Robustness of quantum discord to sudden death in nuclear magnetic resonance*

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We investigate the dynamics of the entanglement and quantum discord of two qubits in liquid state homonuclear nuclear magnetic resonance. Applying a phenomenological description for nuclear magnetic resonance under a relaxation process, and taking a group of typical parameters of nuclear magnetic resonance, we show that when a zero initial state experiences a relaxation process, its entanglement disappears completely after a sequence of so-called sudden deaths and revivals, while the quantum discord retains remarkable values after a sequence of oscillations. That is to say, the quantum discord is more robust than entanglement.

Keywords: quantum discord, nuclear magnetic resonance, concurrence, sudden death

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1. Introduction

Entanglement is one of the most striking features of the quantum world, and is widely seen as the main resource for quantum information processing.[1,2] However, entanglement is not the only useful type of quantum correlation. It has been found that entanglement is not necessary for deterministic quantum computation with one pure qubit (DQC1),[3−5] but another quantum correlation, introduced in Refs. [6] and [7], called quantum discord, is responsible for the computational efficiency of DQC1. Besides, quantum discord has also been used for some studies such as quantum phase transition,[8−10] and the Grover search process.[11] Recently, the dynamics of quantum discord has attracted more attention, such as the robustness of quantum discord to sudden death under dissipative channels,[12] quantum discord in Heisenberg models,[13] the non-Markovian effect of quantum discord,[14] thermal quantum discord in an XXZ model,[15] thermal quantum discord in Heisenberg models,[16] quantum discord in cavity QED,[17] and quantum discord in a two-mode vacuum cavity.[18]

Nuclear magnetic resonance (NMR) has been used to demonstrate the basic concepts of quantum information processing, and has been leading the field both in terms of number of qubits and control techniques.[19,20] Studies of dynamic characteristics and its implementation for the quantum information processing of NMR systems have also received a great deal of attention in recent years.[21−27]

Due to its fundamental and practical significance, we wish to investigate the dynamical behaviours of entanglement and quantum discord of two qubits in NMR systems. The rest of this paper is organized as follows. In Section 2, we present a brief review of entanglement, quantum discord, and liquid state two-qubit NMR systems. In Section 3, we investigate the dynamics of entanglement and quantum discord of two-qubit NMR without relaxation. In Section 4, applying a phenomenological method, we investigate the dynamics of entanglement and quantum discord of two-qubit NMR with relaxation. Section 5 contains the conclusion.

2. Entanglement, quantum discord, NMR systems

We first recall some basics about entanglement, quantum discord, and NMR.

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2.1. Concurrence of two-qubit states

For two quantum systems A and B, described by the Hilbert spaces $H^{\rm A}$ and $H^{\rm B}$, respectively, and the composite system AB is described by the Hilbert space $H^{\rm A}\otimes H^{\rm B}$. A state ρ on $H^{\rm A}\otimes H^{\rm B}$ is called a separable state if and only if it can be written in the form

$$\rho=\sum_{i=1}^{n}p_i\rho_i^{\rm A}\otimes\rho_i^{\rm B}, \tag{1}$$

where n is a positive integer, $p_i\geq 0$, $\sum_{i=1}^{n}p_i=1$, $\{\rho_i^{\rm A}\}_{i=1}^{n}$ are density operators on $H^{\rm A}$, $\{\rho_i^{\rm B}\}_{i=1}^{n}$ are density operators on $H^{\rm B}$. A state is called an entangled state if it is not a separable state. For two qubits, one of the most widely used entanglement measures is the so-called concurrence, we will use it as the quantifier in this paper. For two-qubit state ρ, the concurrence of ρ is defined as[28]

$$C(\rho)=\max\{0,\lambda_1-\lambda_2-\lambda_3-\lambda_4\}, \tag{2}$$

where λ_1, λ_2, λ_3, λ_4 are the square roots of the eigenvalues of the matrix $\rho\widetilde{\rho}$ in decreasing order, and

$$\widetilde{\rho}=(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y), \tag{3}$$

σ_y is the Pauli matrix, ρ^* is the complex conjugate of ρ. For convenience, in Eq. (2) we denote

$$\overline{C}(\rho)=\lambda_1-\lambda_2-\lambda_3-\lambda_4, \tag{4}$$

and call it pseudo-concurrence.

2.2. Quantum discord of two-qubit states

The quantum discord of ρ on $H^{\rm A}\otimes H^{\rm B}$ (with respect to system A) is defined as[6,7]

$$D_{\rm A}(\rho)=S(\rho^{\rm A})-S(\rho^{AB})+\inf_{\{|\alpha\rangle\langle\alpha|\otimes I\}_{\alpha=1}^{n_{\rm A}}}\sum_{\alpha=1}^{n_{\rm A}}p_\alpha S(\rho_\alpha^{\rm B}). \tag{5}$$

In Eq. (5), $S(\cdot)$ is the entropy function, that is, for example, $S(\rho^{\rm A})=-{\rm tr}_{\rm A}(\rho^{\rm A}\log_2\rho^{\rm A})$; $n_{\rm A}={\rm dim}H^{\rm A}$, the inf is taken over all projective measurements $\{|\alpha\rangle\langle\alpha|\otimes I\}_{\alpha=1}^{n_{\rm A}}$ on system A, that is, $\{|\alpha\rangle\}_{\alpha=1}^{n_{\rm A}}$ is an arbitrary orthonormal basis for $H^{\rm A}$. We use I to denote the identity operators on $H^{\rm A}$ or on $H^{\rm B}$, $p_\alpha={\rm tr}(|\alpha\rangle\langle\alpha|\otimes I\rho|\alpha\rangle\langle\alpha|\otimes I)$, $\rho_\alpha^{\rm B}={\rm tr}_A(|\alpha\rangle\langle\alpha|\otimes I\rho|\alpha\rangle\langle\alpha|\otimes I)/p_\alpha$.

Quantum discord captures more correlation than entanglement in the sense that quantum discord can exist even when there is no entanglement. This can be seen in Eq. (1) and the fact[6]

$$D_{\rm A}(\rho)=0\Longleftrightarrow\rho=\sum_{\alpha=1}^{n_{\rm A}}p_\alpha|\alpha\rangle\langle\alpha|\otimes\rho_\alpha^{\rm B}, \tag{6}$$

where, $\{|\alpha\rangle\}_{\alpha=1}^{n_{\rm A}}$ is an arbitrary orthonormal basis for $H^{\rm A}$, $\{\rho_\alpha^{\rm B}\}_{\alpha=1}^{n_{\rm A}}$ are density operators on $H^{\rm B}$, $p_\alpha\geq 0$, $\sum_{\alpha=1}^{n_{\rm A}}p_\alpha=1$. Comparing Eq. (1) with Eq. (6), it is obvious that $D_{\rm A}(\rho)=0$ implies ρ being a separable state.

Equation (5) is difficult to optimize, and even for two-qubit states up to now only a few special states were found to allow analytical solutions.[29,30] Another measure of quantum discord, called geometrical measure, was introduced as[31,32]

$$D_{\rm A}^{\rm G}(\rho)=\inf_{\sigma}{\rm tr}[(\rho-\sigma)^2], \tag{7}$$

where the inf is taken over all σ that $D_{\rm A}(\sigma)=0$. Evidently,

$$D_{\rm A}(\rho)=0\Longleftrightarrow D_{\rm A}^{\rm G}(\rho)=0. \tag{8}$$

One of the most elegant results about the definition of $D_{\rm A}^{\rm G}(\rho)$ is that it allows analytical expressions for all two-qubit states. More specifically, for any two-qubit state ρ which can be written as

$$\rho=\frac{1}{4}\Big(I\otimes I+\sum_{i=1}^{3}x_i\sigma_i\otimes I+\sum_{j=1}^{3}y_jI\otimes\sigma_j+\sum_{i,j=1}^{3}T_{ij}\sigma_i\otimes\sigma_j\Big), \tag{9}$$

then we have,[31]

$$D_{\rm A}^{\rm G}(\rho)=\frac{1}{4}\left(\sum_{i=1}^{3}x_i^2+\sum_{i,j=1}^{3}T_{ij}^2-\lambda_{\max}\right), \tag{10}$$

where $\sigma_1=\sigma_x$, $\sigma_2=\sigma_y$, $\sigma_3=\sigma_z$, are Pauli matrices; $\{x_i\}_{i=1}^3$, $\{y_j\}_{j=1}^3$, $\{T_{ij}\}_{i,j=1}^3$ all are real number sets; $\lambda_{\max}$ is the largest eigenvalue of the matrix $xx^{\rm t}+TT^{\rm t}$, $x=(x_1,x_2,x_3)^{\rm t}$, and T is the matrix (T_{ij}), where t means transpose.

In Eq. (9), it holds that

$$\begin{aligned}x_i&={\rm tr}[\,\rho\sigma_i\otimes I],\\ y_j&={\rm tr}[\,\rho I\otimes\sigma_j],\\ T_{ij}&={\rm tr}[\,\rho\sigma_i\otimes\sigma_j].\end{aligned} \tag{11}$$

2.3. Two qubits in liquid state NMR

The Hamiltonian of two qubits in NMR (we focus on the liquid state homonuclear NMR) is well described by[19]

$$H(t) = H_{\rm sys} + H_{\rm rf}(t), \tag{12}$$

$$H_{\rm sys} = -\frac{\hbar\omega_1}{2}\sigma_z\otimes I - \frac{\hbar\omega_2}{2} I\otimes\sigma_z + \hbar J\sigma_z\otimes\sigma_z, \tag{13}$$

$$H_{\rm rf}(t) = -\frac{\hbar g_1}{2}[\cos(\omega t)\sigma_x\otimes I - \sin(\omega t)\sigma_y\otimes I] - \frac{\hbar g_2}{2}[\cos(\omega t) I\otimes\sigma_x - \sin(\omega t) I\otimes\sigma_y]. \tag{14}$$

In $H_{\rm sys}$, $\omega_1 = B_0\gamma_1$, $\omega_2 = B_0\gamma_2$, are called Lamor frequencies of two qubits (two nuclei), containing the chemical shifts. B_0 is the static magnetic field along z direction, γ_1 and γ_2 are gyromagnetic ratios of the two qubits. Typical values of ω_1 and ω_1 are a few 10^8 Hz, and chemical shifts range from a few 10^3 Hz to a few 10^4 Hz. J describes the spin–spin couplings including direct dipole–dipole coupling and indirect through-bond coupling. Spin–spin couplings are very small in NMR compared to Lamor frequencies, for instance, a few 10^2 Hz.

In $H_{\rm rf}(t)$, $g_1 = B_1\gamma_1$, $g_2 = B_1\gamma_2$, B_1 is the applied magnetic field rotating in x–y plane at frequency ω, at or near the Lamor frequencies ω_1, ω_2. Typical values of g_1, g_2 are up to 10^5 Hz.

3. Dynamics of entanglement and quantum discord of two qubits in NMR without relaxation

We study the dynamics of two-qubit NMR without relaxation. When the evolution time is much shorter than the relaxation time scales, the system can be approximated as an isolated system, and it evolves according to the Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle. \tag{15}$$

Notice that $H_{\rm rf}(t)$ in Eq. (14) is time-dependent, in order to cancel the time t in Hamiltonian $H(t)$, we put

$$|\Psi(t)\rangle = \exp\left[\frac{\mathrm{i}\omega t}{2}(\sigma_z\otimes I + I\otimes\sigma_z)\right]|\varphi(t)\rangle. \tag{16}$$

This transformation is widely used in NMR theory, sometimes called the method of rotating frame. With this transformation, we have

$$\mathrm{i}\hbar\frac{\partial}{\partial t}|\varphi(t)\rangle = \widetilde{H}|\varphi(t)\rangle, \tag{17}$$

$$\widetilde{H} = -\frac{\hbar(\omega_1-\omega)}{2}\sigma_z\otimes I - \frac{\hbar(\omega_2-\omega)}{2} I\otimes\sigma_z + \hbar J\sigma_z\otimes\sigma_z - \frac{\hbar g_1}{2}\sigma_x\otimes I - \frac{\hbar g_2}{2} I\otimes\sigma_x, \tag{18}$$

$$|\Psi(0)\rangle = |\varphi(0)\rangle. \tag{19}$$

In the above reductions, we have used the facts that for any real number λ,

$$\exp(\mathrm{i}\lambda\sigma_i) = \cos(\lambda) I + \mathrm{i}\sin(\lambda)\sigma_i, \quad i = 1,2,3; \tag{20}$$

$$\exp(-\mathrm{i}\lambda\sigma_z)\sigma_x\exp(\mathrm{i}\lambda\sigma_z) = \cos(2\lambda)\sigma_x + \sin(2\lambda)\sigma_y; \tag{21}$$

$$\exp(-\mathrm{i}\lambda\sigma_z)\sigma_y\exp(\mathrm{i}\lambda\sigma_z) = \cos(2\lambda)\sigma_y - \sin(2\lambda)\sigma_x. \tag{22}$$

Since $\widetilde{H}$ is time-independent, together with Eqs. (16) and (19), we obtain

$$|\varphi(t)\rangle = \exp\left(\frac{t}{\mathrm{i}\hbar}\widetilde{H}\right)|\varphi(0)\rangle, \tag{23}$$

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle, \tag{24}$$

$$U(t) = \exp\left[\frac{\mathrm{i}\omega t}{2}(\sigma_z\otimes I + I\otimes\sigma_z)\right] \times \exp\left(\frac{t}{\mathrm{i}\hbar}\widetilde{H}\right). \tag{25}$$

Suppose the initial state is $|00\rangle$, that is

$$\rho = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{26}$$

then we have

$$C(\rho) = 0, \quad D_{\rm A}^{\rm G}(\rho) = 0. \tag{27}$$

After time t, ρ will evolve into $\rho(t)$ as

$$\rho(t) = U(t)\rho U^{\dagger}(t), \tag{28}$$

where $\dagger$ means Hermitian conjugate. Then $C(\rho(t))$ and $D_{\rm A}^{\rm G}(\rho(t))$ can be calculated by using Eq. (2) and Eq. (10).

Figure 1 shows $\overline{C}(\rho(t))$ and $D_{\rm A}^{\rm G}(\rho(t))$ at $t = 10^{-5}$ s, $t = 10^{-3}$ s, $t = 10^{-1}$ s. Where we take a group of typical parameters of NMR as $(\omega_1+\omega_2)/2 = 3\times10^8$ Hz, $(\omega_1-\omega_2)/2 = 10^4$ Hz, $J = 3\times10^2$ Hz, $g_1/2 = 5\times10^4$ Hz, $g_2/2 = 5\times10^4$ Hz. From Fig. 1 we see that, only near or at the frequency $(\omega_1+\omega_2)/2 = 3\times10^8$ Hz, entanglement and quantum discord are remarkably generated and evolved. It is

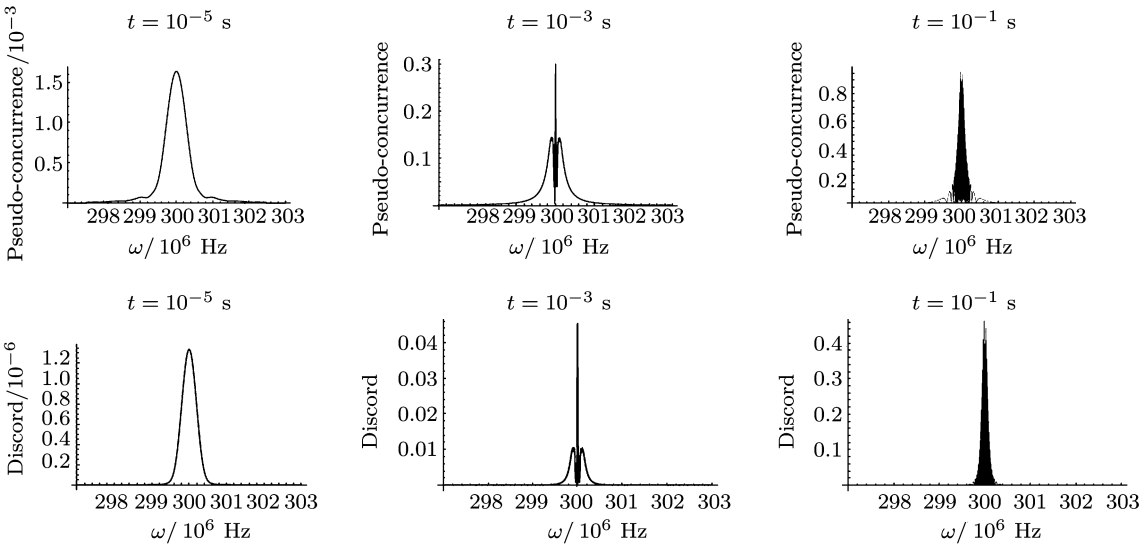


Fig. 1. Pseudo-concurrence and quantum discord at time $t = 10^{-5}$ s, $t = 10^{-3}$ s, $t = 10^{-1}$ s.

a common phenomenon in NMR that the applied magnetic field B_1 can effectively influence or control the nuclear spins at or near the resonance frequencies.

Figure 2 shows $\overline{C}(\rho(t))$ and $D_{\rm A}^{\rm G}(\rho(t))$ at the frequency $(\omega_1+\omega_2)/2 = 3\times10^8$ Hz, and we take the parameters of NMR as those in Fig. 1. From Figs. 1 and 2 we see that the dynamical behaviours of entanglement and quantum discord are similar in the evolution where we ignored the relaxation.

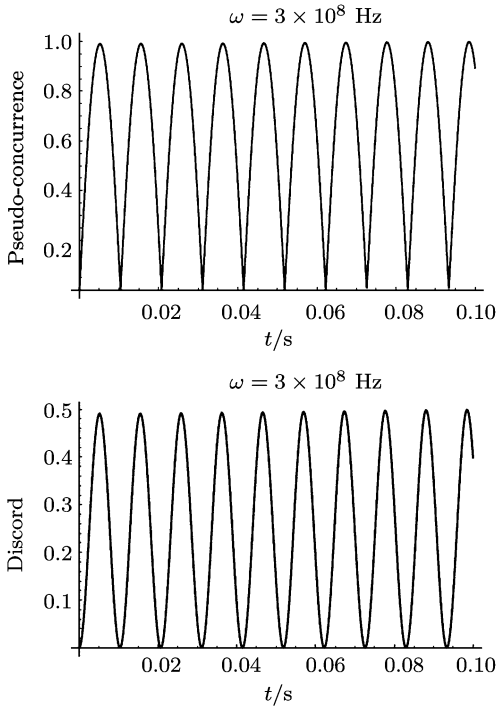


Fig. 2. Pseudo-concurrence and quantum discord at frequency $\omega = (\omega_1+\omega_2)/2 = 3\times10^8$ Hz.

4. Dynamics of entanglement and quantum discord of two qubits in NMR with relaxation

When the evolution time becomes comparable to the relaxation time scales, the relaxation effects must be taken into account. The relaxation process in NMR can be described in a phenomenological way (Refs. [33]; [20], 2.10)

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \frac{1}{\mathrm{i}\hbar}[H(t),\rho(t)] - R, \tag{29}$$

$$R_{ij} = \frac{\rho_{ij}(t)-\rho_{ij}}{T_{ij}}, \tag{30}$$

$$T_{ij} = \delta_{ij}T_1 + (1-\delta_{ij})T_2, \tag{31}$$

where T_1 is the energy relaxation rate, T_2 is the phase randomization rate. Theoretical calculations and experimental measurements for T_1 and T_2 have been well developed. The typical values of T_1 is tens of second, and T_2 is easily on the order of 1 second or more.

Similar to Eq. (16), we let

$$\rho(t) = \exp\left[\frac{\mathrm{i}\omega t}{2}(\sigma_z\otimes I + I\otimes\sigma_z)\right]\sigma(t) \times \exp\left[-\frac{\mathrm{i}\omega t}{2}(\sigma_z\otimes I + I\otimes\sigma_z)\right]. \tag{32}$$

Obviously, we have

$$\rho(0) = \sigma(0). \tag{33}$$

By using Eqs. (20)–(22), after some straightforward calculations, then equation (29) becomes

$$\frac{\mathrm{d}\sigma(t)}{\mathrm{d}t} = \frac{1}{\mathrm{i}\hbar}[\widetilde{H}, \sigma(t)] - \begin{pmatrix} \frac{\sigma_{11}(t)}{T_1} & \frac{\sigma_{12}(t)}{T_2} & \frac{\sigma_{13}(t)}{T_2} & \frac{\sigma_{14}(t)}{T_2} \\ \frac{\sigma_{21}(t)}{T_2} & \frac{\sigma_{22}(t)}{T_1} & \frac{\sigma_{23}(t)}{T_2} & \frac{\sigma_{24}(t)}{T_2} \\ \frac{\sigma_{31}(t)}{T_2} & \frac{\sigma_{32}(t)}{T_2} & \frac{\sigma_{33}(t)}{T_1} & \frac{\sigma_{34}(t)}{T_2} \\ \frac{\sigma_{41}(t)}{T_2} & \frac{\sigma_{42}(t)}{T_2} & \frac{\sigma_{43}(t)}{T_2} & \frac{\sigma_{44}(t)}{T_1} \end{pmatrix} + \begin{pmatrix} \frac{\sigma_{11}(0)}{T_1} & \frac{\sigma_{12}(0)}{T_2}\mathrm{e}^{-\mathrm{i}\omega t} & \frac{\sigma_{13}(0)}{T_2}\mathrm{e}^{-\mathrm{i}\omega t} & \frac{\sigma_{14}(0)}{T_2}\mathrm{e}^{-2\mathrm{i}\omega t} \\ \frac{\sigma_{21}(0)}{T_2}\mathrm{e}^{\mathrm{i}\omega t} & \frac{\sigma_{22}(0)}{T_1} & \frac{\sigma_{23}(0)}{T_2} & \frac{\sigma_{24}(0)}{T_2}\mathrm{e}^{-\mathrm{i}\omega t} \\ \frac{\sigma_{31}(0)}{T_2}\mathrm{e}^{\mathrm{i}\omega t} & \frac{\sigma_{32}(0)}{T_2} & \frac{\sigma_{33}(0)}{T_1} & \frac{\sigma_{34}(0)}{T_2}\mathrm{e}^{-\mathrm{i}\omega t} \\ \frac{\sigma_{41}(0)}{T_2}\mathrm{e}^{2\mathrm{i}\omega t} & \frac{\sigma_{42}(0)}{T_2}\mathrm{e}^{\mathrm{i}\omega t} & \frac{\sigma_{43}(0)}{T_2}\mathrm{e}^{\mathrm{i}\omega t} & \frac{\sigma_{44}(0)}{T_1} \end{pmatrix}.$$

This is a first-order linear nonhomogeneous ordinary differential equation with constant coefficients in the functions $\sigma_{11}(t), \sigma_{12}(t), \sigma_{13}(t), \sigma_{14}(t), \sigma_{21}(t), \ldots, \sigma_{44}(t)$. If we denote $\sigma(t)$ and the last matrix in this equation by

$$\boldsymbol{\sigma}(t) = (\sigma_{11}(t), \sigma_{12}(t), \sigma_{13}(t), \sigma_{14}(t), \sigma_{21}(t), \ldots, \sigma_{44}(t))^{\mathrm{t}},$$
$$\boldsymbol{f}(t) = \left(\frac{\sigma_{11}(0)}{T_1}, \frac{\sigma_{12}(0)}{T_2}\mathrm{e}^{-\mathrm{i}\omega t}, \frac{\sigma_{13}(0)}{T_2}\mathrm{e}^{-\mathrm{i}\omega t}, \frac{\sigma_{14}(0)}{T_2}\mathrm{e}^{-2\mathrm{i}\omega t}, \frac{\sigma_{21}(0)}{T_2}\mathrm{e}^{\mathrm{i}\omega t}, \ldots, \frac{\sigma_{44}(0)}{T_1}\right)^{\mathrm{t}},$$

we can obtain a compact form

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\sigma}(t) = A\boldsymbol{\sigma}(t) + \boldsymbol{f}(t), \tag{34}$$

where A is a 16×16 matrix which is independent of t (the explicit expression of A, see Appendix A). So the general solution of Eq. (34) is

$$\boldsymbol{\sigma}(t) = \exp(tA)\boldsymbol{\sigma}(0) + \int_0^{\mathrm{t}} \exp[(t-s)A]\boldsymbol{f}(t)\,\mathrm{d}s. \tag{35}$$

We now take the zero initial state $|00\rangle$ for ρ, hence we obtain

$$\boldsymbol{\sigma}(0) = (1, 0, 0, 0, 0, ..., 0)^{\mathrm{t}}, \tag{36}$$
$$\boldsymbol{f}(t) = (\frac{1}{T_1}, 0, 0, 0, 0, ..., 0)^{\mathrm{t}}. \tag{37}$$

Then by calculating Eqs. (35), (32), (2), and (10), we will obtain $C(\rho(t))$ and $D_{\mathrm{A}}^{\mathrm{G}}(\rho(t))$. From the discussion about Figs. 1 and 2, in Fig. 3 below we only discuss the dynamics of $\overline{C}(\rho(t))$ and $D_{\mathrm{A}}^{\mathrm{G}}(\rho(t))$ at frequency $(\omega_1 + \omega_2)/2 = 3 \times 10^8$ Hz, and we take the parameters of NMR as those in Fig. 1, and $T_1 = 20$ s, $T_2 = 1$ s.

From Fig. 3 we see that, when the evolution time is much shorter than the relaxation scales, the dynamics of entanglement and quantum discord are similar. However, for a longer time, namely, near relaxation scales, the entanglement manifests a sequence of sudden deaths and revivals, and finally disappears completely. At the same time, the quantum discord, after a sequence of oscillations, still retains remarkable values.

In a dynamical process, the phenomenon of entanglement disappearing in a finite time is named entanglement sudden death.[34] This is an interesting phenomenon in contrast to the usual intuition that decoherence should be completed in an infinite time. Also, this is an important phenomenon for quantum information processing since much of quantum information processing relies on entanglement.[1,2]

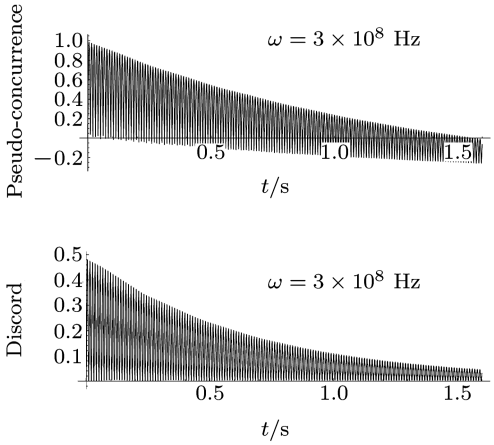


Fig. 3. Pseudo-concurrence and quantum discord at frequency $\omega = (\omega_1 + \omega_2)/2 = 3 \times 10^8$ Hz.

5. Conclusion

In summary, we have investigated the dynamics of entanglement and quantum discord of two qubits in liquid state homonuclear NMR systems. We showed that the dynamical behaviours of entanglement and quantum discord are similar in short time where relax-

ation effects can be neglected. But when the time gets long enough to be comparable to the relaxation rates, the entanglement manifests the phenomenon of sudden deaths and revivals, and finally disappears completely, while the quantum discord retains remarkable values. That is to say, quantum discord is more robust than entanglement against the relaxation processes. Hence quantum algorithms based only on quantum discord may be more robust than those based on entanglement.

Appendix A: explicit expression of matrix A in Eq. (34)

For simplicity, we put $(\omega_1+\omega_2)/2=a$, $(\omega_1-\omega_2)/2=b$, $J=c$, $g_1/2=d$, $g_2/2=e$, then

$$
\begin{aligned}
A = \mathrm{i}\{&\{0,-e,-d,0,e,0,0,0,d,0,0,0,0,0,0,0\},\\
&\{-e,a-y-b-2c,0,-d,0,e,0,0,0,d,0,0,0,0,0,0\},\\
&\{-d,0,a-y+b-2c,-e,0,0,e,0,0,0,d,0,0,0,0,0\},\\
&\{0,-d,-e,2a-2y,0,0,0,e,0,0,0,d,0,0,0,0\},\\
&\{e,0,0,0,-a+y+b+2c,-e,-d,0,0,0,0,0,d,0,0,0\},\\
&\{0,e,0,0,-e,0,0,-d,0,0,0,0,0,d,0,0\},\\
&\{0,0,e,0,-d,0,2b,-e,0,0,0,0,0,0,d,0\},\\
&\{0,0,0,e,0,-d,-e,a-y+b+2c,0,0,0,0,0,0,0,d\},\\
&\{d,0,0,0,0,0,0,0,-a+y-b+2c,-e,-d,0,e,0,0,0\},\\
&\{0,d,0,0,0,0,0,0,-e,-2b,0,-d,0,e,0,0\},\\
&\{0,0,d,0,0,0,0,0,-d,0,0,-e,0,0,e,0\},\\
&\{0,0,0,d,0,0,0,0,0,-d,-e,a-b+2c-y,0,0,0,e\},\\
&\{0,0,0,0,d,0,0,0,e,0,0,0,-2a+2y,-e,-d,0\},\\
&\{0,0,0,0,0,d,0,0,0,e,0,0,-e,-a-b-2c+y,0,-d\},\\
&\{0,0,0,0,0,0,d,0,0,0,e,0,-d,0,-a+b-2c+y,-e\},\\
&\{0,0,0,0,0,0,0,d,0,0,0,e,0,-d,-e,0\}\}\\
&-\text{diagonal matrix}\left[\left\{\frac{1}{T_1},\frac{1}{T_2},\frac{1}{T_2},\frac{1}{T_2},\frac{1}{T_2},\frac{1}{T_1},\frac{1}{T_2},\frac{1}{T_2},\frac{1}{T_2},\frac{1}{T_2},\frac{1}{T_1},\frac{1}{T_2},\frac{1}{T_2},\frac{1}{T_2},\frac{1}{T_2},\frac{1}{T_1}\right\}\right].
\end{aligned}
$$

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