Thermal modeling and the optimized design of metal plate cooling systems for single concentrator solar cells*

Cui Min(崔 敏)[a)†], Chen Nuo-Fu(陈诺夫)[b)], and Deng Jin-Xiang(邓金祥)[a)]

[a)] *College of Applied Sciences, Beijing University of Technology, Beijing* 100124, *China*
[b)] *New and Renewable Energy of Beijing Key Laboratory, North China Electric Power University, Beijing* 102206, *China*

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A metal plate cooling model for 400× single concentrator solar cells was established. The effects of the thickness and the radius of the metal plate, and the air environment on the temperature of the solar cells were analyzed in detail. It is shown that the temperature of the solar cells decreased sharply at the beginning, with the increase in the thickness of the metal plate, and then changed more smoothly. When the radius of the metal plate was 4 cm and the thickness increased to 2 mm or thicker, the temperature of the solar cell basically stabilized at about 53 °C. Increasing the radius of the metal plate and the convective transfer coefficient made the temperature of the solar cell decrease remarkably. The effects of Al and Cu as the metal plate material on cooling were analyzed contrastively, and demonstrated the superiority of Al material for the cooling system. Furthermore, considering cost reduction, space holding and the stress of the system, we optimized the structural design of the metal plate. The simulated results can be referred to the design of the structure for the metal plate. Finally, a method to devise the structure of the metal plate for single concentrator solar cells was given.

Keywords: metal plate, modeling, concentrator solar cells, cooling

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1. Introduction

Tandem solar cells using a concentrator provide the possibility to realize a very efficient PV (photovoltaic) system at low cost. In such systems, the cost of the generated electricity can be significantly reduced by replacing the expensive PV cell area with less expensive optical material.[1] The highest conversion efficiency[2] of 41.6% for GaInP/GaAs/Ge multijunction cells under the standard spectrum at 364 suns has been reported by King *et al.*[3] However, when the concentration of sunlight gets onto the cells, only a fraction of the concentrated sunlight absorbed by the cells is converted into electrical energy. A larger fraction of the concentrated sunlight is converted into thermal energy in the cells, which may lead them to have high temperatures with no cooling operations. Under 400× concentration with no cooing aid, the cell temperature will get up to about 1200 °C.[4] High temperature will lead the performance of the solar cell to deteriorate sharply. It was reported that the conversion efficiency decreases from 18.4% to 4.4% as the temperature increases from 27 °C to 227 °C.[5] Therefore, cooling is indispensable for the concentrator PV system.

Passive and active coolings are the two competing methods for removing heat from high illumination PV cells. Passive cooling[6−9] for the concentrator solar cell only uses metal plates or heat sinks to dissipate heat from the solar cells to the ambient by radiation and convection. Active cooling[10,11] usually involves circulating cooling water through a thermally conductive tube upon which the cells are mounted. Passive cooling has the advantage of cost efficiency over active cooling.[1] For single concentrator solar cell systems, passive cooling with a metal plate or heat sink can suffice because of the very small size of the cell (a few square millimeters).[12] Contrasting with heat sinks, metal plates have edges on a simple structure, convenient maintenance and are lightweight, especially at lower concentrations (lower than 500×).

Metal plates have been used in many concentration systems[6,9,12] to dissipate heat. However, detailed thermal modeling analyses of metal plate cooling for single concentrator solar cells, not to mention the optimized design of the metal plates, are seldom

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†Corresponding author. E-mail: mcui@bjut.edu.cn

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reported. In this work, a metal plate cooling model is established. The effects of the thickness and the radius of the metal plate, and the air environment on the temperature of the solar cells are analyzed in detail. Furthermore, for cost reduction, the structural design of the metal plate is optimized. A method to devise the structure of the metal plate for single concentrator solar cells is given, and this method is very serviceable for the cooling project of concentrator solar cell systems.

2. Thermal modeling

A single concentrator solar cell with metal plate cooling is shown in Fig. 1(a). In the figure, the lenses or mirrors concentrate the sunlight onto the solar cell to increase the light density, and a metal plate is used to dissipate heat from the solar cell to the atmosphere. The solar cell and the metal plate are adhered by high thermally conductive and good electrically conductive material, such as silver paste. The metal plate can therefore be used as one electrode to output the electric energy generated by the solar cell, with another electrode situated at the upper surface of the cell. Because the area of the solar cell is very small, we assume that the temperature of the back surface of the solar cell is equal to that of the contact point with the metal plate.

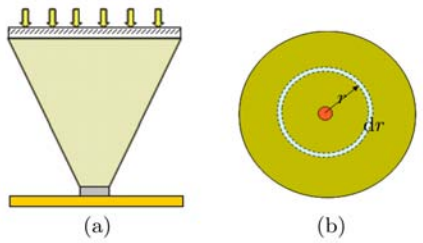


Fig. 1. (colour online) Model illustration of single concentrator solar cells with metal plate cooling (a), and a top view of the solar cell mounted on the metal plate (b) (not to scale).

For convenience of analysis, the shapes of the solar cell and the metal plate are both designed as circles, as shown in Fig. 1(b). In the model, the solar cell is located in the center of the metal plate (the radius of the solar cell $r_c = 2$ mm) in accordance with common practice.

Under continuous illumination, a small amount of the concentrated sunlight is converted into electricity by the solar cell. The large amount of remaining sunlight will enable the center of the metal plate to have persistent heating, which will be conducted to other places in the plate in the radial direction. Finally, the heat in the plate will be dissipated into the ambient through radiation and convection. Therefore, the heat conduction in the plate is fit for steady heat conduction, with the heat source described in a cylindrical coordinate system as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\Phi}{k}=0, \quad (1)$$

where T is the temperature of the metal plate varying with the radius r, Φ is the heat produced by the heat source in unit time and unit volume, and k is the thermal conductivity of the plate material. In the model, it is satisfied that

$$\frac{\partial T}{\partial \theta}=\frac{\partial T}{\partial z}=0. \quad (2)$$

Then, the steady heat conduction Eq. (1) can be simplified as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial T}{\partial r}\right)+\frac{\Phi}{k}=0. \quad (3)$$

The temperature of the metal plate decreases in the radial direction because of heat conduction and heat dissipation. Therefore, heat source Φ is negative. We take the infinitesimal element surface ($\mathrm{d}s = 2\pi r\,\mathrm{d}r$) of the plate and analyze, as shown in Fig. 1(b). In $\mathrm{d}s$, the total dissipated heat (Φ_s) is

$$\Phi_s = 2\mathrm{d}s\varepsilon\sigma(T^4-T_0^4)+2\mathrm{d}sh(T-T_0), \quad (4)$$

where the first term represents the power dissipated through radiation with the surface area $\mathrm{d}s$ (2 denotes the upper and lower surface), the surface emissivity ε, the Stephan–Boltzmann constant σ, and the ambient temperature T_0. The second term characterizes heat dissipated through convection, which depends on the surface area, the convective transfer coefficient h and the temperature difference.

Therefore, heat source Φ can be derived as

$$\begin{aligned}\Phi &= -\frac{\Phi_s}{\mathrm{d}V}=-\frac{2\mathrm{d}s\varepsilon\sigma(T^4-T_0^4)+2\mathrm{d}sh(T-T_0)}{\mathrm{d}sD}\\ &= -\frac{2}{D}[\varepsilon\sigma(T^4-T_0^4)+h(T-T_0)], \quad (5)\end{aligned}$$

where $\mathrm{d}V$ is the infinitesimal element volume and D is the thickness of the metal plate. The heat radiation term of Φ can be linearized as

$$\begin{aligned}T^4-T_0^4 &= (T-T_0)(T^3+T^2T_0+TT_0^2+T_0^3)\\ &\approx 4T_0^3(T-T_0). \quad (6)\end{aligned}$$

We define

$$m^2=\frac{8\varepsilon\sigma t_0^3+2h}{Dk}, \quad (7)$$

then,

$$\Phi = -m^2 k(T - T_0). \tag{8}$$

Therefore, Eq. (3) can be rewritten as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial T}{\partial r}\right) - m^2(T - T_0) = 0. \tag{9}$$

The solution to Eq. (9) needs two boundary conditions. In the model, because the flank area of the plate is extremely small, heat is mainly dissipated from the upper and lower surfaces. Thus the flank of the plate is approximately adiabatic. Then the boundary conditions are the first and second kinds, as follows:

$$T\,|_{r=r_c} = T_c\ , \quad \frac{\mathrm{d}T}{\mathrm{d}r}\,|_{r=R} = 0, \tag{10}$$

where T_c is the temperature of the solar cell and R is the radius of the metal plate. It must be added that because of the small size and thin thickness of the solar cell, we assume that the solar cell is at an equable temperature.

In terms of the excess temperature θ ($\theta = T - T_0$), the conduction equation and boundary conditions become

$$\begin{aligned}
&\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\partial\theta}{\partial r}\right) - m^2\theta = 0 \quad (r_c < r < R),\\
&\theta\Big|_{r=r_c} = \theta_c = T_c - T_0,\\
&\frac{\mathrm{d}\theta}{\mathrm{d}r}\Big|_{r=R} = 0.
\end{aligned} \tag{11}$$

The solution to Eq. (11) is found to be[13]

$$\theta = AI_0(mr) + BK_0(mr), \tag{12}$$

where I_0 and K_0 are Bessel functions, and A and B are two coefficients. According to the boundary conditions, A and B are derived as

$$A = \theta_c \frac{K_1(mR)}{K_1(mR)I_0(mr_c) + K_0(mr_c)I_1(mR)},$$

$$B = \theta_c \frac{I_1(mR)}{K_1(mR)I_0(mr_c) + K_0(mr_c)I_1(mR)}. \tag{13}$$

Then, from Eqs. (12) and (13), we can see that if we obtain the excess temperature of the solar cell θ_c, we can determine the temperature at each point of the metal plate.

According to the one-dimensional heat conduction equation along the radial direction of the metal plate and Eqs. (12) and (13), we have

$$\begin{aligned}
&Q\\
&= -ks\frac{\mathrm{d}\theta}{\mathrm{d}r}\Big|_{r=r_c}\\
&= -k2\pi r_c D[AmI_1(mr_c) - BmK_1(mr_c)]\\
&= 2\pi r_c Dkm\theta_c \frac{K_1(mr_c)I_1(mR) - K_1(mR)I_1(mr_c)}{K_1(mR)I_0(mr_c) + K_0(mr_c)I_1(mR)},
\end{aligned} \tag{14}$$

where Q is the heat energy at the center of the metal plate transferred from the solar cell and s is the heat conduction area.

On the basis of energy conservation law, the concentrated sunlight is partly converted to electrical energy by the solar cells, with its other fraction becoming heat energy, which is transferred to the metal plate. Therefore, Q can also be described as

$$Q = (1 - \eta)C\tau\alpha q A_0, \tag{15}$$

where η is the conversion efficiency of the solar cell, C is the geometric concentration ratio of the concentrator lens, τ is the transmissivity of the lens, α is the surface absorptivity of the cell, q is the sun energy density (AM1.5), and A_0 is the area of the solar cell.

Table 1. Some of the parameters used in the model.

Parameters	Description	Value
C	Geometric concentration ratio of the lens	400
τ	Transmissivity of the optical collector	0.85
α	Surface absorptivity of the cell	0.85
q/mW·cm^{-2}	Energy density (AM1.5)	100
η/%	Conversion efficiency of the solar cell (for GaInP/GaAs/Ge triple solar cells)	30
r_c/mm	Radius of the solar cell	2
k/W·mK^{-1}	Heat conductivity	Cu 401[14] Al 237[14]
ε/W·m^{-2}K^{-4}	Surface emissivity of the metal plate	0.9 (black paint)[14]
σ/W·m^{-2}K^{-1}	Stephan-Boltzmann constant	5.67×10^{-8}
h	Natural convective transfer coefficient	5
T_0/°C	Ambient temperature	27

Then, from Eqs. (14) and (15), we can obtain the temperature of the solar cell as

$$T_{\rm c} = \frac{(1-\eta)C\tau\alpha qA_{\rm c}}{2\pi r_{\rm c}Dkm} \times \frac{K_1(mR)I_0(mr_{\rm c}) + K_0(mr_{\rm c})I_1(mR)}{K_1(mr_{\rm c})I_1(mR) - K_1(mR)I_1(mr_{\rm c})} + T_0. \quad (16)$$

And from Eqs. (12), (13) and (16), we can obtain the temperature at any place of the metal plate as

$$T = (T_{\rm c} - T_0)\frac{K_1(mR)I_0(mr) + I_1(mR)K_0(mr)}{K_1(mR)I_0(mr_{\rm c}) + K_0(mr_{\rm c})I_1(mR)} + T_0. \quad (17)$$

Table 1 shows the values of some of the parameters in the model. We use the Mathematica tool to simulate the effects of some of the key parameters' variations on the cooling system, with the results shown as follows.

3. Results and discussion

Figure 2 shows the temperature variation of the solar cell $T_{\rm c}$ with the thickness of the metal plate D made of the aluminum (Al). The radius of the metal plate R (3 and 4 cm) is studied with the other values of parameters shown in Table 1. We can see for different R, that $T_{\rm c}$ has the same variation tendency with D: $T_{\rm c}$ decreases with the increase in D, decreasing sharply at the beginning and then varying smoothly. When R is 4 cm (the area of the metal plate is equal to that of the concentrated lens) and D increases to 2 mm or thicker, $T_{\rm c}$ basically stabilizes at about 53 °C. Therefore, if a 2 cm thickness of the metal plate can be safely installed in the concentrator system of the solar cell, anything thicker than 2 cm will be useless for the decrease in $T_{\rm c}$, and instead will waste the material of the metal plate and increase the cost of the system. So, in the actual design of the thickness of the metal plate in the concentrated system, the temperature of the solar cell, the stress of the plate and the cost should all be considered together.

The effect of the variation in the radius of the metal plate on the temperature of the solar cell is shown in Fig. 3, in which we choose $D = 2$ mm and Al as the plate material. As shown in Fig. 3, $T_{\rm c}$ decreases remarkably with the increase in the radius of the metal plate. $T_{\rm c}$ reduces from about 53 °C to room temperature only when R increases from 4 to 9 cm. However, in the practical case, the radius cannot be infinitely large because of space holding and cost. Then, the volume of the concentrator system and the temperature of the solar cell should both be taken into account for designing the radius of the metal plate.

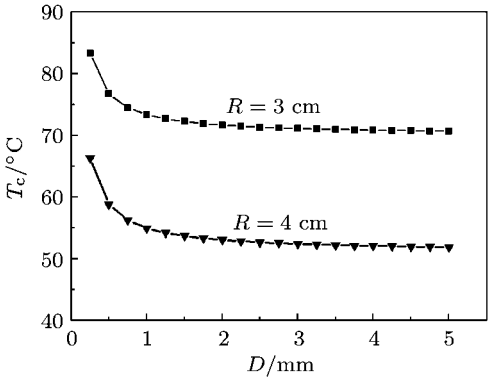


Fig. 2. The temperature of the solar cell, $T_{\rm c}$, against the thickness, D, of the Al metal plate for different radii of 3 and 4 cm.

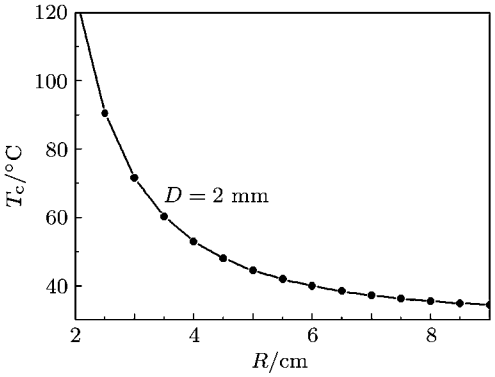


Fig. 3. The variation in temperature of the solar cell, $T_{\rm c}$, with the radius of the Al plate, R.

The above analyses are on the basis of natural convection. However, along the normal operation of the concentrator solar cell system, wind exists and can improve air convection.[15] The effect of convective transfer coefficient h on the temperature of the solar cell is shown in Fig. 4. We can see that $T_{\rm c}$ decreases with the increase in h. When h increases from 5 W/m^{2}K (force 0 wind at speeds between 0 and 0.2 m/s) to 10 W/m^{2}K (force 1 wind at speeds between 0.3 and 1.5 m/s) and 15 W/m^{2}K (force 2 wind at speeds between 1.6 and 3.3 m/s), $T_{\rm c}$ decreases from 53 to 45 and 41 °C, respectively. A gentle breeze will therefore help cool the concentrated solar cell system if the stability of the system is ensured.

Figure 5 shows the detailed temperature distribution on the metal plate with aluminum (Al) and copper (Cu) as the component materials under natural convection. Because of the higher heat conductivity

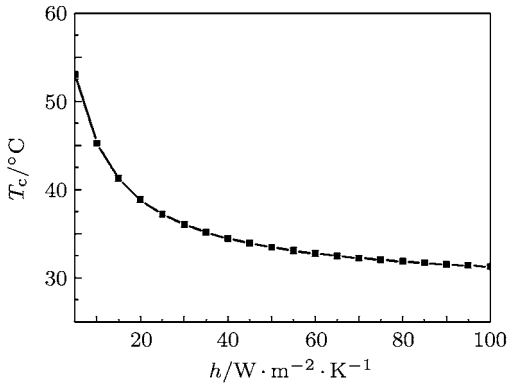


Fig. 4. Solar cell temperature T_c versus convective transfer coefficient h ($D = 2$ mm, $R = 4$ cm, Al).

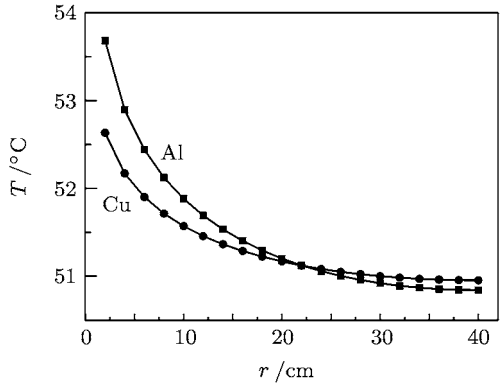


Fig. 5. The temperature distribution on the metal plate ($D = 1.5$ mm, $R = 4$ cm).

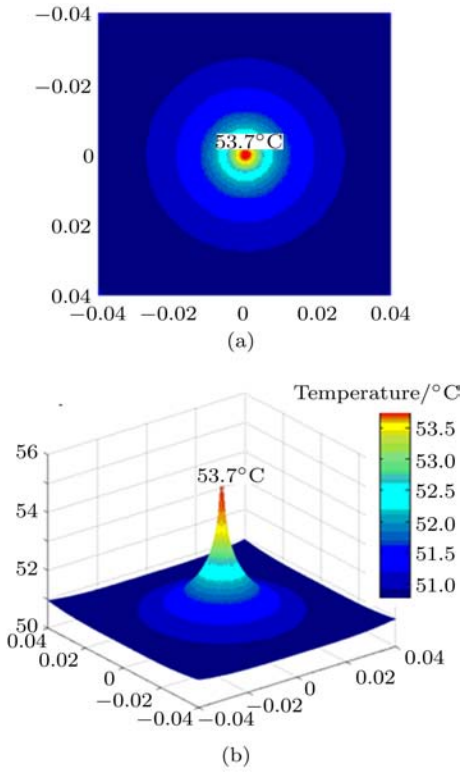


Fig. 6. (colour online) A temperature cloud picture of the Al plate ($D = 1.5$ mm, $R = 4$ cm). (a) The top view and (b) a three-dimensional diagram.

of Cu than Al, the temperature difference between the center and the edge of the Cu plate (1.6 °C) is smaller than that of the Al plate (2.8 °C). The heat dissipation of the Al plate is slightly worse than that of the Cu plate, with a plate center temperature of 53.7 °C (Al plate) and 52.6 °C (Cu plate). However, we know that Cu has been ranked as a rare metal because of its high cost, and its weight is triple that of Al. Therefore, considering cost and the system's load, Al has a growing tendency to be the metal plate material. A temperature cloud picture of the Al plate has also been obtained, as shown in Fig. 6. The temperature decreases from the center to the edge. The edge temperature of 51 °C is slightly higher than the room temperature, 30 °C, so heat transfer from the metal plate to air is very important. Consequently, wind and heat sinks are very serviceable for the cooling of high concentrator solar cell systems.

4. Optimized design

During the structural design of the metal plate, the temperature of the solar cell and the cost reduction of the plate should both be considered. For cooling the concentrator solar cells, the radius and thickness of the metal plate should be large enough, however, this will increase the cost of the system. Therefore, we can first fix the operation temperature of the solar cell and then reduce the cost of the plate to the lowest possible. Usually, the density of the plate metal is well distributed, so the volume of the plate can be, on behalf of the cost, as

$$V = \pi R^2 D, \tag{18}$$

where V is the volume of the metal plate.

We set the operation temperature of the solar cell at 50 °C. According to Eqs. (16) and (18), the relations between the cost of the metal plate and its radius and thickness are shown in Fig. 7. We can see that cost is lowest when the radius is 5.1 cm and the thickness is 0.5 mm. However, a radius of 5.1 cm may be too large and so needs more space to hold, and a thickness of 0.5 mm may be too thin, which is easy to break. The lowest cost may therefore not be the optimal choice when considering the space and stress of the system. The lower cost with $R = 4.5$ cm and $D = 1$ mm may be the optimal design for the system. Other data in Fig. 7 can be referred to the design of the structure for the metal plate, for which both the space holding and stress of the system should be taken into account.

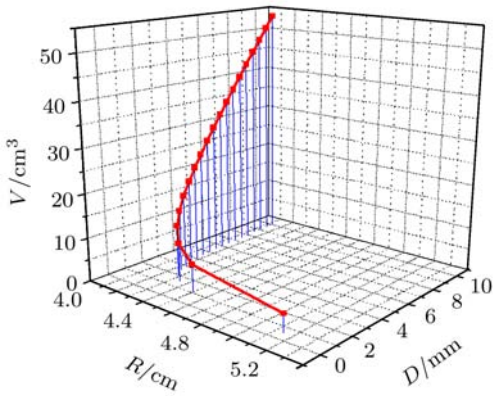


Fig. 7. (colour online) Cost versus both the radius and the thickness of the metal plate.

According to the simulation model and cost analyses above, a flow chart for designing the metal plate cooling of the concentrator solar cells is given, as shown in Fig. 8. Firstly, the operational temperature of the solar cell should be set. Secondly, the range of the radius and thickness of the plate should be determined according to space holding and stress demands. Thirdly, in terms of the relationship of cost with the radius and thickness of the plate, as shown in Fig. 7, the lowest cost in the permitted range should be chosen in the second step. Finally, the optimal structural design for the metal plate is accomplished.

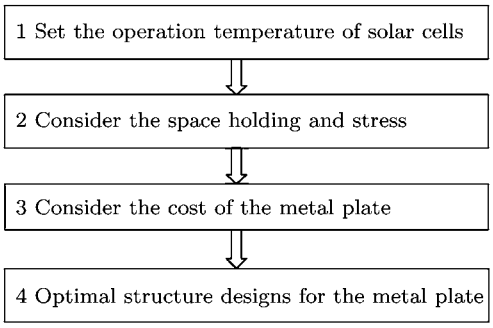


Fig. 8. A flow chart for designing the metal plate to cool the concentrator solar cells.

5. Conclusions

A metal plate cooling model for 400× single concentrator solar cells was established. The effects of the thickness and radius of the metal plate, and the air environment on the temperature of the solar cells were analyzed in detail. The effects on cooling of Al and Cu as the metal plate material were analyzed contrastively, and demonstrated the superiority of Al in the cooling system. Furthermore, considering cost reduction, space holding and the stress of the system, the structural design of the metal plate was optimized. Finally, a method to devise the structure of the metal plate for single concentrator solar cells was given. The results in this work are very useful for designing the structure of the metal plates.

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