Fano resonance and wave transmission through a chain structure with an isolated ring composed of defects*

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We consider a discrete model that describes a linear chain of particles coupled to an isolated ring composed of N defects. This simple system can be regarded as a generalization of the familiar Fano–Anderson model. It can be used to model discrete networks of coupled defect modes in photonic crystals and simple waveguide arrays in two-dimensional lattices. The analytical result of the transmission coefficient is obtained, along with the conditions for perfect reflections and transmissions due to either destructive or constructive interferences. Using a simple example, we further investigate the relationship between the resonant frequencies and the number of defects N, and study how to affect the numbers of perfect reflections and transmissions. In addition, we demonstrate how these resonance transmissions and refections can be tuned by one nonlinear defect of the network that possesses a nonlinear Kerr-like response.

Keywords: wave transmission, Fano resonance, defect, bistability

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1. Introduction

The Fano-type resonance, which arises from the interference between a localized state and a continuum band,[1] has been widely known in numerous physical systems, including atomic photoionization,[2] electron and neutron scattering,[3] Raman scattering,[4] photoabsorption in quantum well structures,[5,6] electron transport through mesoscopic systems,[7−10] Aharonov–Bohm interferometers,[11,12] quantum dots,[13−19] resonant light propagation through photonic-crystal waveguides,[20−25] and nanoscale structures.[26−30] In the corresponding transmission properties, the interference effect can lead to either perfect transmission or perfect reflection, producing a sharp asymmetric response.

One of the simplest models that describes the physics and the main features of the Fano resonance is the so-called Fano–Anderson model.[31] It describes a linear array with nearest-neighbor interaction, and the interaction with one or several defect states through the nearest neighbor. This simple discrete model describes the basic physics of the Fano resonance in a rather simple way.[29] In particular, analytical results for the wave transmission can be derived for the model, which may serve as a guide for the analysis of more complicated physical models associated with the Fano resonance. Reference [30] studied a complicated physical model composed of a linear array coupled to an N-defect chain. It revealed the conditions for perfect reflections and perfect transmissions due to either destructive or constructive interferences, and associated them with the Fano resonances.

As is well known, the actual physical systems are generally very complex, therefore in this paper we also study the Fano resonances of a complicated physical

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system that is composed of linear arrays of interacting elements coupled to an isolated ring composed of N defects (see Fig. 1). The system can be described by the generalized discrete Fano–Anderson model, which allows us to find exact solutions for the wave transmission coefficient and the conditions for perfect reflections and perfect transmissions due to either destructive or constructive interferences. Using the results, we demonstrate and explain the basic principles of resonant scattering under the condition of Fano resonances. Making use of a simple example, we further investigate the relationship between the resonant frequencies and the number of site N, and study how to affect the numbers of perfect reflections and perfect transmissions. We also reveal how resonance transmissions and refections can be tuned by introducing nonlinear defects into the network. These interesting features can deepen our fundamental understanding of Fano resonance transport in complex systems, such as quantum dots, photonic crystals and complex waveguide arrays. The results can also be applied to complex physical systems.

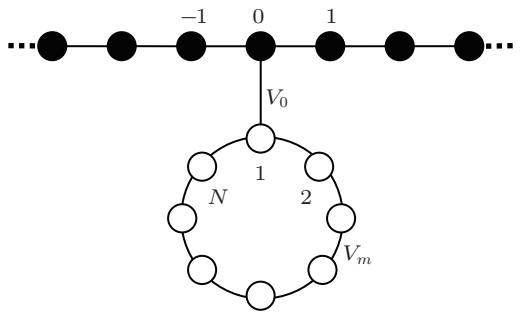


Fig. 1. Schematic diagram of the generalized Fano–Anderson model. The array of black circles denotes a linear chain, the isolated ring consists of N defects, V_0 is the coupling between the main chain and the first defect state, and V_m indicates the coupling of defect states $m-1$ and m.

2. Model and formalism

The model for the system shown in Fig. 1 can be described by the following Hamiltonian:

$$\begin{aligned} H_{\rm L} &= H_0 + H_{\rm R} + H_{\rm I}, \\ H_0 &= C\sum_n \left(\phi_n \phi_{n-1}^* + {\rm c.c.}\right), \\ H_{\rm R} &= \sum_{m=1}^{N-1} \left(E_m|\varphi_m|^2 + V_m\varphi_m\varphi_{m+1}^* + {\rm c.c.}\right) \\ &\quad + \left(E_N|\varphi_N|^2 + V_N\varphi_N\varphi_1^* + {\rm c.c.}\right), \\ H_{\rm I} &= V_0\phi_0\varphi_0^* + {\rm c.c.}, \end{aligned}$$

where the asterisk denotes the complex conjugation. This model describes the interaction of two subsystems. One subsystem is a straight linear chain with complex field amplitude ϕ_n at site n and nearest-neighbor coupling C. This subsystem supports the propagation of plane waves with dispersion $\omega_q = 2C\cos q$, and it is characterized by the frequency band with a continuum spectrum. The other subsystem is a finite inhomogeneous ring of N elements described by wave functions φ_m, which acts as the complex localized defect, with E_m being the energy associated with the m-th element. The isolated ring is attached to the main array. We assume that the defect sites are coupled through the nearest-neighbor interaction with strength V_m.

From the lattice Hamiltonian, we can derive the equations of motion in the frequency domain

$$\begin{aligned} \omega\phi_n &= C(\phi_{n-1} + \phi_{n+1}) + V_0\varphi_1\delta_{n,0}, \\ \omega\varphi_1 &= E_1\varphi_1 + V_0\phi_0 + V_1\varphi_2 + V_N\varphi_N, \\ \omega\varphi_2 &= E_2\varphi_2 + V_1\varphi_1 + V_2\varphi_3, \\ &\vdots \\ \omega\varphi_{N-1} &= E_{N-1}\varphi_{N-1} + V_{N-2}\varphi_{N-2} + V_{N-1}\varphi_N, \\ \omega\varphi_N &= E_N\varphi_N + V_{N-1}\varphi_{N-1} + V_N\varphi_1. \end{aligned} \tag{1}$$

In terms of the Gramer principle, we can obtain

$$\varphi_1 = \frac{V_0 D_2(\omega)}{D(\omega)}\phi_0, \tag{2}$$

where the denominator $D(\omega) = \det(\omega I - H_{\rm R})$, the numerator $D_2(\varepsilon) = \det(\omega I - H_{\rm R2})$, and $H_{\rm R}$, $H_{\rm R2}$ are given by

$$H_{\rm R} = \begin{bmatrix} E_1 & V_1 & 0 & \ldots & 0 & 0 & V_N \\ V_1 & E_2 & V_2 & \ldots & 0 & 0 & 0 \\ . & . & . & \ldots & . & . & . \\ . & . & . & \ldots & . & . & . \\ . & . & . & \ldots & . & . & . \\ 0 & 0 & 0 & \ldots & V_{N-2} & E_{N-1} & V_{N-1} \\ V_N & 0 & 0 & \ldots & 0 & V_{N-1} & E_N \end{bmatrix}, \tag{3}$$

$$H_{\rm R2} = \begin{bmatrix} E_2 & V_2 & 0 & \ldots & 0 & 0 & 0 \\ V_2 & E_3 & V_3 & \ldots & 0 & 0 & 0 \\ . & . & . & \ldots & . & . & . \\ . & . & . & \ldots & . & . & . \\ . & . & . & \ldots & . & . & . \\ 0 & 0 & 0 & \ldots & V_{N-2} & E_{N-1} & V_{N-1} \\ 0 & 0 & 0 & \ldots & 0 & V_{N-1} & E_N \end{bmatrix}. \tag{4}$$

Equation (2) allow us to eliminate the additional degrees of freedom from system (1) and obtain a system of equations for describing the wave propagation in the main array with only an effective localized defect

$$\omega\phi_n = C(\phi_{n-1} + \phi_{n+1}) + \frac{D_2(\omega)}{D(\omega)} V_0^2 \phi_0 \delta_{n0}. \quad (5)$$

According to Eq. (5), the strength of the effective scattering potential depends on the frequency of the incoming wave. If frequency ω of the incoming particle coincides with one of the eigenfrequencies of the defect ring of N sites described by Eq. (3), the effective scattering potential becomes infinitely large, and this will lead to a total reflection of the incoming wave.

In order to calculate the transmission coefficient for the system, we consider the following boundary conditions:

$$A_n = \begin{cases} I\mathrm{e}^{\mathrm{i}qn} + r\mathrm{e}^{-\mathrm{i}qn}, & n < 0, \\ t\mathrm{e}^{\mathrm{i}qn}, & n > 0, \end{cases} \quad (6)$$

where I, r and t represent the incoming, the reflected and the transmitted wave amplitudes, respectively. For such a plane wave, we use the well-known transfer-matrix approach, and thus the transmission coefficient $T = |t|^2/|I|^2$ can be obtained as

$$T = \frac{\alpha_q^2}{\alpha_q^2 + 1}, \quad (7)$$

where

$$\alpha_q = \frac{c_q D(\omega)}{V_0^2 D_2(\omega)}, \quad c_q = 2C \sin q. \quad (8)$$

3. Linear and nonlinear results and discussion

For the model, there are two scattering channels, the continuum band without defects and the effective discrete states with defects, so the system should demonstrate a resonant scattering. The transmission coefficient (7) is written in a form similar to that of the Fano formula, and thus it allows us to associate it with the Fano resonance. From the results of Eqs. (7) and (8), we can draw a conclusion that there are N frequencies for the perfect reflections when $\alpha_q = 0$ and $N - 1$ frequencies for the perfect transmissions when $\alpha_q = \pm\infty$. In order to find these frequencies, we make the following analysis. For the perfect reflections, we can calculate a discrete spectrum of oscillatory frequencies of the isolated complex defect through cutting the coupling between the main array and the subsystem of the defect ring (i.e. $V_0 = 0$). For the perfect transmissions, the frequencies can be obtained by eliminating the 1 site from the defect ring (i.e. $V_1 = 0$ and $V_N = 0$) and calculating the oscillatory eigenfrequencies of the remaining parts. In general, both the perfect reflections and the perfect transmissions excite some eigenstates of the complex defect, so they correspond to the resonant scattering.

In the following, we consider a simple example to investigate the relationship between the resonant frequencies and the number of site N, and further study how to affect the numbers of perfect reflections and perfect transmissions. For the simple case, the complex Fano defect consists of a homogeneous ring of defects, and the energy of all sites and the couplings between them are constants, i.e. $E_m = E$ and $V_m = V$. In this case, function $D(\omega)$ vanishes (i.e. $T = 0$) at

$$\omega_k = E + 2V \cos\frac{2k\pi}{N}, \quad k = 0, 1, \ldots, N-1, \quad (9)$$

function $D_2(\omega)$ vanishes at

$$\omega_m = E + 2V \cos\frac{m\pi}{N}, \quad m = 1, \ldots, N-1. \quad (10)$$

Obviously, degenerated eigenfrequencies may exist in Eq. (9), so the number of perfect reflections is less than or equal to site number N, as shown in Fig. 2. Equation (10) changes to Eq. (9) when $m = 2k$, therefore the condition of perfect transmission (i.e. $T = 1$) is modified to

$$\omega_n = E + 2V \cos\frac{(2n-1)\,\pi}{N}, \quad n = 1, \ldots, \left[\frac{N}{2}\right], \quad (11)$$

where $\left[\frac{N}{2}\right]$ represents taking the integer part. The number of perfect transmissions is $\left[\frac{N}{2}\right]$, as shown in Fig. 2. This example shows a very interesting property of the Fano resonance. When site number N is even, the spectrum of the Fano transmission is strictly symmetric, when site number N is odd, it is asymmetric. By adding or removing two sites of the additional defect ring, the transmission coefficients can be changed from one to zero for some particular frequencies. That is to say, the perfect transmissions can become the perfect reflections at some particular frequencies after adding two sites to the ring. These interesting features can deepen our fundamental understanding of the Fano resonance transport in complex systems such as quantum dots, photonic crystals and complex waveguide arrays.

Now we discuss how the presence of nonlinear defects in the discrete network may be employed to manage and tune the response of the Fano resonance. We

consider the general case of the N-defect Fano resonance and assume that one of the defects possesses a Kerr-type nonlinear response that can contribute as an additional nonlinear term into the system Hamiltonian

$$H_{\rm NL} = H_{\rm L} + \lambda|\varphi_1|^4. \tag{12}$$

Here we choose the first defect to be nonlinear due to its specific role in the transmission properties and the resonant reflections. In the presence of the nonlinear defect, the equations of motion can be easily derived to be

$$\omega\phi_n = C(\phi_{n-1} + \phi_{n+1}) + V_0\varphi_1\delta_{n0}, \tag{13}$$

$$\frac{D(\omega)}{D_2(\omega)}\varphi_1 = \lambda|\varphi_1|^2\varphi_1 + V_0\phi_0. \tag{14}$$

These equations are equivalent to the equations describing an effective single nonlinear defect. Making use of the approach developed earlier in Ref. [29], we can obtain the transmission coefficient as

$$T = \frac{x^2}{x^2+1}\,, \tag{15}$$

where x is a real solution of the cubic equation

$$(x^2+1)(x-\alpha_q) - \gamma_q = 0, \tag{16}$$

with the parameter

$$\gamma_q = \lambda c_q^3 I^2/V_0^4. \tag{17}$$

Here, I is the input intensity.

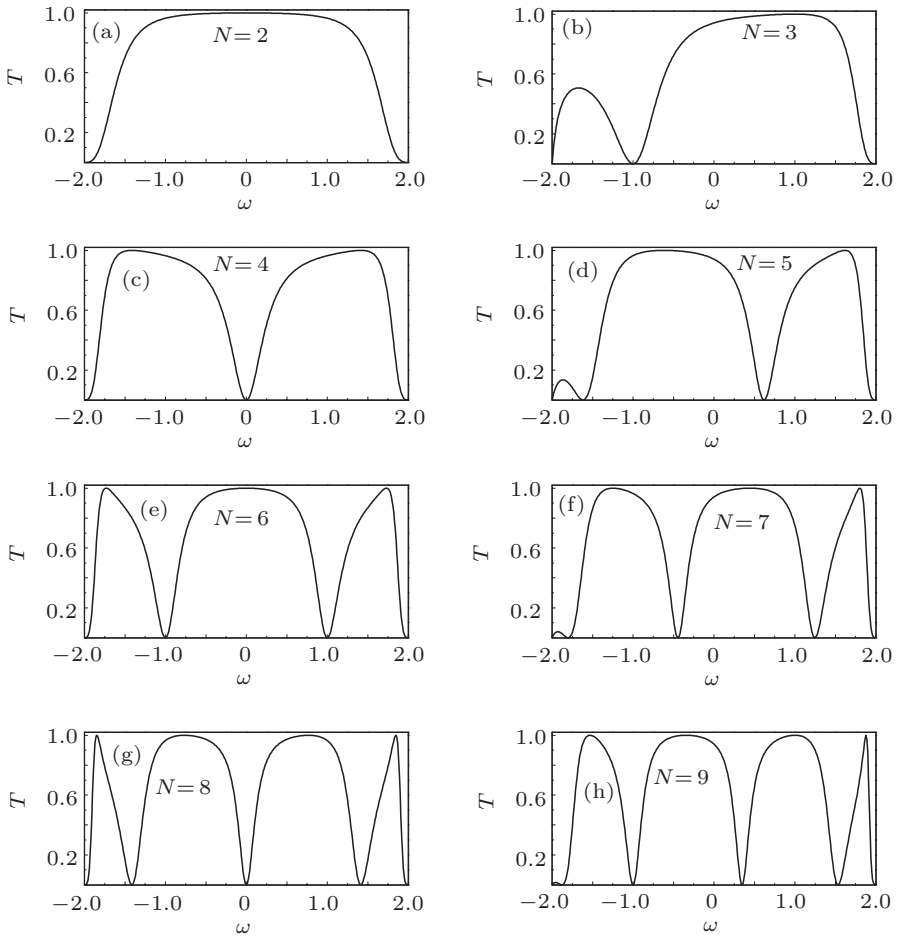


Fig. 2. Transmission T versus frequency ω for $N = 2$–9 ((a)–(h)) with the degenerated energies $E_m = E$. The parameters are $C = 1$, $V_m = 1$ and $E = 0$.

From Eq. (14), we can see that the nonlinearity shifts the frequency of the localized mode and therefore shifts the position of the Fano resonance. Figure 3 shows that the Fano blockade points (the positions of total reflection) move toward the lower frequency as the nonlinear parameter increases. Transmission (15) vanishes at $x = 0$ when $\alpha_q = -\gamma_q$. When $\gamma_q = 0$, only one real solution of Eq. (15) exists (i.e. $x = \alpha_q$), and

leads to the linear result (7) obtained above. In the nonlinear regime ($\gamma_q \neq 0$), there may exist up to three real solutions, and the output may become a multivalued function of the input, which leads to the bistability of the wave transmission[32] under the condition of $|\alpha_q| > \sqrt{3}$, as already mentioned in Ref. [29]. The scattering resonance bistable transmission can be observed under some conditions, as shown in the example presented in Fig. 3. The system allows several perfect transmissions inside the transmission spectrum. The perfect transmission $T = 1$ corresponds to the condition $x = \infty$ or $\alpha_q = \infty$ according to Eq. (15). Although the nonlinearity can shift the frequency of the localized mode, the frequencies of the resonant reflections and the resonant transmissions are shifted nonuniformly. Figure 3 shows that the perfect reflections will be changed, while the perfect transmissions will not be unchanged or will only be modified slightly.

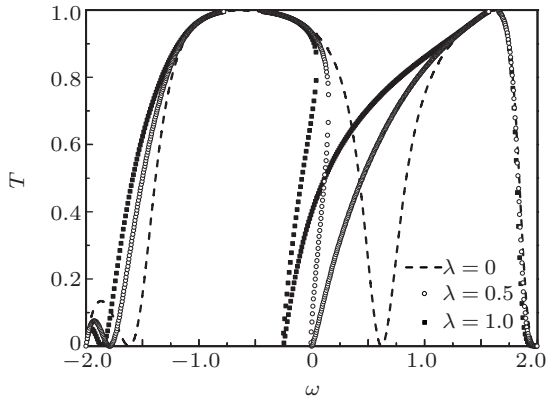


Fig. 3. Nonlinear transmission T through the $N = 5$ defect ring versus ω frequency for different nonlinear λ parameters with degenerated energies $E_m = E$. The parameters are $C = 1$, $V_m = 1$, $E = 0$ and $I = 1$.

In the nonlinear regime, the transmission coefficient depends on two parameters, wave frequency ω and input intensity I. From Eq. (17), we can see that nonlinear parameter λ has the same role as the square of the input intensity. So the transmission coefficient T versus frequency ω for changing input intensity I in Fig. 4 is similar to that for changing nonlinear parameter λ in Fig. 3. In general, the nonlinear parameter is relatively more difficult to control than the input intensity in the experiment. This feature would allow us to achieve a simple tuning of the nonlinear Fano resonance by changing the intensity of the incoming wave. In Fig. 5, we plot transmission T as a function of input intensity I for three different fixed wave ω frequencies in the present of the nonlinear defect. As ω increases, the phenomenon of the bistable transmission disappears, and the Fano blockade point moves toward the lower input intensity. For a given wave frequency ω, similar resonance transmission variation exists when the nonlinear parameter increases.

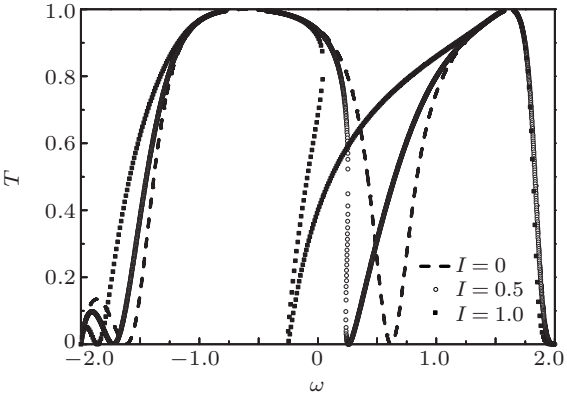


Fig. 4. Nonlinear transmission T through the $N = 5$ defect ring versus ω frequency for different input intensity I with degenerated energies $E_m = E$. The parameters are $C = 1$, $V_m = 1$, $E = 0$ and $\lambda = 1$.

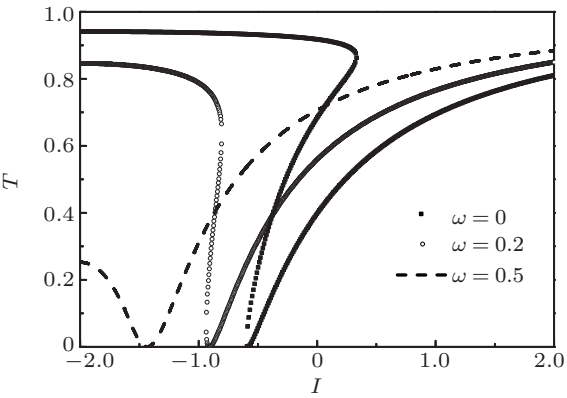


Fig. 5. Nonlinear transmission T through the $N = 5$ defect ring versus input intensity I for different ω frequencies with degenerated energies $E_m = E$. The parameters are $C = 1$, $V_m = 1$, $E = 0$ and $\lambda = 1$.

4. Conclusion

We investigated the Fano resonance transport of particles across a linear chain of particles coupled to an isolated ring composed of N defects. The analytical transmission coefficient was obtained, along with the conditions for perfect reflections and transmissions due to either destructive or constructive interference. Making use of a simple example, we investigated the relationship between the resonant frequencies and the number of site N, and studied how to affect the numbers of perfect reflections and transmissions. Furthermore, we studied a nonlinearity-tunable Fano resonance when one defect of the network possessed a nonlinear Kerr-like response. Our results may be useful for studying the Fano resonances and resonant transmissions in complex physical systems.

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