparrow\rangle_c|\downarrow\rangle_1|\downarrow\rangle_3 \end{aligned}$$

$$
\begin{aligned}
&+\beta_3|\uparrow\rangle_c|\uparrow\rangle_1|\uparrow\rangle_3+\beta_4|\uparrow\rangle_c|\uparrow\rangle_1|\downarrow\rangle_3\\
&-\beta_5|\downarrow\rangle_c|\downarrow\rangle_1|\uparrow\rangle_3-\beta_6|\downarrow\rangle_c|\uparrow\rangle_1|\uparrow\rangle_3\\
&-\beta_7|\downarrow\rangle_c|\downarrow\rangle_1|\downarrow\rangle_3-\beta_8|\downarrow\rangle_c|\uparrow\rangle_1|\downarrow\rangle_3,\\
|\psi\rangle_4 = {}&\beta_1|\uparrow\rangle_c|\uparrow\rangle_2|\downarrow\rangle_4+\beta_2|\uparrow\rangle_c|\uparrow\rangle_2|\uparrow\rangle_4\\
&+\beta_3|\uparrow\rangle_c|\downarrow\rangle_2|\downarrow\rangle_4+\beta_4|\uparrow\rangle_c|\downarrow\rangle_2|\uparrow\rangle_4\\
&-\beta_5|\downarrow\rangle_c|\uparrow\rangle_2|\downarrow\rangle_4-\beta_6|\downarrow\rangle_c|\downarrow\rangle_2|\downarrow\rangle_4\\
&-\beta_7|\downarrow\rangle_c|\uparrow\rangle_2|\uparrow\rangle_4-\beta_8|\downarrow\rangle_c|\downarrow\rangle_2|\uparrow\rangle_4, \quad (7)
\end{aligned}
$$

where the subscript c means "control out". Similar to the implementation of quantum SWAP gate, a quantum Fredkin gate can be deterministically implemented after the charge detection and the appropriate single-qubit local rotation operations on the spin qubits from the output ports "control out" and "target $2_{\rm out}$-$3_{\rm out}$", or "target $1_{\rm out}$-$3_{\rm out}$", or "target $1_{\rm out}$-$4_{\rm out}$", or "target $2_{\rm out}$-$4_{\rm out}$", as shown in Table 2.

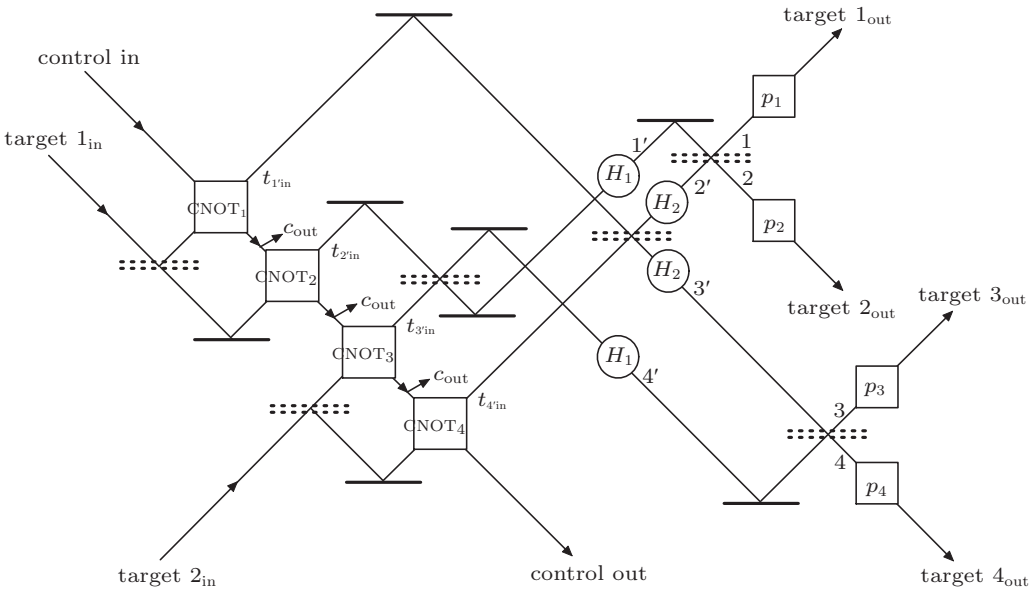


Fig.3. Schematic diagram of a deterministic Fredkin gate. Here CNOT$_k$ ($k = 1, 2, 3, 4$) denotes controlled-NOT gate, which was previously proposed in Fig. 3 of Ref. [22].

Table 2. The obtained output states from the spin qubit output ports and the single-qubit local rotation operations required for achieving the quantum Fredkin gate corresponding to different measurement values of charge detectors p_1, p_2, p_3, and p_4, respectively. Here $|\psi\rangle_j$ ($j = 1, 2, 3, 4$) are denoted by Eq. (6), and σ_z^β are Pauli operators on the spin qubit output from the ports β.

Results of charge detection				Output states	Operations
p_1	p_2	p_3	p_4		
0	1	1	0	$\|\psi\rangle_1$	$I^c \otimes I^2 \otimes I^3$
1	0	0	1	$\|\psi\rangle_2$	$I^c \otimes \sigma_x^1 \otimes \sigma_x^4$
1	0	1	0	$\|\psi\rangle_3$	$\sigma_z^c \otimes \sigma_x^1 \otimes I^3$
0	1	0	1	$\|\psi\rangle_4$	$\sigma_z^c \otimes I^2 \otimes \sigma_x^4$

We now briefly analyze and discuss the characteristics of our scheme. Firstly, to achieve our scheme experimentally, the polarizing beam splitters and the charge detectors required in our scheme are two elementary ingredients, and they have both been realized experimentally by means of the point contacts in a two-dimensional electron gas.[24−27] The charge detectors in our scheme can be realized by using a point contact made of a quantum dot with a resonant conductance. In this case the charge detectors can distinguish occupation number unity from occupation number 0 or 2 according to whether it is resonant or off resonant. Secondly, we may encode the qubits into the spin degrees of freedom of, for example, the conduction electrons in a quantum dot (QD) system. Such conduction electrons have recently been proposed in Ref. [28] to convert spin parity into charge information by resonant tunneling between two QDs when the spins are antiparallel. Thirdly, the CNOT gate, which was previously proposed by Beenakker *et*

al.,[22] used in our scheme is nearly deterministic if the charge detector counts the number of electrons in a mode, and fully deterministic if it only measures the parity of that number. Furthermore, similar setups have also been used to entangle two particles of any type with arbitrarily high efficiency, to test quantum statistics through entanglement-induced spin correlations, and to probe complementarity in two-particle interference.[21] Both the fidelities and success probabilities of the SWAP and Fredkin gates could reach unity under ideal conditions. In a real experiment, however, some impractical factors will affect the experimental performance, which call for further study. The earlier experiments, such as Refs. [29] and [30] suggested that it would be possible to test our results in the near future.

In conclusion, we have proposed a scheme to implement quantum SWAP and Fredkin gates based on polarizing beam splitters and local single-spin rotations in combination with the charge detectors. Our scheme is deterministic without the need for qubit–qubit interaction. Comparing with linear optics implementation of quantum SWAP and Fredkin gates, our scheme is computationally more powerful. Furthermore, the construction of quantum SWAP and Fredkin gates provides a potential way for quantum computation and quantum information processing with fermions.

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