Quantum state swap for two trapped Rydberg atoms*

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The quantum swap gate is one of the most useful gates for quantum computation. Two-qubit entanglement and a controlled-NOT quantum gate in a neutral Rydberg atom system have been achieved in recent experiments. It is therefore very interesting to propose a scheme here for swapping a quantum state between two trapped neutral atoms via the Rydberg blockade mechanism. The atoms interact with a sequence of laser pulses without individual addressing. The errors of the swap gate due to imprecision of pulse length, finite Rydberg interaction, and atomic spontaneous emission are discussed.

Keywords: neutral atom, swap gate, Rydberg blockade

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Quantum information processing based on a neutral Rydberg atom system has attracted much interest recently.[1−7] Rapid progress has been made in recent entanglement experiments with neutral atoms via the Rydberg blockade mechanism.[8−11] Entanglement of the hyperfine ground states of two atoms can be realized in ways that the atoms are individually controlled[10] and the atoms are addressed by common laser beams.[11] The experimental approaches were discussed in those pioneering theoretical works on fast quantum gates.[12−15] In addition, implementation of a mesoscopic Rydberg gate based on electromagnetically induced transparency[16] and preparation of a Greenberger–Horne–Zeilinger state using asymmetric Rydberg interaction in an optical lattice[17] were also investigated recently. The methods can be generalized for implementing quantum phase gates[18−20] and performing quantum simulations in many-body condensed matter physics.[21,22]

Any logic operations within a quantum computer can be expressed as a sequence of one- and two-qubit universal sets of quantum gates.[23] A two-qubit quantum swap gate described by

$$\begin{aligned}|0\rangle_1|0\rangle_2 &\rightarrow |0\rangle_1|0\rangle_2,\\ |1\rangle_1|1\rangle_2 &\rightarrow |1\rangle_1|1\rangle_2,\\ |0\rangle_1|1\rangle_2 &\rightarrow |1\rangle_1|0\rangle_2,\end{aligned}$$

$$|1\rangle_1|0\rangle_2 \rightarrow |0\rangle_1|1\rangle_2, \quad (1)$$

is a universal quantum gate and is useful for quantum state transfer, i.e.,

$$(\alpha|0\rangle_1+\beta|1\rangle_1)\otimes|k\rangle_2 \rightarrow |k\rangle_1\otimes(\alpha|0\rangle_2+\beta|1\rangle_2), \quad (2)$$

where $k = 0$ or 1. It is known that a two-qubit swap operation usually requires no less than three CNOT gates, which can be then decomposed into six Hadamard gates and three C-phase (π) gates.[24] However, in this paper, we show that a quantum swap gate between two neutral atoms can be directly implemented without decomposing into several universal gates via the Rydberg blockade mechanism. The atoms are exposed to a sequence of laser pulses without individual addressing. The effect of imprecise laser pulses, imperfect Rydberg interaction, and atomic spontaneous emission on the gate fidelity are discussed.

Consider two atoms 1 and 2 held in two separate dipole traps, e.g., far-off resonance traps or optical tweezers.[8−11] The atoms are assumed to be trapped in the vibrational ground state of the trapping potential. The relevant single-atom energy level is shown in Fig. 1. The atomic ground states $|0\rangle$ and $|1\rangle$ are encoded as logic zero and one, respectively. A smooth laser pulse couples $|0\rangle$ and $|1\rangle$ to the Rydberg state $|p\rangle$ with Rabi frequency Ω_p. The Rydberg excitations

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$|0\rangle \leftrightarrow |s\rangle$ and $|1\rangle \leftrightarrow |s\rangle$ are stimulated by two laser pulses with Rabi frequencies Ω_+ (phase ϕ_+) and Ω_- (phase ϕ_-), respectively. Double excitation of Rydberg state $|s\rangle$ is blockaded due to the fact that the strong Rydberg interaction between $|s\rangle$ state leads to a large energy gap $U = \hbar\Delta_{ss}$. Besides, strong Rydberg interaction between states $|s\rangle$ and $|p\rangle$ described by

$$H = \sum_{i,j=1,2} \hbar\Delta_{sp}|s\rangle_i\langle s| \otimes |p\rangle_j\langle p| \quad (i \neq j)$$

inhibits them being simultaneously populated, while the interaction between states $|p\rangle$ only gives rise to a small energy shift Δ_{pp}. In general, the Rydberg interaction strength fulfills the condition $(\Delta_{ss}, \Delta_{sp}) \gg \Delta_{pp}$.

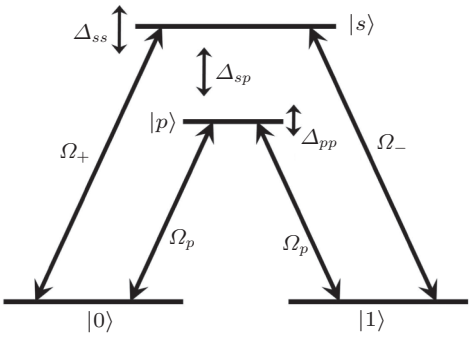


Fig. 1. Atomic level structure. Δ_{ss}, Δ_{sp}, and Δ_{pp} are energy shifts induced by interaction between different Rydberg levels. The interaction strength fulfill Δ_{ss}, $\Delta_{sp} \gg \Delta_{pp}$. The ground states $|0\rangle$ and $|1\rangle$ are resonantly coupled to Rydberg state $|s\rangle$ with Rabi frequencies Ω_+ and Ω_-, respectively. The transitions $|0\rangle \leftrightarrow |p\rangle$ and $|1\rangle \leftrightarrow |p\rangle$ are driven by a smooth Raman laser pulse with Rabi frequency Ω_p.

The realization of quantum state swapping between the two Rydberg atoms consists of three steps, where a sequence of laser pulses are applied, as shown schematically in Fig. 2.

Step 1 The first laser pulse is resonantly coupled to the transition $|0\rangle \leftrightarrow |s\rangle$. The interaction Hamiltonian for the excitation process is $H_+ = \Omega_+ \, \mathrm{e}^{\mathrm{i}\,\phi_+}|0\rangle_k\langle s| + \mathrm{h.c.}$ Only single excitation is allowed due to the strong Rydberg interaction between $|s\rangle$ states. Thus, with $\phi_+ = 0$, the time evolution for the four basis states $\{|0\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |1\rangle_1|1\rangle_2\}$ are given by

$$\begin{aligned}
|0\rangle_1|0\rangle_2 &\to \cos\left(\frac{\Omega_+ t_1}{\sqrt{2}}\right)|0\rangle_1|0\rangle_2 \\
&\quad - \mathrm{i}\sin\left(\frac{\Omega_+ t_1}{\sqrt{2}}\right)\frac{|0\rangle_1|s\rangle_2 + |s\rangle_1|0\rangle_2}{\sqrt{2}}, \\
|0\rangle_1|1\rangle_2 &\to \cos\left(\frac{\Omega_+ t_1}{2}\right)|0\rangle_1|1\rangle_2 \\
&\quad - \mathrm{i}\sin\left(\frac{\Omega_+ t_1}{2}\right)|s\rangle_1|1\rangle_2, \\
|1\rangle_1|0\rangle_2 &\to \cos\left(\frac{\Omega_+ t_1}{2}\right)|1\rangle_1|0\rangle_2 \\
&\quad - \mathrm{i}\sin\left(\frac{\Omega_+ t_1}{2}\right)|1\rangle_1|s\rangle_2, \\
|1\rangle_1|1\rangle_2 &\to |1\rangle_1|1\rangle_2. \qquad (3)
\end{aligned}$$

Likewise, the second laser pulse coupling to the transition $|1\rangle \leftrightarrow |s\rangle$ can be described by the Hamiltonian $H_- = \Omega_- \, \mathrm{e}^{\mathrm{i}\,\phi_-}|1\rangle_k\langle s| + \mathrm{h.c.}$ ($\phi_- = 0$), which determines the time evolution

$$\begin{aligned}
&\frac{|0\rangle_1|s\rangle_2 + |s\rangle_1|0\rangle_2}{\sqrt{2}} \\
&\quad \to \cos\left(\frac{\Omega_- t_2}{2}\right)\frac{(|0\rangle_1|s\rangle_2 + |s\rangle_1|0\rangle_2)}{\sqrt{2}} \\
&\quad\quad - \mathrm{i}\sin\left(\frac{\Omega_- t_2}{2}\right)\frac{(|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2)}{\sqrt{2}}, \\
|0\rangle_1|1\rangle_2 &\to \cos\left(\frac{\Omega_- t_2}{2}\right)|0\rangle_1|1\rangle_2 \\
&\quad - \mathrm{i}\sin\left(\frac{\Omega_- t_2}{2}\right)|0\rangle_1|s\rangle_2, \\
|1\rangle_1|0\rangle_2 &\to \cos\left(\frac{\Omega_- t_2}{2}\right)|1\rangle_1|0\rangle_2 \\
&\quad - \mathrm{i}\sin\left(\frac{\Omega_- t_2}{2}\right)|s\rangle_1|0\rangle_2, \\
|1\rangle_1|1\rangle_2 &\to \cos\left(\frac{\Omega_- t_2}{\sqrt{2}}\right)|1\rangle_1|1\rangle_2 \\
&\quad - \mathrm{i}\sin\left(\frac{\Omega_- t_2}{\sqrt{2}}\right)\frac{(|1\rangle_1|s\rangle_2 + |s\rangle_1|1\rangle_2)}{\sqrt{2}}. \qquad (4)
\end{aligned}$$

Note that the $|1\rangle \leftrightarrow |s\rangle$ transition is blockaded if there is one population in the Rydberg state $|s\rangle$. By choosing the interaction time appropriately so that $\Omega_+ t_1/2 = \Omega_- t_2/2 \simeq 6\pi$, the basis states after the two laser pulses become

$$\begin{aligned}
&|0\rangle_1|0\rangle_2 \to \Phi_{s0}, \quad |0\rangle_1|1\rangle_2 \to |0\rangle_1|1\rangle_2, \\
&|1\rangle_1|0\rangle_2 \to |1\rangle_1|0\rangle_2, \quad |1\rangle_1|1\rangle_2 \to \Phi_{s1}, \qquad (5)
\end{aligned}$$

where $\Phi_{sk} = (|k\rangle_1|s\rangle_2 + |s\rangle_1|k\rangle_2)/\sqrt{2}$, $k = 0, 1$. The error for the transformation is less than 10^{-3}. Given the imprecision of the pulse length, the fidelity for the first step approximates to

$$\begin{aligned}
F \simeq \frac{1}{16}\Bigg\{ &\cos\left(\frac{\Delta x_+}{\sqrt{2}}\right)\left[\cos\left(\frac{\Delta x_-}{2}\right)\right. \\
&\left. - \sqrt{2}\sin\left(\frac{\Delta x_-}{2}\right)\right] \\
&+ \sqrt{2}\cos\left(\frac{\Delta x_+}{2}\right)\left[\sin\left(\frac{\Delta x_-}{2}\right)\right. \\
&\left. - \sqrt{2}\cos\left(\frac{\Delta x_-}{2}\right)\right]
\end{aligned}$$

$$-\frac{1}{\sqrt{2}}\sin\left(\frac{\Delta x_+}{2}\right)\left[\sin\left(\frac{\Delta x_-}{\sqrt{2}}\right)+\sin\left(\frac{\Delta x_+}{\sqrt{2}}\right)\right]+\cos\left(\frac{\Delta x_-}{\sqrt{2}}\right)\Bigg\}^2, \quad (6)$$

where $x_+ = \Omega_+ t_1$, $x_- = \Omega_- t_2$. Δx_+ and Δx_- are the deviations due to the imprecise interaction time. The fidelity versus Δx_+ and Δx_- is shown in Fig. 3(a). The elaborate control of the Rabi pulse length can be experimentally achieved with the precision on the order of tens of nanoseconds.[8,9] In addition, the fidelity of the transformation may be limited by finite Rydberg interaction Δ_{ss} leading to double Rydberg excitation. This deficiency can be readily solved as follows: The two atoms are firstly stimulated to a higher lying Rydberg state $|r\rangle$, in which the Rydberg interaction Δ_{rr} is much larger. Thus, only one atom can be excited. After that, the atom in state $|r\rangle$ is transferred to the state $|s\rangle$. Thus, the first step can be well realized.

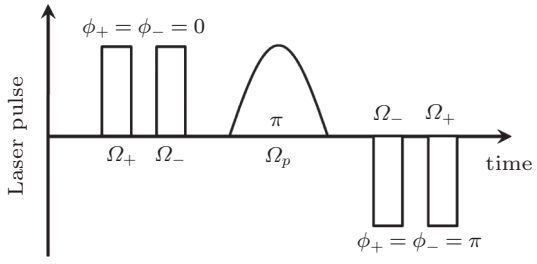


Fig. 2. Sequence of laser pulses. The first two short laser pulses applied in the first step drives the atomic transitions $|0\rangle \leftrightarrow |s\rangle$ and $|1\rangle \leftrightarrow |s\rangle$ (with the phase of the coupling strength $\phi_+ = \phi_- = 0$). After that, a smoothly sinusoidal laser pulse is applied coupling to the transitions $|0\rangle \leftrightarrow |p\rangle$ and $|1\rangle \leftrightarrow |p\rangle$. The last two laser pulses are same to the first two laser pulses but $\phi_+ = \phi_- = \pi$.

Step 2 The following transformation is implemented in the second step,

$$\Phi_{s0} \to \Phi_{s0}, \quad |0\rangle_1|1\rangle_2 \to |1\rangle_1|0\rangle_2,$$
$$|1\rangle_1|0\rangle_2 \to |0\rangle_1|1\rangle_2, \quad \Phi_{s1} \to \Phi_{s1}. \quad (7)$$

The atoms interact with a smooth laser pulse coupling to transitions $|0\rangle \leftrightarrow |p\rangle$ and $|1\rangle \leftrightarrow |p\rangle$. The interaction can be resonant or dispersive for different basis states.

(i) ***Resonant*** (Transfer): the interaction Hamiltonian is $H_{01} = (\Omega_p/2)(|0\rangle_k\langle p| + |1\rangle_k\langle p|) + \text{h.c.}$, which gives the time evolution for the states $|0\rangle_j$ and $|1\rangle_j$ $(j = 1, 2)$,

$$|0\rangle_j \to \frac{1+\cos(\Omega_p t/\sqrt{2})}{2}|0\rangle_j + \frac{\cos(\Omega_p t/\sqrt{2})-1}{2}|1\rangle_j - \frac{\mathrm{i}}{\sqrt{2}}\sin\left(\frac{\Omega_p t}{\sqrt{2}}\right)|p\rangle_j,$$

$$|1\rangle_j \to \frac{1+\cos(\Omega_p t/\sqrt{2})}{2}|1\rangle_j + \frac{\cos(\Omega_p t/\sqrt{2})-1}{2}|0\rangle_j - \frac{\mathrm{i}}{\sqrt{2}}\sin\left(\frac{\Omega_p t}{\sqrt{2}}\right)|p\rangle_j, \quad (8)$$

here Ω_p is treated as a time-independent constant for convenience and is a smoothly sinusoidal pulse in the numerical calculation. Setting $\Omega_p t/\sqrt{2} = \pi$, the basis states $|0\rangle_1|1\rangle_2$ and $|1\rangle_1|0\rangle_2$ become

$$|0\rangle_1|1\rangle_2 \to |1\rangle_1|0\rangle_2,$$
$$|1\rangle_1|0\rangle_2 \to |0\rangle_1|1\rangle_2. \quad (9)$$

Noted that the time evolution of the system contains a fraction of double Rydberg excitation state $|p\rangle_1|p\rangle_2$, which gives rise to a weak Rydberg interaction $H_p = \Delta_{pp}\ |p\rangle_1\langle p| \otimes |p\rangle_2\langle p|$ and imperfect swapping. For $\Delta_{pp} \ll \Omega_p$, the error can be calculated by the first-order perturbation theory and the modified time evolution operator for states $|0\rangle_1|1\rangle_2$ and $|1\rangle_1|0\rangle_2$ is

$$U_p = I - \mathrm{i}\,\Delta_{pp}\sum_{m_1,m_2}\sum_{m_1',m_2'} c_{m_1}c_{m_2}c^*_{m_1'}c^*_{m_2'} \times \frac{\mathrm{e}^{\mathrm{i}(w_{m_1}+w_{m_2}-w_{m_1'}-w_{m_2'})}-1}{\mathrm{i}(w_{m_1}+w_{m_2}-w_{m_1'}-w_{m_2'})} \times (|m_1\rangle\langle m_1'|)\otimes(|m_2\rangle\langle m_2'|), \quad (10)$$

where $c_{m_j} = \langle m_j|p_j\rangle$, $j = 1, 2$. w_{m_j} and $|m_j\rangle$ are eigenvalues and the corresponding eigenstates of the Hamiltonian H_{01}, respectively. Therefore, we find the transfer error $E_{\text{tr}} = 0.6(\Delta_{pp}/\Omega_p)^2$ for the initial state $|0\rangle_1|1\rangle_2$ or $|1\rangle_1|0\rangle_2$. The extra phase due to Δ_{pp} can be neglected if $\Delta_{pp}/\Omega_p \ll 1$. The transferring probability P_{tr} as a function of $\Delta_{pp}/\Omega_p^{\max}$ is shown in Fig. 3(b), where $\Omega_p^{\max}$ is the peak amplitude of the Rabi pulse. The Raman laser profile in the numerical simulation is $\Omega = \Omega_P/\sqrt{2} = (2\pi/T)\sin^2(\pi t/T)$, where $T = 0.71$ μs and $\int_0^T \Omega\,\mathrm{d}t = \pi$. It can be seen that P_{tr} decreases as $\Delta_{pp}/\Omega_p^{\max}$ increases and saturates for $\Delta_{pp}/\Omega_P^{\max} > 2$.

(ii) ***Dispersive*** (Blocking): for the new basis states Φ_{sk} $(k = 0, 1)$ after the first step, the Rydberg state $|s\rangle$ is populated by one of the two atoms. If the Rydberg interaction between states $|s\rangle$ and $|p\rangle$ induced energy shift Δ_{sp} is much larger than Ω_p and spontaneous decay rate γ_p of $|p\rangle$, the time evolution for Φ_{sk} can be described by the dispersive interaction Hamiltonian $H_{\text{spe}} = (\Omega_p^2/4\Delta_{sp})|0\rangle_j\langle 1| + \text{h.c.}$ after adiabatically eliminating the medium state $|p\rangle$,

$$|0\rangle_j \to \cos\left(\frac{\Omega_p^2 t}{4\Delta_{sp}}\right)|0\rangle_j - \mathrm{i}\sin\left(\frac{\Omega_p^2 t}{4\Delta_{sp}}\right)|1\rangle_j,$$
$$|1\rangle_j \to \cos\left(\frac{\Omega_p^2 t}{4\Delta_{sp}}\right)|1\rangle_j - \mathrm{i}\sin\left(\frac{\Omega_p^2 t}{4\Delta_{sp}}\right)|0\rangle_j. \quad (11)$$

The probability for leaving the basis states Φ_{sk} approximates to $E_b = (\pi\Omega_p/2\Delta_{sp})^2$. However, a more stringent analysis is needed if Δ_{sp} is not strong enough. In this case, the time evolution for the atoms initially in the state $|0\rangle_j$ can be more strictly depicted by

$$\begin{aligned}
&\frac{1}{\sqrt{2}}\left[1+\mathrm{e}^{-\mathrm{i}Vt/2}\left(\cos\left(\frac{\Omega' t}{2}\right)\right.\right.\\
&+\mathrm{i}\frac{V}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right)\Big)\Big]|0\rangle_j\\
&+\frac{1}{\sqrt{2}}\left[-1+\mathrm{e}^{-\mathrm{i}Vt/2}\left(\cos\left(\frac{\Omega' t}{2}\right)\right.\right.\\
&+\mathrm{i}\frac{V}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right)\Big)\Big]|1\rangle_j\\
&-\mathrm{i}\,\mathrm{e}^{\mathrm{i}Vt/2}\frac{\Omega_p}{\Omega'}\sin\left(\frac{\Omega' t}{2}\right)|p\rangle_j, \qquad (12)
\end{aligned}$$

where $\Omega' = \sqrt{2\Omega_p^2 + V^2}$. $V = \Delta_{sp}$ is the finite energy shift. The probability of finding the atoms in the state $|0\rangle_j$ is

$$\begin{aligned}
P_0 = \frac{1}{2\Omega'^2}\Bigg[&\Omega'^2 + \frac{\Omega_p^2}{2} - \frac{\Omega_p^2}{2}\cos\left(\frac{\sqrt{2}\Omega'\pi}{\Omega_p}\right)\\
&+\Omega'^2\cos\left(\frac{V\pi}{\sqrt{2}\Omega_p}\right)\cos\left(\frac{\Omega'\pi}{\sqrt{2}\Omega_p}\right)\\
&+V\Omega'\sin\left(\frac{V\pi}{\sqrt{2}\Omega_p}\right)\sin\left(\frac{\Omega'\pi}{\sqrt{2}\Omega_p}\right)\Bigg]. \qquad (13)
\end{aligned}$$

Thus, the error due to imperfect blockade is $E_\mathrm{b} = 1 - P_0$. We show E_b as a function of $V/\Omega_p^{\max}$ in Fig. 3(c). The imperfect blockade error is less than 0.01 for $V/\Omega_p^{\max} \geq 5.6$. The extra phase introduced in this case can be eliminated by appropriately tuning the phase of the laser pulses during the first and third step.

The spontaneous decay γ_p of the Rydberg state $|p\rangle$ leads to decoherence and therefore introduces an additional error during the state transfer between $|0\rangle$ and $|1\rangle$. The error is determined by the time-averaged state population of $|p\rangle$ and can be given by $E_\mathrm{se} = \pi\gamma_p/\sqrt{2}\Omega_p$ for the resonance case. For the dispersive case, the effective decay rate approximates to $\Omega_p^2\gamma_p/\Delta_{sp}^2$, which can be neglected if $\gamma_p/\Delta_{sp} \ll 1$. In addition, mechanical interaction between the atoms comes about if they simultaneously reside in the Rydberg states, which may lead to atomic motion and momentum shift. However, in a recent experiment,[8,9] Gaëtan *et al.* have proved that the atomic motion can be frozen if the excitation time is of the order of 0.1 μs.

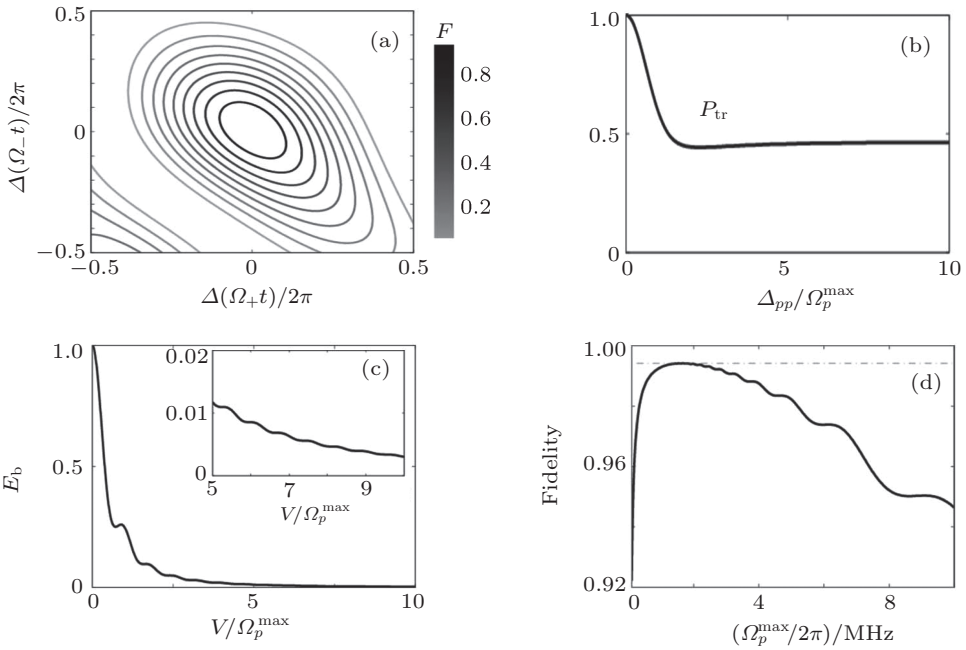


Fig. 3. (a) The fidelity of transformations in the first step versus imprecision of pulse length, (b) the success probability for transferring process in the second step as a function of $\Delta_{pp}/\Omega_p^{\max}$, (c) operation error due to imperfect blockade in the second step as a function of asymmetric Rydberg interaction $V/\Omega_p^{\max}$, (d) the fidelity of quantum swap gate versus Rabi frequency $\Omega_p^{\max}/2\pi$. The initial atomic state is supposed to be $(1/\sqrt{2})(|0\rangle_1 + |1\rangle_1) \otimes (1/\sqrt{2})(|0\rangle_2 + |1\rangle_2)$ in panels (a) and (d) for simplicity. In panels (b) and (c), we set $\Omega_p^{\max}/2\pi = 2$ MHz.

Step 3 Turn on the lasers H_+ and H_- in reverse order with $\phi_- = \phi_+ = \pi$. In contrast to the first step, the atoms in the Rydberg state $|s\rangle$ are transferred to initial ground state and the swapping gate is successfully implemented,

$$\Phi_{s0} \to |0\rangle_1|0\rangle_2, \quad |1\rangle_1|0\rangle_2 \to |1\rangle_1|0\rangle_2,$$
$$|0\rangle_1|1\rangle_2 \to |0\rangle_1|1\rangle_2, \quad \Phi_{s1} \to |1\rangle_1|1\rangle_2. \tag{14}$$

In analogy to the recent entanglement experiment,[11] our scheme can be implemented in the same experimental setup with Rubidium atoms. We encode the hyperfine ground states $|5s_{1/2}, F = 1, m_F = 0\rangle$ and $|5s_{1/2}, F = 2, m_F = 2\rangle$ as logic 0 and 1, and choose the high-lying Rydberg states $|41s_{1/2}, m = 1/2\rangle$ and $|40p_{3/2}, m = 1/2\rangle$ as $|s\rangle$ and $|p\rangle$. The atoms are separated by a distance $d = 3$ μm and are addressed by common laser beams, as demonstrated by Wilk *et al.*[11] The detailed laser driving scheme is as follows. In the first step, the atomic transition $|0\rangle \leftrightarrow |s\rangle$ can be made via a two-photon transition process, in which a left-circularly polarized laser and a π polarized laser are used to drive the transitions $|5s_{1/2}, F = 1, m_F = 0\rangle \leftrightarrow |5p_{1/2}, F = 2, m_F = 1\rangle$ and $|5p_{1/2}, F = 2, m_F = 1\rangle \leftrightarrow |41s_{1/2}, m = 1/2\rangle$, respectively. Both lasers are detuned from the medium energy level $|5p_{1/2}, F = 2, m_F = 1\rangle$. The ground state $|1\rangle$ is kept unperturbed. In order to realize the transition $|1\rangle \leftrightarrow |s\rangle$, we drive the atomic transitions $|5s_{1/2}, F = 2, m_F = 2\rangle \leftrightarrow |5p_{1/2}, F = 2, m_F = 1\rangle$ and $|5p_{1/2}, F = 2, m_F = 1\rangle \leftrightarrow |41s_{1/2}, m = 1/2\rangle$ with a right-circularly polarized laser and a π polarized laser, respectively. In the second step, coupling from the ground states to the Rydberg state $|40p_{3/2}, m = 1/2\rangle$ can be done by a one photon transition, in which we can use a linear polarized laser with a relatively short wave length.

The strong Rydberg interaction Δ_{sp} between the two-atom states $|sp\rangle$ and $|ps\rangle$ are classical-like dipole interaction. It is angle-dependent and has a zero-value when the angle between the dipoles is $\beta = \cos^{-1}(1/\sqrt{3})$. However, the interaction of dipoles with angular zeroes can be recovered by applying a weak external magnetic field to both atoms. In this way, the atomic Zeeman states with different magnetic quantum numbers become degenerate and thereby couple to each other. The weak Rydberg interaction Δ_{pp} is induced by the Förster process[25] when both atoms are excited to Rydberg state $|p\rangle$. Δ_{sp} has a scaling of $1/R^3$, while Δ_{pp} has a scaling of $1/R^6$ in the van der Waals limit, where R denotes the practical interatomic distance. It has been proved that the angle-dependent ratio Δ_{sp}/Δ_{pp} is larger than 150 for arbitrary angles.[17] We then characterize the interactions, after averaging all spatial angles between dipoles, by $(\overline{\Delta}_{sp}(d), \overline{\Delta}_{pp}(d))/2\pi = (14.4, 0.019)$ MHz for simplicity.[17] Furthermore, the interaction between Rydberg states $|s\rangle_1$ and $|s\rangle_2$ has a strength $\Delta_{ss}(d)/2\pi = 3.7$ MHz. It is isotropic and is proportional to $1/R^6$.[26] Therefore, the Rydberg interaction strengths fulfill the condition $(\Delta_{ss}, \Delta_{sp}) \gg \Delta_{pp}$ that we are interested in.[1,27] Assume that the initial atomic state is $\frac{1}{2}(|0\rangle_1|0\rangle_2+|0\rangle_1|1\rangle_2+|1\rangle_1|0\rangle_2+|1\rangle_1|1\rangle_2)$ and take into account the finite lifetime $\tau_p = 57$ μs for state $|p\rangle$, we obtain the fidelity of our scheme as a function of Rabi frequency $\Omega_p^{\max}/2\pi$ after neglecting the errors in the first and third steps, as shown in Fig. 3(d). The maximal fidelity $F^{\max} = 0.9942$ can be achieved for $\Omega_p^{\max}/2\pi \approx 1.6$ MHz. Given the imprecision error with perfect laser pulses in the first and third steps, the fidelity is still larger than 0.99.

In the above discussion, we assume that the atoms are in a motional ground state, but the fidelity of the quantum swapping gate may decrease due to atomic motion. However, it has been demonstrated that gate operation based on Rydberg blockade is not directly sensitive to the center of mass motion.[1,10,28] Thus, trapping the atoms in the potential ground state is not necessary in principle. But it does help to lengthen the coherence times and reduce the gate errors.

In summary, we have proposed an experimentally feasible scheme for implementing two-atom swap gate via asymmetric Rydberg blockade. The gate errors due to imprecision of interaction time and imperfect blockade are discussed. The idea can be generalized for mesoscopic atomic ensembles with $|0\rangle$ and $|1\rangle$ encoded respectively by $|0\rangle_1|0\rangle_2\cdots|0\rangle_N$ and $|1\rangle_1|1\rangle_2\cdots|1\rangle_N$. The teleportation of a quantum state may also be realized in the context of Rydberg interaction.

References

[1] Saffman M, Walker T G and Mølmer K 2010 *Rev. Mod. Phys.* **82** 2313

[2] Zhao B, Müller M, Hammerer K and Zoller P 2010 *Phys. Rev.* A **81** 052329

[3] Li D, Chen A X and Zhang J S 2011 *Chin. Phys.* B **20** 110304

[4] Han Y, He B, Heshami K, Li C Z and Simon C 2010 *Phys. Rev.* A **81** 052311

[5] Gorshkov A V, Otterbach J, Fleischhauer M, Pohl T and Lukin M D 2011 *Phys. Rev. Lett.* **107** 133602

[6] Mølmer K, Isenhower L and Saffman M 2011 *J. Phys.* B: *At. Mol. Opt. Phys.* **44** 184016

[7] Chen A X 2011 *Opt. Express* **19** 2037

[8] Urban E, Johnson T A, Henage T, Isenhower L, Yavuz D D, Walker T G and Saffman M 2009 *Nat. Phys.* **5** 110

[9] Gaëtan A, Miroshnychenko Y, Wilk T, Chotia A, Viteau M, Comparat D, Pillet P, Browaeys A and Grangier P 2009 *Nat. Phys.* **5** 115

[10] Isenhower L, Urban E, Zhang X L, Gill A T, Henage T, Johnson T A, Walker T G and Saffman M 2010 *Phys. Rev. Lett.* **104** 010503

[11] Wilk T, Gaëtan A, Evellin C, Wolters J, Miroshnychenko Y, Grangier P and Browaeys A 2010 *Phys. Rev. Lett.* **104** 010502

[12] Jaksch D, Cirac J I, Zoller P, Rolston S L, Côté R and Lukin M D 2000 *Phys. Rev. Lett.* **85** 2208

[13] Lukin M D, Fleischhauer M, Cote R, Duan L M, Jaksch D, Cirac J I and Zoller P 2001 *Phys. Rev. Lett.* **87** 037901

[14] Saffman M and Walker T G 2002 *Phys. Rev.* A **66** 065403

[15] Saffman M and Walker T G 2005 *Phys. Rev.* A **72** 042302

[16] Müller M, Lesanovsky I, Weimer H, Büchler H P and Zoller P 2009 *Phys. Rev. Lett.* **102** 170502

[17] Saffman M and Mølmer K 2009 *Phys. Rev. Lett.* **102** 240502

[18] Wu H Z, Yang Z B and Zheng S B 2010 *Phys. Rev.* A **82** 034307

[19] Wu H Z, Yang Z B and Zheng S B 2010 *Commun. Theor. Phys.* **54** 835

[20] Ni B B, Gu Y J, Chen X D, Liang H H, Lin X and Lin X M 2010 *Chin. Phys.* B **19** 090316

[21] Weimer H, Müller M, Lesanovsky I, Zoller P and Büchler H P 2010 *Nat. Phys.* **6** 382

[22] Reslen J 2011 *J. Phys.* B: *At. Mol. Opt. Phys.* **44** 195505

[23] Bennett C H and DiVincenzo D P 2000 *Nature* **404** 247

[24] Vatan F and Williams C 2004 *Phys. Rev.* A **69** 032315

[25] Förster T 1948 *Ann. Phys.* **437** 55

[26] Saffman M and Walker T G 2008 *Phys. Rev.* A **77** 032723

[27] Saffman M and Mølmer K 2008 *Phys. Rev.* A **78** 012336

[28] Saffman M and Walker T G 2005 *Phys. Rev.* A **72** 022347